Efficient Use of Induced Strain Actuators in Aeroelastic Active Control

Victor Giurgiutiu*, Zaffir Chaudhry**, Craig A. Rogers***
Virginia Polytechnic Institute and State University,
Blacksburg, VA 24061-0261, USA

Efficient Static Design

Consider a stacked actuator of nominal (free-stroke) displacement \( u_{ISA} \), and internal stiffness \( k_i \). During static operation, the total induced energy, \( E_{ISA} \), gets divided between the internal and external subsystems: part of it, \( E_e \), is transmitted to the external application, while the rest, \( E_i \), remains stored in the internal compressibility of the stack. The value of \( E_{ISA} \), and its repartition between \( E_e \) and \( E_i \), depends on the stiffness ratio \( r = k_e / k_i \). It can be easily shown (Giurgiutiu, Chaudhry, and Rogers, 1993) that

\[
E_{ISA}(r) = \frac{r}{1+r} E_{ref}, \quad E_e(r) = \frac{r^2}{(1+r)^2} E_{ref}, \quad E_i(r) = \frac{r^2}{(1+r)^2} E_{ref}, \quad E_{ref} = \frac{1}{2} k_i u_{ISA}^2
\] (1)

Dividing through by \( E_{ref} \) yields the non-dimensional energy coefficients \( E_{ISA}'(r), E_e'(r), E_i'(r) \). Figure 1a (Giurgiutiu, Chaudhry, and Rogers, 1994a) shows their variation with the stiffness ratio, \( r \). It can be seen that the externally delivered energy, \( E_e' \), reaches a maximum at \( r = 1 \). Figure 1b shows the variation of the energy transmission efficiency \( \eta(r) \) with the stiffness ratio \( r \). At the stiffness-mismatch point, the energy transmission efficiency is only 50%. Figure 1b also illustrates another important concept, viz. that of conjugate stiffness ratios. Consider \( r_1 = 1/4 \) and \( r_2 = 4 \). The corresponding external energy delivery is, in both cases, the same: \( E(r_1) = E(r_2) = 0.16 E_{ref} \). But the energy transmission efficiency is widely different, viz. \( \eta(r_1) = 80\% \) while \( \eta(r_2) = 20\% \). The soft design (\( r_1 = 1/4 \)) is 4 times more efficient than the stiff design (\( r_2 = 4 \)). Under static conditions, the best use of an actuator is made when the internal and external stiffness are matched. The maximum attainable energy delivery is \( (E_e)_{max} = \frac{1}{4} E_{ref} \). If stiffness match cannot be ensured, a “soft” design (\( r < 1 \)), will always be more efficient than a “stiff” design (\( r > 1 \)).

Efficient Dynamic Design

Figure 2 presents an ISA stack coupled with a external dynamic system consisting of mass, spring and damper. In aerodynamic control, the mass, spring and damper values vary with the operating frequency, and also with airspeed and length scale. For the present exploratory study, constant mass, spring and damper values are considered. Hence, the equivalent dynamic stiffness can be written as: \( k_e(\omega) = (-\omega^2 m_e + i\omega c_e + k_e) \). Assuming the natural frequency of the external system as \( \omega_0 \), one expresses the complex dynamic stiffness as:

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*Visiting Professor, Engineering Science and Mechanics Department, Member AIAA, AHS, RAeS
**Research Scientist, Center for Intelligent Material Systems and Structures Member AIAA, ASME
***Professor, Director of the Center for Intelligent Material Systems and Structures Member AIAA, ASME
\[ \bar{k}_e(\omega) = \left(1 - \frac{\omega^2}{\omega_0^2}\right) + i \frac{2\zeta}{\omega_0} \omega p \right) + i \frac{2\zeta}{\omega_0} \right) k_e = \left(1 - p^2 \right) + i \frac{2\zeta}{\omega_0} \right) k_e, \tag{2} \]

