INFLUENCE OF FIBER COATING AND INTERPHASE ON THE DESIGN OF POLYMERIC COMPOSITE STRENGTH - ANALYTICAL PREDICTIONS

Engineering Science and Mechanics Department, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061-0261, USA

ABSTRACT

The paper discusses the effects of tough compliant thermoplastic fiber coatings on the composite strength under tension, compression, shear, transverse, and thermal loading. Various strength prediction models are reviewed and used to estimate the influence of the compliant fiber coating on the strength values. Parameter studies for various relative coating stiffness and thickness are performed. It is found that some models predict that a compliant fiber coating may reduce the overall static strength of the composite. However, a more detailed analysis points out that such reduction may be compensated by the more subtle improvements induced by the coating in the statistical strength of the processed fibers and in the reduction of stress concentration around the broken fiber through spreading of the effect over a larger number of adjacent fibers. Reduction in residual stresses due to thermal differences between processing and operating temperatures are also predicted.

INTRODUCTION

Modification of fiber-matrix interface/interphase through compliant fiber coating is considered a viable and economic way to achieve polymeric matrix composites with improved impact performance, fracture toughness, durability and fatigue resistance. Such materials bring superior performance and reduced cost to the coming generation airframe and aircraft engines structures. Under support from ARPA/McDonnell Douglas Aircraft Co., the Virginia Tech NSF-STC for High Performance Polymeric Adhesives and Composites has undertaken an industrial cooperative study to achieve affordable polymeric composites capable of higher impact resistance, longer durability and lower cost. One line of research is to use tough compliant fiber coatings of functionally terminated thermoplastics to induce tough behavior in carbon fiber epoxy composites and to increase their durability. The purpose of the present research is to investigate the effect that tough compliant thermoplastic coatings will have on the time-zero strength properties of the composite. The results of our studies performed with a battery of predictive models, are presented here.

Constitutive Materials Data

A carbon fiber — epoxy resin composite with 60% fiber content

Table 1 Material Properties of PANEX 33/UDEL/EPON 828 composite system

<table>
<thead>
<tr>
<th>Property</th>
<th>Fiber, PANEX 33</th>
<th>Coating, Udel</th>
<th>Matrix, EPON 828</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius, ( \mu \text{m} )</td>
<td>3.26</td>
<td>3.335 - 3.634</td>
<td>4.008</td>
</tr>
<tr>
<td>Young's Modulus, GPa</td>
<td>228</td>
<td>2.48</td>
<td>3.45</td>
</tr>
<tr>
<td>Shear modulus, GPa</td>
<td>20.0</td>
<td>0.905</td>
<td>1.287</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.20</td>
<td>0.37</td>
<td>0.34</td>
</tr>
<tr>
<td>Thermal expansion, ppm/°C</td>
<td>-0.30</td>
<td>6.0</td>
<td>48.1</td>
</tr>
<tr>
<td>Tensile strength, MPa</td>
<td>3700</td>
<td>70</td>
<td>71</td>
</tr>
<tr>
<td>Compression strength, MPa</td>
<td>2500</td>
<td>150</td>
<td>180</td>
</tr>
<tr>
<td>Shear strength, MPa</td>
<td>45</td>
<td>30</td>
<td>35</td>
</tr>
</tbody>
</table>

volume fraction is considered. The matrix was Shell’s Epon 828 epoxy resin. Texaco’s Jeffamine DU-700, and Aldrich’s m-phenylenediamine were used as curing agents. The fibers are Zoltek’s PANEX 33 carbon fibers. A compliant coating is assumed to be applied to the carbon fibers. Thermoplastic resins (e.g. Amoco’s Udel) were assumed as candidate coating materials. For parametric studies, we further assumed that the coating stiffness can be varied between 25% and 100% of the surrounding matrix stiffness. The coating thickness was allowed to vary between 1% and 50% of the half interstitial space between the fibers. The material properties of the constituents are presented in Table 1.

Geometry of the Inter-fiber Region

The geometry of the inter-fiber region consisting of fiber, coating (interphase), and matrix is shown in Figure 1. Evaluations of the available inter-fiber spacing for typical fiber volume fractions were performed using the hexagonal closed-packed array. It was found that
the inter-fiber spacing is typically very small, i.e., \( \sim 1.5 \) \( \mu m \) for PANEX 33 fibers at \( \nu_f = 60\% \) volume fraction.

**TENSION MODELS BASED ON BUNDLE STRENGTH AND INEFFECTIVE LENGTH**

Rosen\(^2\) and Tsai and Hahn\(^4\) used Coleman's\(^5\) bundle strength concept to determine the strength of a composite. The strength of the fibers is assumed to have a Weibull statistical distribution. The damage around a broken fiber does not spread further than a finite axial distance called ineffective length.

**Rosen's Analysis of Tension Strength**

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Figure 2 Rosen's concept of ineffective length, \( \delta \), around a broken fiber Rosen, 1964a, b)

Rosen\(^2\) assumes that a stress diffusion process takes place at the tip of a broken fiber(Figure 2). Shear stresses in the surrounding matrix contribute to the gradual built-up of the axial load in the broken fiber, until, some distance away from the broken tip, the far field fiber stress, \( \sigma_f^\infty \), is recovered. The ineffective length, \( \delta \), is defined as the distance from the broken fiber tip within which the axial stress built up in the broken fiber, \( \sigma(z) \), has reached a ratio, \( \phi \), of the far field value \( \sigma_f^\infty \). Rosen took \( \phi = 90\% \), but values of 95\%, 99\%, or 99.9\% can also be used.

Applying weakest link statistical theorems, Rosen\(^2\) calculated the distribution function \( \lambda(\sigma) \) of average fiber stress at composite stress \( \sigma_c \). By setting \( d\lambda(\sigma_c)/\sigma_c = 0 \), Rosen\(^2\) determined the critical value of composite stress, \( \sigma_c^* \), at which no further increase in average fiber stress is possible. This value, \( \sigma_c^* \), called "statistical strength" by Rosen\(^2\), is the tensile strength of the composite. Expressions were developed for the statistical strength, \( \sigma_c^* \), and for the ratio \( \sigma_c^*/\bar{\sigma}_L \) between the statistical strength, \( \sigma_c^* \), of the composite, and the mean strength, \( \bar{\sigma}_L \), of individual fibers of the same length \( L \).

**Fiber Strength Statistical Data**

Zhuang and Wightman\(^1\) measured the fiber tensile strength of Zoltek PANEX 33 carbon fibers at several gauge lengths and a cross head speed of 10 \( \mu m/sec \). The results of these tests are given in Table 2 as fiber tensile strengths as a function of gauge length. The parameters of the Weibull distribution corresponding to fiber strength statistical data were estimated using Maximum Likelihood Estimator and the Method of Moments. Representative values for the Weibull shape parameter, \( \alpha \), were found around \( \alpha = 5 \).

