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Shear horizontal wave excitation and reception with shear horizontal piezoelectric wafer active sensor (SH-PWAS)

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Abstract

This article discusses shear horizontal (SH) guided-waves that can be excited with shear type piezoelectric wafer active sensor (SH-PWAS). The paper starts with a review of state of the art SH waves modelling and their importance in non-destructive evaluation (NDE) and structural health monitoring (SHM). The basic piezoelectric sensing and actuation equations for the case of shear horizontal piezoelectric wafer active sensor (SH-PWAS) with electromechanical coupling coefficient $d_{35}$ are reviewed. Multiphysics finite element modelling (MP-FEM) was performed on a free SH-PWAS to show its resonance modeshapes. The actuation mechanism of the SH-PWAS is predicted by MP-FEM, and modeshapes of excited structure are presented. The structural resonances are compared with experimental measurements and showed good agreement. Analytical prediction of SH waves was performed. SH wave propagation experimental study was conducted between different combinations of SH-PWAS and regular in-plane PWAS transducers. Experimental results were compared with analytical predictions for aluminum plates and showed good agreement. 2D wave propagation effects were studied by MP-FEM. An analytical model was developed for SH wave power and energy. The normal mode expansion (NME) method was used to account for superpositioning multimodal SH waves. Modal participation factors were presented to show the contribution of every mode. Power and energy transfer between SH-PWAS and the structure was analyzed. Finally, we present simulations of our developed wave power and energy analytical models.

Keywords: shear horizontal (SH), piezoelectric wafer active sensor (PWAS), structural health monitoring (SHM), wave power and energy, dispersion wave speeds

(Some figures may appear in colour only in the online journal)

1. Introduction

A conventional piezoelectric wafer active sensor (PWAS) is a thin and rectangular or circular wafer that is poled in thickness direction with electrodes on the top and bottom surfaces. Those types of PWAS transducers are either used in in-plane or thickness mode. In the in-plane mode, applying an electric field in thickness direction causes the sensor lateral dimensions to increase or decrease, and a longitudinal strain will occur $\varepsilon_l = d_{13}E_l$, where $d_{13}$ is the piezoelectric coupling coefficient measured in [nm kV$^{-2}$]. Thickness mode is a mode that occurs simultaneously with extension mode, but dominates at higher frequencies in MHz. In the thickness mode, strain in the thickness direction will occur $\varepsilon_t = d_{33}E_t$, where $d_{33}$ is the piezoelectric coupling coefficient in thickness direction. A different mode of oscillation can be achieved when the applied electric field direction is perpendicular to the poling direction, and it is referred to as shear mode. The common piezoelectric coupling coefficient known for this mode is defined as $d_{15}$; however, this coupling coefficient occurs only when the electric current is applied in $E_t$ direction and the poling is along thickness direction. The shear coupled
PWAS presented in this study is associated with the $d_{35}$ coupling coefficient, in which electric current is applied across thickness (i.e., in $x_3$ direction), and the poling is in $x_1$ direction. A few studies considered this ($d_{35}$-mode) [1]. For most piezoelectric materials, the coupling coefficients associated with shear mode have the largest value of all coefficients [2–4]. The higher values of shear coupling coefficients make SH-PWAS superior in actuation and sensing [5]. SH waves are also preferable because the first symmetric mode is non-dispersive in isotropic materials, i.e., wave speed is constant at different frequencies. On the other hand, one of the important disadvantages of SH-PWAS is that thicker transducers are needed to sustain and generate the shear actuation. Due to the high density of piezoceramic materials ($\approx 7600 \text{ kg m}^{-3}$ for APC850 piezoceramic Navy II type), using shear mode piezoelectric elements increases the mass of the system.

Shear mode piezoelectric transducers were used as an actuator element in a cantilever adaptive sandwich beam setup [6]. The stress distribution under mechanical and electrical loading was investigated across thickness and length. A similar study on using shear-type piezoelectric as a shear Bender was studied by [7]. A piezoelectric device was also used for designing torsional actuators generating angular displacement [2], where the torsional element consists of different segments, and the neighbouring segments are of opposite poling. The application of a torsional actuator was applied in a later study by [8] to control rotor blade trailing edge flaps.

For SHM and NDE applications, shear horizontal (SH) guided waves showed high potential for quantitative detection of structure defects [9–11]. In [11], it was shown that SH wave mode conversion occurs at the damage from fundamental incident S0, and it was shown that SH0 could be used for quantitative identification of delamination in composite beams. In another application, SH polarized waves were used for evaluating the quality of bonding between the transducer and the structure [12]. This can be compared to the method of using an imaginary component of PWAS impedance analysis to test the bonding between the transducer and the structure [13]. Shear horizontal waves are usually associated with electromagnetic acoustic transducers, or EMAT [14], where SH waves were used for detection of weld defects. SH waves excited by EMAT have shown superiority over conventional shear vertical (SV) and longitudinal waves for detection of weld defects [15]; however, it was suggested that piezoelectric-based transducers generating SH would show better acoustic generation than EMAT [15]. Also, one point to consider is that EMAT needs conductive structures, while PWAS can be used for conductive metallic structures and non-conductive composites (e.g., glass fibre reinforced polymers), not including the fact that SH-PWAS is inexpensive. In terms of effectiveness, EMAT always showed reliability for detecting damages, especially magnetostriective MsS®, which have been developed by Southwest Research institute (http://www.swri.org).