where \( p = \frac{\omega}{\omega_0} \) is the frequency ratio, and \( \zeta \) is the internal damping of the external system. The real and imaginary parts of the complex dynamic stiffness signify in-phase and out-of-phase force reactions. At low frequency (\( p \to 0 \)), the inertial and damping terms in the dynamic stiffness expression vanish, and the static stiffness \( k_e \) is predominant. This is the quasi-static dynamic operation. At higher frequencies, but still below external resonance (\( 0 < p < 1 \)), the effective stiffness is reduced by \( (1 - p^2) \), while the dissipative (out-of-phase) component grows as \( 2\zeta p \). As the mechanical resonance is approached (\( p \to 1 \)), the reactive inertial forces balance the reactive elastic forces, and hence the real part of the dynamic stiffness vanishes. At resonance (\( p = 1 \)), only the imaginary part of the dynamic stiffness is non-zero i.e., dissipative forces are predominant. Above resonance (\( p > 1 \)), the inertial forces dominate, and stiffness magnitude increases parabolically. A phase shift of 90° is recorded at resonance, and an overall phase shift of 180° takes place as the operating point passes from sub-resonance to post-resonance. Using the complex dynamic stiffness, Giurgiutiu, Chaudhry, and Rogers, 1994b defined the dynamic stiffness ratio, \( \overline{r}(\omega) = \frac{k_e(\omega)}{\bar{k}_e}, \quad \bar{k}_e = k_e(1+\eta) \). The dynamic stiffness ratio is a frequency dependent complex quantity.

The ISA is driven by alternating voltage, \( V(t) \), and current, \( I(t) \), which induce an alternatively varying strain. The resulting dynamic displacement, \( u(x, t; \omega) \) varies with time and position. Neglecting wave propagation effects inside the stack yields a linear variation of the displacement along the stack length. Displacement compatibility and force balance between the stack and the external mechanical impedance are imposed at \( x = l \). The dynamic displacement expression takes the form (Giurgiutiu, Chaudhry, and Rogers, 1994b):

\[ \hat{u}(x,t;\omega) = \frac{l}{1 + \overline{r}(\omega)} \frac{x}{l} \hat{u}_{ISA} \sin \omega t. \tag{3} \]

where \( \hat{u} \) signifies motion amplitude, and \( \hat{u}_{ISA} \) is the free-stroke amplitude. At the interface between the internal and external systems, we obtain the external displacement:

\[ u_e(t;\omega) = \frac{l}{1 + \overline{r}(\omega)} \hat{u}_{ISA} \sin \omega t, \quad \overline{u}_e = \frac{l}{1 + \overline{r}(\omega)} \hat{u}_{ISA}. \tag{4} \]

where \( \overline{u} \) signifies complex motion amplitude. Using the dynamic stiffness concept we extend the stiffness match concept from static analysis into dynamic analysis. The dynamic stiffness concept allows direct analytical continuation between the static and dynamic regimes. To get an incremental understanding of the salient points, three situations will be progressively considered: (a) the quasi-static dynamic operation; (b) the undamped dynamic operation; and (c) the damped dynamic operation.

**Quasi-static Dynamic Operation**

Under quasi-static dynamic operation, damping and inertial effects are neglected (\( \bar{k}_e(\omega) = k_e \)). Hence, no phase shift occurs, and all displacement and force amplitudes are real:

\[ u_{ISA}(t) = \hat{u}_{ISA} \sin \omega t, \quad u_e(t) = \hat{u}_e \sin \omega t, \quad u_i(t) = \hat{u}_i \sin \omega t, \quad F(t) = k_e \hat{u}_e(t) \tag{3} \]

The energy principles developed for static operation are directly translated into power principles. Use instantaneous power expression \( P(t) = F(t) \cdot \dot{u}(t) = \hat{P} \sin 2\omega t, \quad \hat{P} = \frac{1}{2} \hat{F} \cdot \hat{u}, \) to get:
\[ \hat{P}_{ISA}(r) = \frac{r}{1+r} P_{ref}, \quad \hat{P}_e(r) = \frac{r}{(1+r)^2} P_{ref}, \quad \hat{P}_i(r) = \frac{r^2}{(1+r)^2} P_{ref}, \quad P_{ref} = \omega E_{ref} = \omega \frac{1}{2} k_i \mu_{ISA}^2 \]  

(6)

A plot of these expressions vs stiffness ratio, \( r \), is given in Figure 3. The plot closely resembles that of Figure 1, only that instead of energy coefficients we have power coefficients. For quasi-static dynamic operation, the maximum power delivery from an ISA device is achieved when the internal and external stiffness are matched. Under quasi-static dynamic operation, the static stiffness match principle still applies.