**Interface Shear Stress**

In order to analyze the interfacial adhesion, Zhuang and Wightman\(^1\) conducted single fiber fragmentation tests of PANEX 33 carbon fibers embedded in EPON 828 epoxy resin. The interface shear strength \( \tau_v \) (stress transfer coefficients) between PANEX 33 fibers and EPON 828/ DU-700 was \( \tau_v = 32.1 \) MPa. For EPON 828/mPDA the interface shear strength was 48.4 MPa.

**Table 2 Tensile Strengths of PANEX 33 at several gauge lengths**

<table>
<thead>
<tr>
<th>Gauge Length (mm)</th>
<th>1</th>
<th>11</th>
<th>20</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile Strength (GPa)</td>
<td>3.7</td>
<td>3.2</td>
<td>3.0</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>( \pm 1.0 )</td>
<td>( \pm 0.7 )</td>
<td>( \pm 1.0 )</td>
<td>( \pm 0.6 )</td>
</tr>
</tbody>
</table>

Figure 1 Geometry of a fiber/interphase/ matrix system at 60\% fiber volume fraction and 50\% interphase thickness.

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Figures 3 and 4 illustrate the composite strength and bundle strength, respectively, normalized to the mean fiber strength, versus Weibull shape parameter and normalized to the average fiber strength. The plots for both composite and bundle strength show the influence of the fiber failure, the composite strength, and the bundle strength on the overall structural integrity. The normalized composite strength, $\sigma_c/\bar{\sigma}_L$, and the normalized bundle strength, $R(\varepsilon)$, are given in the figures for various values of the relative ineffective length, $\delta L$. The implication of these plots is that, for low values of Weibull shape parameter, i.e., high dispersion in the virgin fiber strength, the strength of the composite can be significantly larger than the strength of the naked fibers of the same length. From a manufacturing point of view, this observation can have an important consequence. The mechanical behavior of virgin fibers with high strength dispersion can be significantly improved by incorporating the fibers in a matrix material that confers, to the resulting composite, a small relative ineffective length, $\delta L$.

**Tsai and Hahn’s Analysis of Tension Strength**

Tsai and Hahn further developed Rosen’s concept of statistical fiber strength. They discussed three possible situations that may arise in the composite tensile failure. In one extreme case, called the “transverse crack propagation failure mode”, the cracks created at the fiber breaks propagate through the matrix across the neighboring fibers and produce a sudden failure of the composite. This mode, associated with a brittle matrix and a strong fiber/matrix interface, that facilitates transfer of crack from matrix into fiber, is not very common with polymeric composites. In the other extreme, called the “longitudinal damage growth mode”, the matrix and/or fiber-matrix interface yield extensively along the broken fibers which become separated from the intact fibers. The composite behaves like a bundle of dry fibers, with the bundle strength:

$$X_b(L) = E_f \varepsilon_0 (L \cdot \alpha \cdot \varepsilon)^{-1/\alpha}$$

where $L$ is the normalized length of the fibers in the bundle, equal for all the fibers. The parameters $\alpha$ and $\varepsilon_0$ are parameters of Weibull's statistical distribution, $\alpha$.

which signifies the fraction of the fibers in the bundle that are still unbroken at a loading strain $\varepsilon$. Multiplying $R(\varepsilon)$ by fiber strain and fiber modulus yields the nominal stress in the fiber (i.e., the load value divided by the original cross-section):

$$\sigma(\varepsilon) = E_f \cdot \varepsilon \cdot R(\varepsilon)$$

Figure 4 Bundle strength normalized to average fiber strength versus Weibull shape parameter, $\alpha$

As strain increases, and more and more fibers break, a point is reached where no further increase in stress is possible, i.e., $d\sigma/d\varepsilon = 0$. Solving for $\varepsilon$ yields the strain at failure of the fiber bundle. Multiplying the strain at failure by the fiber modulus yields the strength of the fiber bundle,

$$X_b(L) = E_f \varepsilon_0 (L \cdot \alpha \cdot \varepsilon)^{-1/\alpha}$$

The average strength of the individual fibers in the bundle, $X_f$, is given by the average value relation of Weibull’s distribution, i.e.,

$$X_f(L) = E_f \cdot \varepsilon_0 \cdot L^{-1/\alpha} \cdot \Gamma\left(1 + \frac{1}{\alpha}\right)$$

Eliminating $E_f$, $\varepsilon_0$, and $L$ between the expressions for $X_f(L)$ and $X_b(L)$, yields the bundle strength in terms of the average fiber strength, i.e.,

$$X_b = X_f (\alpha \cdot \varepsilon)^{-1/\alpha} / \Gamma\left(1 + \frac{1}{\alpha}\right)$$

A plot of $X_b$ normalized to $X_f$, versus the Weibull shape parameter, $\alpha$, is given in Figure 4. As the dispersion of the fibers narrows, $\alpha$ increases, and the bundle strength approaches the average fiber strength. At practical
values, $5 < \alpha < 10$, the bundle strength is only 65% - 75% of the average fiber strength.

![Graph showing tensile strength of the composite versus ratio of fiber length to ineffective length for various values of the Weibull shape parameter, $\alpha$.](image)

**Figure 5** Tensile strength of the composite versus ratio of fiber length to ineffective length, for various values of the Weibull shape parameter, $\alpha$.

The third mode of failure considered by Tsai and Hahn\(^4\) falls between the transverse crack propagation mode and the longitudinal damage growth mode. Through stress diffusion at the broken fiber tip, the load in the broken fiber is recovered over a finite fiber length. Thus, the effect of a fiber break is contained within a small ineffective length region of length $\delta 2$ from the broken tip. If the fiber is broken at both ends, the effective length of the fiber is reduced by $\delta$. Outside the ineffective length, the composite is "unaware" of the fiber break. Hence, one can model this physical case as a bundle of fibers of length $\delta$ instead of $L$. Introducing the fiber volume fraction $\nu_f$, and recalling that the average fiber strength is assumed to be measured for the total length, $L$, Tsai and Hahn\(^4\) wrote the composite tension strength as

$$X = \nu_f X_f \left( \frac{1}{(\alpha \cdot e)^{1/\alpha}} \Gamma \left( \frac{L}{\delta}, 1 + \frac{1}{\alpha} \right) \right),$$

Note that $L$ and $\delta$ are independent parameters. Plots of the composite strength normalized to the weighted average fiber strength, $X/(\nu_f X_f)$, are given, for various $\alpha$, in Figure 5. Increase of the ratio between fiber length and ineffective length, $\delta$, leads to increase of composite strength. For fixed values of fiber length, this result can be achieved by decreasing the ineffective length. This effect is more pronounced for fibers with large dispersion in strength values (small $\alpha$ values). For fibers with low dispersion values (large $\alpha$ values), the variation of composite strength with ineffective length is much less pronounced.

![Plot showing elastic ineffective length versus fiber to matrix stiffness ratio, $E_f/E_m$, for three values of the stress efficiency parameter, $\phi$.](image)

**Figure 6** Plot of Rosen’s actual elastic ineffective length versus fiber to matrix stiffness ratio, $E_f/E_m$, for three values of the stress efficiency parameter, $\phi$.