Nevertheless, fibre optics were also used for detecting SH waves, [16] used fibre optic sensors for detection of SH0 wave type generated from mode conversion from excited Lamb waves, and this was used for detecting delamination in CFRP composites.

SH-PWAS can be used as an alternative for spiral wave ultrasonic conventional transducers in applications such as detecting weep holes in wing spars [17, 18]. This application is not necessarily used for weep holes, but it is used for holes with fasteners. SH-PWAS can be used as a torsional wave exciter and/or recipient for pipe crack damage detection. SH-PWAS can also be used in applications that require the detection of shear waves for finding adhesive shear stiffness [12].

This study focuses on generation of SH waves by piezoelectric wafer-type shear transducer; we call it SH-PWAS. The objectives are: (i) predictive modelling of SH-PWAS response and (ii) performing extensive experimental studies to support the predictive models. The study is structured into three main parts. In the first part, basic piezoelectric sensing and actuation equations for the case of SH-PWAS are discussed. A finite element model was performed to show how the transducer resonates. FEM is also used to show the excitability of SH waves and the axial-flexural waves. The second part presents generation and reception of SH waves and compares dispersion wave speeds with analytical predictions. Experimental studies investigate (1) different possible pitch–catch configurations between SH-PWAS and regular in-plane PWAS and (2) directivity of SH-PWAS and its effect on wave amplitudes. Moreover, FEM was performed to show the 2D effects associated with excitation and reception of SH waves. Finally, in the third part, predictive models for power and energy of SH waves were analytically developed based on the normal modes theory. In this study, it is shown how to superimpose SH wave power when multi-modes exist (typically at high excitation frequencies and in thick structures). Power is the time rate of change of wave energy. We developed energy models to verify our analytical models by showing that potential energy and kinetic energy are equal. Modelling the total wave power is important to better understand the power consumption of the electric source for practical SHM and NDT systems. Also, the developed power model shows how different modes contribute to the total power.

2. SH-PWAS constitutive relations and actuation mechanism

2.1. Constitutive relations

Most literature mentioned earlier dealt with shear dielectric coupling coefficient $d_{15}$; however, this is only applicable if the electric field ($E_x$) is applied in the in-plane direction and the piezoelectric poling is in the thickness direction. In such a case, the transducer electrodes are installed on two of the vertical sides. In our model and in FEM simulations, we use
\[ S_{ij} = s_{ij}^{E} T_{ij} + d_{ij} E_{k} \quad (1) \]
\[ D_{i} = d_{i} T_{ij} + \epsilon_{i}^{T} E_{k} \quad (2) \]

where \( S_{ij} \) are the strain components, \( s_{ij}^{E} \) denotes compliance matrix under a constant electric field condition, \( T_{ij} \) are stress components, \( d_{ij} \) are piezoelectric coupling coefficients, \( E_{k} \) represents the electric field vector, \( D_{i} \) is the electric displacement vector, and \( \epsilon_{i}^{T} \) are electric permittivity constants of the PWAS material. Equation (1) is considered the piezoelectric converse effect, where an applied electric field will result in induced strains. Equation (2) is the direct piezoelectric mechanism, where applied stresses will result in output electrical displacements. In the contracted Voigt contraction form, equations (1), (2) can be written as

\[
\begin{bmatrix}
S_{1} \\
S_{2} \\
S_{3} \\
2S_{4} \\
2S_{5} \\
2S_{6}
\end{bmatrix}
= \begin{bmatrix}
s_{11}^{E} & s_{12}^{E} & s_{13}^{E} \\
s_{21}^{E} & s_{22}^{E} & s_{23}^{E} \\
s_{31}^{E} & s_{32}^{E} & s_{33}^{E} \\
s_{44}^{E} & s_{55}^{E} & s_{66}^{E} \\
S_{4} & S_{5} & S_{6}
\end{bmatrix}
\begin{bmatrix}
T_{1} \\
T_{2} \\
T_{3} \\
T_{4} \\
T_{5} \\
T_{6}
\end{bmatrix}
\]

\[
\begin{bmatrix}
D_{1} \\
D_{2} \\
D_{3} \\
D_{4} \\
D_{5} \\
D_{6}
\end{bmatrix}
= \begin{bmatrix}
d_{31} & d_{32} & d_{33} & d_{35} \\
d_{41} & d_{42} & d_{43} & d_{45} \\
d_{51} & d_{52} & d_{53} & d_{55} \\
d_{61} & d_{62} & d_{63} & d_{65}
\end{bmatrix}
\begin{bmatrix}
E_{1} \\
E_{2} \\
E_{3} \\
E_{4} \\
E_{5} \\
E_{6}
\end{bmatrix}
\]