**Undamped Dynamic Operation**

Many dynamic utilizations of ISA technology do not take place under quasi-static conditions. For mechanical resonances in the 10 to 50 Hz frequency range, inertial forces cannot be neglected. As the mechanical resonance of the external system is approached, the reactive inertial forces are subtracted from the reactive elastic forces, and the effective stiffness ratio is modified. A system with statically matched stiffness will not retain its optimal condition under full dynamic operation. This is illustrated in Figure 4. For a statically matched system, the external power coefficient at \( p = 0 \) (quasi-static operation) is optimum, i.e. it has the maximum possible value, 0.25. As the frequency increases, its power coefficient decreases. At \( p = 1 \) (i.e. at external resonance), the power coefficient becomes zero. This behavior is expected, since the equivalent dynamic stiffness of the external system decreases with frequency and upsets the static stiffness match. At resonance, the equivalent dynamic stiffness is virtually zero, hence no force is transmitted, and thus no power. For a soft external system (\( r = 1/4 \)), a similar behavior is observed. The starting point at \( p = 0 \) (quasi-static conditions) is lower, since the unmatched static stiffness gives less than optimal quasi-static behavior. For the conjugate stiff system (\( r = 4 \)), the behavior is entirely different. Under quasi-static conditions, the power coefficient of the conjugate stiff and soft systems was the same. As frequency increases, the stiff system drastically departs from the soft system. Its power coefficient increases, while that of the soft system decreases. This behavior is explained by the dynamic softening of the stiff system. As frequency increases, its stiffness decreases towards that of the internal system. When the external dynamic stiffness equals the internal stiffness, dynamic stiffness match is attained. The frequency value at which dynamic stiffness match occurs is given by

\[ P_{match}(r) = \sqrt{1 - (1/r)} \]  

(7)

Figure 5 shows the variation of the dynamic stiffness match frequency with static stiffness ratio. For large static stiffness ratios, the dynamic stiffness match frequency approaches asymptotically the value 1. For moderate static stiffness ratios, values between 0 and 1 are obtained. For example, for a static stiffness ratio \( r = 4 \), the dynamic stiffness match takes place at \( P_{match}(4) = \sqrt{3}/2 = 0.866 \). This value corresponds to the local maximum of the power coefficient, as show in Figure 4.

The dynamic stiffness match principle is a powerful design tool. Depending on the type of design, either the static stiffness ratio, or the operating
frequency ratio can be modified to yield an operating point as close as possible to the optimum condition. Adequate use of the dynamic stiffness match principle in ISA systems design can yield significant weight savings and increased performance.

The Damped Dynamic System

For a system that presents internal and external losses, the dynamic stiffness is a complex quantity with real and imaginary components. The dynamic stiffness match principle has to take into account the additional aspects of complex power analysis. We define:

\[ P^* = \frac{1}{2} \vec{F} \cdot \vec{u}_e, \quad \vec{F} = -\vec{k}_e \cdot \vec{u}_e, \quad \vec{u}_e = i\omega \cdot \vec{u}_e, \]

\[ P_{\text{rating}} = |P^*| \cos \varphi = \arg (P^*), \quad P_{\text{av}} = \frac{|P^*|}{2} \cos \varphi \] (8)

The complex power \( P \) varies with the frequency parameter \( p \), and the static stiffness ratio, \( r \), as:

\[ \bar{P}(r, p) = -i \frac{\bar{r}(r, p)}{[1 + \bar{r}(r, p)]^2} P_{\text{ref}} \] (9)

Figure 6 presents the variation of power rating with frequency. It can be seen that, as expected, the statically matched system, and the soft system, display a decrease in power rating with frequency. The power rating of the stiff system increases until the dynamic stiffness match is attained; then it starts to decrease. These trends are consistent with the behavior displayed by the undamped system (Figure 5). An aspect specific to the damped system is the power dissipation factor, \( \cos \varphi \) (Figure 7). For a statically matched system, the power dissipation factor is positive (\( \cos \varphi > 0 \)) and increases with frequency, as expected. Similar behavior is displayed by the soft system. The stiff system presents an unexpected behavior: below dynamic stiffness match, its power dissipation factor is negative (\( \cos \varphi < 0 \)). Further aspects of this remarkable phenomenon are presented in Figure 8. As the external damping is increased from \( \zeta = 5\% \) to \( \zeta = 10\% \), the amplitude of the negative power dissipation factor increases as well. Increasing the internal damping from \( \eta = 0 \) to \( \eta = 3\% \) has an opposite effect, and decreases the negative power dissipation factor. Further investigation into this possible feedback instability phenomenon, accompanied by experimental tests, are required.

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Bibliography