**Ineffective length predictions**

The prediction of ineffective length, $\delta$, plays a crucial role in the statistical modeling of composite strength. Two situations are discussed next: Rosen’s elastic diffusion model\(^2\), and Tsai and Hahn's plastic yield model\(^4\).

**Rosen’s Ineffective Length for Perfect Elastic Behavior**

For perfect elastic behavior, Rosen\(^2\) performed a simplified analysis of the gradual built up of stresses in the broken fiber. In Cox’s classical stress diffusion model\(^6\), transfer of the load from the surrounding matrix into the broken fiber is effected by the shear stress at the fiber/matrix interface. A built-up of fiber stress develops until displacement compatibility between the fiber and the matrix is attained. The fiber stress becomes:

$$\sigma_f(z) = \left( 1 - \frac{\cosh \eta z}{\cosh \eta L} \right) \sigma_f^0,$$

where $z$ is the axial coordinate measured from the center of the fiber of length $2L$, assumed to be broken at both ends. Assuming average shear strain in the matrix region, and expressing the matrix radius, $r_m$, as function of the volume fraction, $\nu_f$, via the concentric cylinders model, Rosen\(^2,3\) expressed the stress diffusion parameter, $\eta$, in the form:

$$\eta = \frac{1}{r_f} \left( 2 \frac{C_m \nu_f^{1/2}}{E_f} \right)^{1/2}$$

where, for consistency, we have added a factor of 2 in the numerator within the parentheses. (This action is in
line with other authors, for example, Gao, and Reifsnider\textsuperscript{8}) Rosen's developed an expression for the ineffective length, $\delta$, in terms of the diffusion strength, $\eta$, and the efficiency parameter, $\phi$, of the form:

$$\delta = \frac{1}{\eta} \cdot \ln \frac{1}{1 - \phi}$$

![Graph showing interfacial shear stress vs. axial coordinate, z, micro-meters.](image)

Figure 7 Axial variation of the interfacial shear stress towards the tip of a 10 mm fiber. Note axial position at which the strength limits of AS4/EPON-828 and PANEX33/EPON-828 interfaces are exceeded ($G'_{m} = 25\% - 100\% G_{m}$)

Upon substitution of the expression for $\eta$, one gets the equivalent of Rosen's expression for the elastic ineffective length normalized with respect to the fiber diameter:

$$\frac{\delta}{d_f} = \frac{1}{2} \left[ \frac{E_f}{2G_m} \left( v_f^{-1/2} - 1 \right) \right]^{1/2} \ln \frac{1}{1 - \phi}.$$  

A plot of Rosen's elastic ineffective length, in $\mu$m, against fiber-to-matrix stiffness ratio, $E_f/G_m$, is given in Figure 7. The stiffness ratio, $E_f/G_m$, was allowed to vary in the range 200 to 1000, by decreasing the inter-fiber shear modulus from 1000% to around 20% of the original matrix modulus. The increase of Rosen's ineffective length with the compliance of the equivalent material inter-fiber is apparent. The effect is more pronounced as the stress efficiency parameter, $\phi$, is made closer to 1.

One shortcoming of perfectly elastic analysis, is that interfacial shear stress required towards the end of the broken fiber grossly exceed the available fiber/matrix interface strength. As an illustration, Figure 8 shows that, with small numerical variations, the shear stress values in the last 50 $\mu$m towards the fiber tip exceed the interface shear strength for both AS4/EPON-828 and PANEX-33/EPON-828 systems. This fact suggests that the analysis should consider the yield in the fiber/matrix interface.

**Ineffective Length for Perfectly Plastic Interface**

Tsai and Hahn\textsuperscript{4} have treated the case of a fully yielded interface (Figure 8). A constant shear stress is assumed at the fiber interface ($\tau = \tau_y$, the yield stress). The condition that the stress built-up in the fiber cannot exceed the average fiber strength is imposed. Due to the statistical nature of the fiber strength, its average strength over the small length, $x_{max}$, corresponding to stress diffusion at the fiber tip, is higher than the strength of the fibers embedded in the composite of length $L$ ($L >> x_{max}$).

![Graph showing simplified stress distribution.](image)

Figure 8 Simplified stress distribution around and inside a broken fiber. A rigid-perfectly plastic behavior is assumed at the interface (Tsai and Hahn\textsuperscript{4})

Denoting by $X_f$ the average strength of fibers of length $L$, Tsai and Hahn\textsuperscript{4} expressed the average strength of the fiber in the small tip region, $x_{max}$, as function of the fiber average strength, $X_f$, the length ratio, $x_{max}/L$, and the shape parameter of the Weibull distribution, $\alpha$.

Solving for $x_{max}$, and taking the ineffective length, $\delta_y$, as twice $x_{max}$, yields the following expression for the normalized ineffective length, $\delta_y/d_f$, in the case of rigid-perfectly plastic behavior at the interface as:

$$\frac{\delta_y}{d_f} = 2 \left( \frac{X_f}{4\tau_y} \right)^{\frac{\alpha+1}{\alpha+1}} \left( \frac{\alpha+1}{\alpha^2} \right)^{\frac{1}{\alpha+1}} \left[ \frac{L}{d_f} \right].$$

(Please note that, irrespective of elastic or plastic assumptions about material response, Tsai and Hahn's\textsuperscript{4} definition of ineffective length differs by a factor of 2 from Rosen's definition\textsuperscript{5}). For the numerical data considered in this paper, Tsai and Hahn's\textsuperscript{4}
ineffective length, $\delta_i$, vs. interface yield stress, $\tau_i$, is plotted in Figure 9. Values of Weibull parameter, $\alpha$, between 5 and 30 were considered. It can be seen that, for fibers with wide statistical dispersion ($\alpha = 5$) the perfectly plastic ineffective length is almost twice as large than for fibers with much less dispersion ($\alpha = 30$). We also notice that the actual values for the perfectly plastic ineffective length presented here are about two times greater than the values of the elastic ineffective length calculated in the previous section using Rosen’s model$^2$ with $\phi = 90\%$. This observation is consistent with the general opinion that the perfectly plastic ineffective length is larger than the elastic ineffective length. However, we note that, depending on how close to unity one takes the stress efficiency function, $\phi$, the asymptotic part of the elastic ineffective length can be made as large as desired. Hence, the direct comparison of the two types of stress variation, decaying exponential, for elastic behavior, and decreasing linear, for plastic behavior, is difficult.

**Ineffective Length for Elastic-Perfectly Plastic Behavior**

A shortcoming of Tsai and Hahn’s perfectly plastic model$^4$ is that the interface is assumed to be yielded everywhere. However, this may not necessarily be true. The distribution of elastic stresses about the tip of a broken fiber presented in Figure 8, showed that the interfacial shear stress has very large values only in a tip region of about 25% of the ineffective length. Hence, the yield of the interface is limited to this region. In real situations, a combination of elastic and plastic behavior takes place. The analysis of the ineffective length in the case of an elastic-plastic behavior at the fiber/matrix interface was mentioned, without many details, in Rosen’s pioneering work$^2$. Taking an interface yield stress equal to 1/20 of fiber strength, Rosen$^2$ derived the shear stress distribution for an elastic-plastic case, and compared it with the shear stress distribution for purely elastic case. These results are reproduced in Figure 10 from Rosen’s original work$^2$. The increase in ineffective length at the fiber tip is apparent. In our case, the ratio of interface strength to fiber strength is around 1/100, i.e., five times lower than in Rosen’s work$^2$. Therefore, it is expected that the increase in ineffective length due to elastic-plastic effects will be even larger than depicted in Figure 10.

Figure 10 Variation of interface shear stress under "elastic", and "elastic-plastic" assumptions (Rosen, 1964b)

An extensive analysis of the elastic-plastic behavior around a broken fiber tip was performed by Gao and Reifsnider$^7$. A remarkable theoretical development based on matched elastic and perfectly plastic solutions is presented, and the reader is referred to this work$^7$ for further information.

**CONTROL OF COMPOSITE BUNDLE STRENGTH THROUGH COATINGS AND INTERPHASES**

We investigated the effect of the fiber coating and interphase on bundle-strength type models by seeing how the parameters $\alpha$ and $\delta$ vary with the application of a compliant fiber coating.

The Weibull shape parameter, $\alpha$, associated with fiber strength statistics is expected to increase with the application of fiber coating. A compliant thermoplastic fiber coating will protect the fibers from surface damage during handling and, hence, will decrease the likelihood of their breaking at low stress levels due to wide spread surface flaws. A thermoplastic surface treatment of the fibers will improve their processability into 2- and 3-dimensional textiles by decreasing the surface friction coefficient and increasing their ability to withstand passage through the looms without accumulating surface damage. The coating will decrease the amount of damage induced by rubbing. The decrease in surface friction coefficient will reduce the tension in the fiber and hence its bending stress during turns in the loom. Though these arguments are more qualitative, and need to be confirmed through experimental evidence, it is clear that there may be an opportunity to increase $\alpha$ through application of compliant thermoplastic coating.
Actual values of such increase must be ascertained through carefully conducted experiments.

The ineffective length, \( \delta \), will change with fiber coating in a variety of ways which are closely inter-related. For Tsai and Hahn’s perfectly plastic model, the ineffective length depends on the interfacial yield stress, \( \tau_y \). The thermoplastic surface coating may decrease the interface yield stress if this parameter is assumed to be dependent only on the dry friction coefficient between fiber/coating/matrix. However, if functionally terminated coating materials are used, as proposed by McGrath et al., the interface yield stress may even increase through the additional chemical bonding. At this stage of research, it is not yet clear whether, on balance, the interface shear strength will decrease, increase, or stay the same after the application of fiber coating. For the present investigation, we assume, that the interface shear strength will not be modified by the application of fiber coating, and hence Tsai and Hahn’s perfectly plastic ineffective length will not be modified by the application of the type of fiber coating considered in this research.

Considering Rosen’s perfectly elastic ineffective length, we can identify trends of behavior when a compliant fiber coating is applied to the fiber surface. An important composite strength with relative inter-fiber shear modulus for perfectly elastic ineffective length is shown in Figure 11 for several values of the Weibull parameter, \( \alpha \). The composite strength at a reduced value of shear modulus was normalized with respect to its value at the current value of shear modulus for EPON 828 epoxy resin, \( G_m = 1.287 \) GPa. Examination of Figure 11, indicates that the variation of the bundle strength with the relative shear modulus is more pronounced at low values of the Weibull parameter \( \alpha \). For the current value \( \alpha = 5 \), the decrease in normalized composite strength is about 2.5% for a 25% decrease in relative shear modulus.

Figure 11 also shows the relative sensitivity of the statistical fiber strength and of the bundle-type composite strength to variations in the fiber Weibull parameter. For example, considering that an addition of fiber coating will increase the Weibull parameter by decreasing the fiber damage during processing, we see from Figure 11 that an improvement in \( \alpha \) would reduce the decrease due to shear modulus reduction. Thus, at 75% relative shear modulus, an increase of \( \alpha \) from 5 to 10 would reduce the strength loss from 2.8% to 1.4%. If the \( \alpha \) increase were from 5 to 15, the strength loss would be almost imperceptible. This predicted behavior is consistent with the observation that, in bundle-strength based models, the fiber behavior is dominant. Hence, we draw the conclusion that the bundle-type composite strength will be decreased slightly by application of compliant fiber coatings, but this decrease may be significantly compensated by the effect of improving the fiber Weibull parameter, \( \alpha \), through the protective effect the fiber coating has during fabrication and lay-up processing of the composite.

**TENSION MODELS BASED ON STRESS CONCENTRATION IN THE ADJACENT FIBERS**

Rosen’s model and Tsai and Hahn’s model do not take into account the stress concentration (over-stress) induced by a broken fiber on the neighboring fibers. It was shown by Batdorf that this effect can significantly affect the tensile strength of the composite. Gao and Reifsnyder studied Batdorf model and researched extensively the influence that modifications in stress concentration factor can have on the composite strength. In their studies, interface, matrix yielding, and irregular fiber spacing effects were considered.

**Batdorf’s model**

Batdorf methodology addresses the formation and growth of multiple fiber fractures and their relation to composite strength. Below, we present the interpretation of Batdorf’s model presented by Gao and Reifsnyder. The weakest link theory is used to determine the number of isolated fiber fracture (singlets or 1-plets), double fracture (doublets or 2-plets), and multiple fractures (multiplets or i-plets) of arbitrary order...
as a function of stress. If the fracture of individual fibers obeys Weibull distribution, a plot of $\ln(Q)$ vs. $\ln(\sigma)$ is a straight line of the slope $k$. Here $\sigma$ is the applied stress, $Q_i$ is the number of "i-plets" formed during the loading process, and $\alpha$ is the Weibull shape parameter. This plot defines a failure envelope which leads to a rational failure criterion based on a Griffith-type instability. Note, however, that Batdorf's formulation requires accurate micromechanical analysis for the ineffective length $\delta$ and the stress concentration factors, $C_i$.

Assuming that the fiber strength is characterized by Weibull's distribution, and that the total number of fibers in the composite is $N$, one expresses the number of isolated broken fibers (singlets) created by the time the stress rises to value $\sigma$ as:

$$Q_1 = NP_f(\sigma) = N\left(1 - \exp\left[-L(\sigma/\sigma_0)^\alpha\right]\right)$$

A singlet becomes a doublet when one of the neighboring fibers breaks. This break will happen due to the over-stressed induced by the broken fiber in its elastic neighborhood. The probability that a neighboring fiber will fail is:

$$P_1 = \left(1 - \exp\left[-\lambda_1(\sigma/\sigma_0)^\alpha\right]\right)$$

where $\lambda_1 = 2 \int_0^{\delta_c} \left[C_i(x)\right] \alpha \, dx$, and $C_i(x)$ is the stress concentration (of a singlet) defined as the ratio of tensile stress in the neighboring fiber to the remote value of fiber stress. If $n_1$ is the number of nearest neighbors of a singlet, then the probability that a given singlet becomes a doublet is

$$n_1\left(1 - \exp\left[-\lambda_1(\sigma/\sigma_0)^\alpha\right]\right).$$

Therefore, the number of doublets, $Q_2$, created in loading to stress $\sigma$ becomes

$$Q_2 = Q_1 n_1\left(1 - \exp\left[-\lambda_1(\sigma/\sigma_0)^\alpha\right]\right).$$

In general, if $Q_i$ is the number of i-plets (i adjacent fibers broken) at stress level $\sigma$, the following iterative relation applies:

$$Q_{i+1} = Q_i n_1\left(1 - \exp\left[-\lambda_i(\sigma/\sigma_0)^\alpha\right]\right)$$

where $\lambda_i = 2 \int_0^{\delta_c} \left[C_i(x)\right] \alpha \, dx$, and $n_1$ is the number of nearest neighbors an i-plet has. The $\sigma$ vs. $Q_i$ diagram is used to determine the strength of the composite (Figure 12). The resulting strength value depends strongly on the Weibull parameter, $\alpha$, and on the stress concentration factor, $C_i$, around the broken fibers. After a fiber breaks, its load must be redistributed to the rest of the fibers as over-stress. If, on one hand, we assume that the redistribution is uniform over all the remaining fibers, one has a global load sharing situation, and the over-stress in each fiber is usually very small.

Figure 12 Failure envelope based on the probability of fiber fracture

If, on the other hand, the strain field disturbance due to a broken fiber does not spread uniformly to all the fibers, a localization of over-stress takes place. Only a limited numbers of fibers around the broken fiber participate in sharing the over-stress, and hence their share of over-stress can be very significant. Other types of non-uniformity in stress distribution may also be present. Hence a stress-concentration effect is present. The sum of the two effects, over-stress and stress concentration, is to be found in the $C_i$ coefficients.

Over-Stress due to a Broken Fiber and a Number of Adjacent Broken Fibers

Simple geometric and equilibrium arguments can be used to assess the over-stress factor. The assumptions regarding the number of active fibers around a broken fiber, or around a core of broken fibers, strongly influence the effective stress concentration coefficient. Figure 13 shows the variation of the over-stress coefficient with the number of active rows of unbroken fibers around a number of adjacent broken fibers. It can be seen that the stress concentration decreases rapidly with the number of active rows. For four broken fibers in the bundle, the stress concentration decreases from 1.4 for only one active row, through 1.15 for two active rows, and to 1.08 for three active rows. For eight adjacent broken fibers, these values are, 1.62, 1.25, and 1.14, respectively.
Figure 13 Variation of stress concentration coefficient with the number of rows of unbroken fibers

**Stress Concentration around a Broken Fiber and a Number of Adjacent Broken Fibers**

On the other hand, the finite width of the fiber, and the fact that the over-strain/stress field is not uniform, produces a difference in over-stress between the side of the fiber that is oriented towards the broken fiber, and the side which is oriented away from the broken fiber. This effect can increase the value of the local over-stress and leads to an actual stress concentration effect. The stress concentration factor that can occur around a broken fiber has values significantly above the over-stress values given by simple statics. Experimental studies of this phenomenon by Carman, Lesko, Razvan, and Reifsnyder, were followed by elasticity analysis by Carman, Lesko, and Reifsnyder.

**Effect of Interface Yield on Stress Concentration**

An important factor in the reduction of stress concentration in the fibers around a core of broken fibers is a partially-yielded fiber matrix interface. In this situation, the stress increase in a broken fiber is limited by the yield stress in the interface. Thus, the stress diffusion process along the fiber is made up of two distinct mechanisms: a constant-shear diffusion process (fiber sliding) over the initial portion of the fiber in which the interface has yielded; and a decreasing-shear process for the remaining portion of the fiber, once the shear stress has decreased below the yield value of the fiber/matrix interface, or of the coating. This diffusion was analyzed by Gao and Reifsnyder for a single row of active fibers around the broken fibers core.

The existence of two mechanisms in the diffusion process leads to the fact that the characteristic diffusion length of the purely elastic analysis, \( 1/\eta \), is no longer directly related to the ineffective length, \( \delta \), of the broken fibers. Gao and Reifsnyder studied this situation, and found that the stress concentration parameter decreases rapidly as the product \( \eta \delta \) increases (Figure 14). This argument also points to the possibility of controlling the stress concentration factor around a broken fiber through the properties of the fiber coating. A fiber coating with a lower yield stress will induce a higher ineffective length and hence will lead to a lower stress concentration.

Figure 14 Stress concentration factor as function of the ratio between the elasto-plastic ineffective length and the elastic stress diffusion characteristic length (Gao & Reifsnyder).  

**Reifsnyder's Model for Tension Strength**

A closed-form approximation to Batdorf's model for composite strength under tension loading was developed by Reifsnyder. It was shown in Figure 14 that the stress concentration (over-stress) factor in the unbroken fibers decays rapidly with increasing ineffective length, \( \delta \). By taking a normalized ineffective length, \( \vec{\delta} \), and assuming \( C = a / \vec{\delta}^2 \), Reifsnyder developed the following closed form expression for strength:

\[
\sigma_c = \sigma_0 \left[ a^m (m+1)^{-1} \left( \frac{1-2m}{\vec{\delta}^2} + \frac{\vec{\delta}}{a^m} \right) \right]
\]

Three parameters were identified, the ineffective length, the stress concentration, and the Weibull parameter. One of the parameters was eliminated by finding its optimum values for peak strength as function of the other two parameters. Hence a surface plot of the optimum values for different values of the other two parameters was generated (Figure 15). The special geometry of the optimum surface is remarkable, and can be invaluable for interphase design optimization.

**CONTROL OF COMPOSITE BATDORF'S STRENGTH THROUGH COATINGS AND INTERPHASES**

**Reduction of Stress Concentration Coefficients through Compliant Coating**

We make the general claim that a compliant coating, having a lower stress-diffusion parameter than the initial matrix, will lead to a reduction in the stress...
concentration coefficients due to one or several broken fibers. The number of concentric rows of unbroken fibers activated in sharing the local over-stress due to a core of broken fibers will be higher for a compliant inter-fiber medium. We base our argument on simple physical observations. The physical phenomenon that must be considered is that of stress diffusion from the broken fiber into the neighboring unbroken fibers. The diffusion takes place simultaneously in radial and axial directions. At a given axial position, \( z \), two situations may be encountered in the radial direction. In the first situation, if the diffusion medium is stiff, i.e. a high \( G_m/E_f \) ratio, the stress diffusion process is intense, and the characteristic diffusion length is short. In this case, the load sharing is spread to the next neighboring row of fibers only, and the stress concentration is expected to be high. In extreme, one attains a very stiff diffusion medium, such as ceramics, in which the stress concentration is so high that it leads to rapid crack propagation in the matrix and in the next nearest fibers.

**The Influence of Stress Concentration Coefficients on Tensile Strength**

For a compliant fiber coating/interphase, the Batdorf's strength will be modified through modification of the stress concentration coefficients and of the ineffective length. Our studies show that a compliant inter-fiber region is likely to improve the composite strength through reduction of the stress concentration coefficients around the broken fibers. To illustrate this, we varied the number of concentric fiber rows participating in the load sharing process (active rows).

The assumption that the number of active rows participating in the load sharing process can be controlled through modifications in the fiber coating and the interphase is implied. The over-stress coefficients, depicted by the three curves of Figure 13, were used sequentially in Batdorf's model. Strength predictions were performed for various coating/matrix compliance combinations resulting in effective modulus of the inter-fiber material equal to 25%, 50%, and 75% of the original matrix modulus. The Weibull parameter was kept constant, i.e. the beneficial effect of coating on the Weibull parameter was not included. To facilitate the parametric study, the effect of coating/interphase compliance on the ineffective length was considered decoupled from the effect of engaging a larger number of unbroken fiber rows into the load sharing process. For each coating/matrix compliance combination, three different values of Batdorf's strength were calculated and plotted (Figure 16).

![Figure 15 Surface plot of the optimum ineffective length vs. stress concentration parameter and Weibull shape parameters](image)

![Figure 16 The tensile strength by Batdorf’s formula with the number of active rows of unbroken fibers for various shear moduli](image)

On the other hand, if the diffusion region is compliant, i.e. the inter-fiber region has a low \( G_m/E_f \) ratio, the diffusion will take place over a long radial distance, and hence a larger number of concentric rows will be activated in the load sharing process. In this case, the stress concentration factor in the neighboring unbroken fibers will be reduced. In extreme, one gets the case of global load sharing. Let's consider a very compliant stress-diffusion medium, such as, for example, fiber reinforced elastomers. When some fibers break, all the remaining unbroken fibers participate in the load sharing process, and the stress concentration factor approaches unity. Real life situations will be between these two extremes. We claim that this phenomenon can be controlled through fiber coating/interphase manipulation. When the compliance of the inter-fiber region (coating/interphase) is increased, the stress
load-sharing rows due to the associated reduction in the stress concentration coefficient. However, this effect may be partially offset by the reduction in bundle strength due to the increased ineffective length associated with increased diffusion length. Further studies are required to correlate directly the value of the coating to matrix stiffness ratio with the corresponding reduction in the stress concentration coefficients and thus identify the optimum stiffness ratio that is likely to give maximum tensile strength performance.

**COMPRESSION STRENGTH OF COATED-FIBER COMPOSITES**

The number of predictive models for compression strength is even larger than for tension strength. Pioneering concepts laid down by Rosen, Tsai and Hahn, have recently been extended by Fleck and Budiansky, and by Xu and Reifsnyder.

![Graph](image)

Figure 16: Comparative plot of compressive strength as predicted by several models

Figure 17 presents a comparison of compression strength predictions through these models, as well as through the law of mixtures. It can be seen that a wide variation in prediction is achieved as the control parameters of each model are varied. This casts additional light on the effectiveness of interphase control. Various compression failure mechanisms, and the associate predictive models are discussed next.

**INFLUENCE OF COATING AND INTERPHASE COMPLIANCE ON COMPRESSION STRENGTH**

**Rosen's Compression Strength Models based on Fiber Microbuckling**

Microbuckling of aligned fiber composites under axial compression is a failure mechanism wherein the fibers undergo cooperative kinking in a narrow band. Observations show typical values for band width, w, in the neighborhood of 10 fiber diameters, and for the inclination of the kink band in the range $\beta \approx 10^\circ - 30^\circ$. On the assumption that the kinking is an elastic bifurcation-buckling phenomenon, and that $\beta = 0$, Rosen derived the widely-quoted formula:

$$\sigma_c = G_m / (1 - v_f)$$

for the shear mode kinking stress, where $G_m$ is the shear modulus of the matrix, and $v_f$ is the fiber volume fraction. It is apparent that the shear-mode compressive strength decreases as the effective stiffness of the inter-fiber region decreases. Rosen extended this model to calculate the compressive strength of a lamina in terms of micro-buckling half-wave length, $L$, amplitude of fiber deflection, $f_0$, fiber thickness, $h$, and longitudinal shear strength of composite, $\tau_c$, in the form:

$$\sigma = \frac{G_l}{1 + \pi \left( \frac{f_0}{L} \frac{G_l}{\tau_c} \right)}$$

where

$$G_l = E_f V_f \left( \frac{h}{L} \right)^2 \left[ 1 + \left( \frac{h}{L} \right)^2 \left( \frac{E_f}{G_f} \right) \frac{1}{4} \right]$$

Rosen also developed a compression strength formula for the extension mode as:

$$\sigma_c = 2v_f \left[ v_f E_m E_f / 3(1 - v_f) \right]^{1/2}$$

**Fleck and Budiansky's Compression Strength Models based on Fiber Microbuckling and Plastic Kinking**

Fleck and Budiansky developed analytical formulae for compressive failure of polymer matrix composites by considering plastic kinking. The kinking mechanism in polymer-matrix and carbon-matrix fiber composites is dominated by fiber misalignment together with plastic shear deformation in the matrix. Fleck and Budiansky considered strain-hardening effects, kink inclination, and applied shear stress, and developed the following formula for composite compressive strength:

$$\sigma_c = G / \left\{ 1 + n \left[ \frac{1}{\gamma_c (n-1)} \right]^{1/n} \right\}^{1/2}$$

where $\phi$ is the initial misalignment, $\gamma_c$ is the elastic shear strain of the composites, and $n$ is the parameter of Ramberg-Osgood representation of matrix non-linear stress-strain law. In Figure 19, the strengths are normalized by the strength from Rosen's formula for shear mode. It is shown that the compressive strength is sensitive to the initial misalignment, and the
sensitivity becomes larger when the coating stiffness increases. The kinking strengths are known to be sensitive to the initial mis-alignment of the fibers. It is common knowledge that, for mis-aligned fibers, kinking stresses are typically only 25% of the microbuckling stresses of composites with perfectly aligned straight, fibers.

![Graph showing normalized strength vs. relative shear stiffness for three values of the relative initial misalignment](image)

Figure 18: Compressive strength prediction vs. relative shear stiffness for three values of the relative initial misalignment, using Fleck and Budiansky’s model\(^\text{14}\).

**Xu and Reifsnider model**

In the Xu and Reifsnider\(^\text{15}\) model, composite compressive failure is studied using a micromechanical model, and the stiffness of the foundation is determined through an elasticity solution of the foundation model problem. An explicit expression has been derived for evaluation of composite longitudinal compressive strength (CLCS) based on the analysis of microbuckling of a representative volume element (RVE) using a beam-on-elastic-foundation model. The effect of fiber slippage is introduced. The fiber buckling wavelength can be determined after the stiffness of the foundation is obtained. It is found that fiber slippage, which is directly related to the interfacial shear strength, lowers the CLCS considerably. The compressive strength of the composite in which fibers have partially slipped can be estimated by

\[
\sigma_{\text{comp}} = \left( \frac{E_f V_f + E_m (1 - V_f)}{E_f} \right) \times \left[ \frac{E_f \frac{r_f^2 \pi^3}{12 L^2} + G_m + \frac{k L^2}{r_f^2 \pi^3} \frac{2 G_m s}{L} - \frac{G_m \sin \frac{2 \pi s}{L}}{2 \pi}}{L} \right]
\]

where \(L\) is the wavelength of the buckled fiber, \(k\) is the normalized stiffness of the foundation, \(s\) is the slippage parameter, \(E_f\) and \(E_m\) are Young’s moduli of fiber and matrix, respectively, and \(G_m\) is the shear modulus of the matrix. The slippage parameter, \(s\), will be zero if the fibers are well bonded to the matrix and there is no slippage at the fiber/matrix interface.

Figure 19 shows predictions made with Xu and Reifsnider (1993) model for a range of relative shear modulus of the equivalent coating/interphase/matrix region. Zero relative slippage between fiber and the surrounding material was assumed. It can be seen that the decrease in shear modulus produces a decrease in the composite compressive strength. Hence, according to this micromechanical model, the addition of compliant fiber coating might not be beneficial to the compressive strength of the composite. On the other hand, the parameter \(r_0\), which represents the ratio between the fiber radius, \(r\), and the characteristic buckling wave length, \(L\), plays a paramount role in this model. Modifications in this parameter can modify significantly the values of the normalized compressive strength. The correlation between the fiber/matrix interphase and the \(r_0\) parameter are not yet clearly established.

![Graph showing variation of Xu and Reifsnider composite compressive strength with relative shear stiffness for three values of the \(r_0\) parameter (\(r_0 = r/L\)].](image)

Figure 21 shows the relationship between the compressive strength and the wavelength parameter, \(r_0\), for several different values of relative stiffness. Interestingly, \(r_0\) has a critical value, \(r_0^{*}\), which minimizes the compressive strength. Since the radius of the fiber is assumed fixed, wavelength decreases as \(r_0\) increases, and hence the right hand arm of the curve seems physically correct. The left hand arm of the curve presented in Figure 20 predicts increase in strength with increasing wavelength; this behavior is not clear at this stage, and will not be used. Hence, for further studies, the parameter \(r_0\) will be considered larger than its critical value, \(r_0^{*}\). Figure 21 shows the
variation of the critical wavelength parameter, $r_0^*$, with the relative shear stiffness. Its value becomes larger with a stiffer coating.

\[ v_0 = f_0 \left[ 1 + \cos \left( \frac{\pi}{l} \right) \right] \]

where $l$ is the half wavelength. If a compressive load is applied, the final deflection of fibers becomes

\[ v = f \left[ 1 + \cos \left( \frac{\pi}{l} \right) \right] \]

where $f$ is the amplitude of the deflection for a given applied load. If we take an infinitesimal representative volume element of length $\Delta x$ and cross-sectional area $A$ at distance $x$, the balance of moments requires that

\[ \frac{dM}{dx} = V + \sigma A \frac{dV}{dx} = 0 \]

where $M$ is the bending moment, $V$ is the shear, and $\sigma$ is the applied compressive stress. Assuming that the bending moment is supported by the structural fibers alone, while the shear is representative of the composite element, yields moment and shear as:

\[ M = E_f I_f \frac{d^2}{dx^2} (v - v_0), \quad V = AG_c \frac{d}{dx} (v - v_0) \]

where $G_c$ is the in-plane shear modulus of the lamina, and $E_f$ and $I_f$ are the fiber stiffness and area moment of inertia, respectively.

\[ \sigma_c = \left[ G_x + E_f \frac{I_f}{A} \left( \frac{\pi}{l} \right)^2 \left( 1 - \frac{f_0}{f} \right) \right] \]

Solving for the applied compressive stress, $\sigma_c$, to the composite

\[ X_c = G_x \left( 1 - \frac{f_0}{f} \right) \]

We realize that the most probable cause for composite failure is due to local shear failure of the matrix leading to kink band formulation. Applying the boundary conditions, Carman Eskandari, and Case obtain the expression for the compression strength of the composite as:

\[ X_c = G_x \left[ 1 + \frac{f_0 \pi G_m K}{l S_m} \right], \quad S_m = G_m \frac{\pi}{l} (f - f_0), \]

where $K$ represents the stress concentration due to geometry and constituent properties. Defining the sensitivity parameter $\alpha = \frac{f_0 G_m}{l S_m}$, we plotted in Figure 23 the relationship between the normalized compression strength versus the relative shear stiffness for various values of the sensitivity parameter.
FIBER COATING EFFECT ON COMPOSITE TRANSVERSE STRENGTH

The common perception for a continuous-fiber composite is that its transverse strength is dominated by the matrix behavior. For example, a Halpin-Tsai type equation for transverse strength can be written in the form:

\[ Y = \left[1 + \nu_f \left(\frac{1}{\eta} - 1\right)\right] X_m, \]

where \( X_m \) is the strength of the isotropic matrix, \( \nu_f \) is the fiber volume fraction, and \( \eta \) is an engineering correction factor to be determined experimentally. The mechanism of composite failure under transverse loading is generally regarded as rapid propagation of matrix fracture after cracks are initiated at point of maximum stress concentration. However, more experimental evidence with polymeric matrix composites points out that matrix micro-cracking around the fibers at points of maximum stress concentration can exist without catastrophic crack propagation to occur. In this situation, cracks can propagate under fatigue and/or environmental loading, and the process becomes more complex.

The stress concentration at the fiber-matrix interface depends on several factors, among which the local stiffness ratio between the fiber and the matrix plays a major role. Carman, Averill, Reifsnider and Reddy\textsuperscript{17} offered a comprehensive explanation of this phenomenon. A concentric cylinder model with an interphase region between the fiber and the matrix is used, and, for the sake of argument, the inner fiber and interphase cylinders are replaced by an equivalent with effective properties. Then, the principal stress in the matrix is examined as the ratio of the effective modulus of the inner equivalent cylinder over the modulus of fiber is varied. In one extreme, if the local modulus of the inner fiber + interphase cylinder is negligible — i.e. if the coating/interphase stiffness is very low compared to the matrix stiffness, — the maximum stress occurs at an angle \( \theta = 90^\circ \), and corresponds to \( \sigma_{\theta}\). This situation is analogous to that of a hole in a 2-D plate.

Figure 23 Variation of maximum principal matrix stress for a Carbon fiber/ Epon 828 composite with various ratios of interphase to matrix stiffness ratio, \( E/E_m \) (Carman, Averill, Reifsnider and Reddy\textsuperscript{17} )

On the other hand, if the local modulus of the inner fiber + interphase cylinder is much stiffer than the modulus of the matrix, the maximum stress occurs at \( \theta = 0^\circ \), and corresponds to \( \sigma_\theta \). This situation is analogous to that of a perfectly bonded rigid inclusion. Since the two extreme situations have different maximum stress values and position, it is intuitively expected that a situation may exist in between when cancellation of competing effects will lead to a lower overall stress value. In a detailed analysis, Carman, Averill, Reifsnider and Reddy\textsuperscript{17} showed that (Figure 24) that variation of the interphase to matrix stiffness ratio from \( E/E_m = 1 \) to \( E/E_m = 0.01 \) can change the situation radically, from the “rigid inclusion” case to the “hole in a flat plate” case. In their studies, Carman, Averill, Reifsnider and Reddy\textsuperscript{17} found that a value \( E/E_m = 0.1 \) is optimum for reducing the stress concentration (see Figure 24). As an added feature of this optimization process, the \( \theta \)-variation of the principal stress for the resulting optimized interphase stiffness is almost flat.

Experimental studies reported by Case and Carman\textsuperscript{18} and by Case, Carman, and Reifsnider\textsuperscript{19} on a scaled up glass/epoxy model composite of 36% fiber volume fraction and on actual Carbon fiber/Epon 828 composites with 60% fiber volume fraction have correlated favorably with the above analysis.

FIBER COATING AND INTERPHASE EFFECTS ON COMPOSITE SHEAR STRENGTH

Carman, Eskandari and Case\textsuperscript{16} have recently evaluated the effect of fiber coatings on the stress distribution in a composite subjected to anti-plane shear. They utilize a
Concentric Cylinder Model (C.C.M.) to demonstrate that a distinct fiber coating exists which minimizes the stress. Optimization criteria are used to minimize the stress concentrations in the matrix with appropriate interphase coatings. The effective shear modulus for the composite is calculated by relating the volume averaged stresses in the element to the applied strain depicted by the following equation:

\[ C_{55}^c = \frac{1}{\nu_c} \sum_n \int \left( \sigma_{rz}^n \cos \theta - \sigma_{\theta z}^n \right) dV_n \]

where \( \sigma_{rz} \) and \( \sigma_{\theta z} \) are the non-zero stresses under anti-plane shear load, and \( n \) represents the phase type including fiber, coating, and matrix. \( C_{55}^c \) is the 55 element of the composite stiffness matrix. The optimum interphase shear modulus that minimizes the stress concentrations is determined by the following closed-form relationship:

\[ 0 = C_{55}^f C_{55}^m - \left( C_{55}^f \right)^2 - \left( \frac{C_{55}^m + C_{55}^f}{r_1^2 / r_2^2} \left( \frac{r_1^2}{r_2^2} - 1 \right) \right) C_{55}^f \]

where \( C_{55}^f, C_{55}^m, C_{55}^c \) are the shear moduli of fiber, interphase coating, and matrix, respectively. By using above equations and iteration technique, we can obtain the optimum interphase shear modulus, and furthermore the effective shear modulus of the composite when the optimum condition is fulfilled.

Figure 24 shows the normalized optimum interphase shear modulus as a function of the relative coating thickness. The normalization was performed with respect to the values obtained at 10% relative coating thickness. At 10% coating, the optimum interphase shear modulus for our numerical example has the value 39.5 MPa. Figure 24 shows that the thicker the coating, the larger the optimum modulus becomes. Physically, for thicker coating, we need stiffer coatings to minimize the stress concentration. We also studied the variation of the normalized effective shear modulus of an optimum composite versus relative coating thickness. For our material system, and 10% coating, the effective shear modulus of the optimum composite has the value 493.3 MPa. This value decreases with coating thickness, but by an insignificant amount.

**Fiber Coating and Interphase Effects on the Residual Thermal Stresses**

Due to different thermal expansion coefficients, residual thermal stresses at the fiber/matrix interface are high and can lead to early development of matrix cracks. By applying a tough compliant coating on the fibers, the residual stresses in the matrix can be reduced. We used the NDSANDS (N Directional Stiffness A N D Strength) computer code of Pagano and Tandon\(^{19}\) to obtain elasticity solutions of the concentric cylinders model comprising the fiber, coating, and matrix under a temperature load \( \Delta T = -200^\circ \text{C} \), typical of hot-cure epoxy technology. The coating thickness is assumed to vary from 1% to 10% of the half inter-fiber distance. Figure 25 shows the variation of the circumferential matrix stress, \( \sigma_{\theta z} \) at the interface against the relative coating thickness. The stress is normalized by its no-coating value. Decrease of the circumferential matrix stress with the increase in coating thickness is observed.

![Figure 24 Normalized optimum interphase shear modulus vs. the relative coating thickness using Carman, Eskandari and Case model](image)

Further studies of the coating effect on the thermal stresses were performed using a thick wall concentric cylinders (TWCC) model representing the fiber, coating and matrix. The radial displacement is expressed in terms of the inner and outer pressures, \( p_1 \) and \( p_2 \), inner and outer radii of the cylinder, \( r_1 \) and \( r_2 \), and material properties such as Young's modulus and Poisson's ratio:

\[
u(r) = \frac{r}{E \left( r_2^2 - r_1^2 \right)} \times \left[ \left( 1 - \nu \right) \left( p_1^2 - p_2^2 \right) + \left( 1 + \nu \right) r_1^2 p_2^2 - \left( r_1 - p_2 \right) \right] \]

Displacement continuity boundary conditions are imposed at the fiber/coating and coating/matrix interfaces. The results obtained with this simpler model, which is much easier to run for parameter studies, are also plotted in Figure 25. We conclude that, according to these results, a thicker and stiffer coating will result in lower stress concentration at the coating/matrix interface.
CONCLUSIONS

1. The fiber coating thickness and properties, and the related fiber-matrix interface/interphase, were found to play a significant role in the predicted tension, compression, transverse, and shear strengths of polymeric matrix fiber composites.

2. The fiber coating can be used as a modifier of the composite properties, and desirable results, such as ductility, fracture toughness, and reduced notch sensitivity, can be achieved with a suitable blend of coating properties.

3. Optimal values for the fiber coating and interphase properties were identified. These values do not necessarily agree with the "rule-of-thumb", thus highlighting the subtle role of the interphase in the final composite strength.

4. Compliant fiber coating and interphase is recommended because it contributes to the spread of load due to a broken fibers over a larger region and hence reduces the stress concentration coefficients in the next neighboring fibers.

5. Again, compliant fiber coatings and interphase regions are recommended because they reduce the residual matrix stresses due to curing thermal cycles, and lead to reduction in matrix cracking, and to improvements in fatigue life under severe environmental conditions.

6. The possibility arises, though it needs further research, to create toughened polymeric matrix composites through interphase control alone, thus creating an economical avenue for greatly improving the processability and productivity of toughened composite manufacturing.

7. New opportunities were identified for strength and durability control, such as controlling Weibull shape parameter of fiber strength statistics by affecting manufacturing changes, by coating the fiber with thermoplastic polymers at the production plant before any damage can be produced through, for example, handling of the fibers in subsequent processing.

However, it was found that an acute need still exists for analytical models that can properly predict the mechanical and chemical properties of the coating and interphase, and for supporting experimental results. Identification of the coating/interphase combination that will produce optimal thermo-mechanical behavior of the composite under static, dynamic and fatigue loading remains a focus area for further research.

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