The mechanical compliance matrix of the piezoelectric material takes an orthotropic form. The electrical permittivity matrix is a diagonal matrix, corresponding to the three possible directions of the applied electric field. The piezoelectric coupling matrix contains coefficient \( d_{33} \) corresponding to the thickness strain, i.e., \( \epsilon_{3} = d_{33} E_{3} \) and coefficients corresponding to the lateral strain, i.e., \( d_{13} \) and \( d_{23} \). Also, the \( d_{34} \) coefficient relates the electric field in lateral direction \( E_{4} \) with shear motion (in 2–3 direction, i.e., \( T_{2}, 2S_{5} \)), while \( d_{35} \) relates the electric field in lateral direction \( E_{5} \) with shear motion (in 1–3 direction, i.e., \( T_{3}, 2S_{6} \)). In our SH-PWAS, provided by APC piezoceramic Int., Ltd. (figure 1(b)), the electric field is applied in \( E_{3} \) direction, resulting in shear motion (in 1–3 direction, i.e., \( T_{3}, 2S_{6} \)). So, the \( d_{35} \) term appears. For the SH-PWAS transducer in this study, we have an electric field in 3-direction, and the poling is in longitudinal direction, as shown in figure 1(a); the two equations (3), (4) reduce to

\[
(2S_{4}) = u_{3} = s_{35} T_{3} + d_{35} E_{3} \quad (5)
\]
\[
D_{3} = d_{35} T_{3} + \epsilon_{3} E_{3} \quad (6)
\]

2.2. Free SH-PWAS response

A multiphysics finite element model was constructed for the free SH-PWAS using COMSOL Multiphysics. The coupled physics incorporates the induced mechanical strain due to an applied electrical field across the thickness in \( x_{3} \) direction.
The SH-PWAS dimensions are 15 mm × 15 mm × 1 mm. One square side of the SH-PWAS was grounded, and a 10 volt was applied to the other side. A sweep of frequencies was performed from 100 kHz to 4 MHz, and the resulting displacement response was reported in figure 2 for two example frequencies: a) at 200 kHz and b) at 950 kHz, where the transducer’s first resonance occurs. PWAS material is APC850; detailed properties can be found on the APC website (http://www.americanpiezo.com). Figure 2 shows a finite element analysis of the vibrations’ modeshapes of SH-PWAS. It can be observed that at low frequency (figure 2(a)), the vibration has a linear shape across the thickness. Whereas at higher frequencies, e.g., ≈1 MHz, (figure 2(b)), the modeshape shows nonlinearities and a more complicated shape.

The modeshape of vibration of the shear horizontal coupled PWAS is important, as it controls the excitation of guided waves in the structure, which is discussed in the next section.

2.3. Bonded SH-PWAS to beam structures

When SH-PWAS is bonded to the structure (figure 3(a)), SH waves are excited in the direction perpendicular to poling direction. Poling direction is the direction where the transducer vibrates when excited by electrical voltage. Because of this actuation mechanism, axial-flexural response can be obtained in the direction of poling $P$. Figure 3(b) shows the shear actuation of the transducer. Normal load transfer as well as bending moment transfer lead to an axial and flexural response in 1-direction.
Electromechanical (E/M) impedance is measured for bonded SH-PWAS on 1 mm thick aluminium beams. A frequency sweep is performed up to 160 kHz to capture structure resonances. The objective is to study structure response when SH-PWAS is bonded in either one of two configurations: (a) transducer poling is parallel to the beam length or (b) transducer poling is perpendicular to the beam length (figure 4).

Finite element models are constructed for bonded SH-PWAS on 1 mm thick aluminium beams. Three models are constructed: (a) 2D model for the case where poling of the SH-PWAS is along the beam length, (b) 3D model for the same case of having poling direction parallel to the beam length, and (c) 3D model for the case of transducer poling perpendicular to the beam length. Figure 5 shows detailed finite element setups. The transducer top electrode is excited by electric voltage of 1 V. A sweep of frequency is performed up to 160 kHz. E/M impedance is calculated for different models and compared with bonded SH-PWAS experimental results.

2.4. Discussion of bonded SH-PWAS electromechanical response results

The electromechanical impedance of bonded SH-PWAS in configuration-1 shows peaks at 42 kHz, 90 kHz, and 136 kHz (figure 6). Structure modeshapes at these frequencies indicate that these frequencies correspond to the axial-flexural response (figure 7). The modeshapes at 42 kHz and 136 kHz are width independent (z-invariant). However, the captured modeshape at 90 kHz has some width coupled vibration.

These results agree with the suggested mechanism of actuation of SH-PWAS for the response in the same direction of poling direction (figure 3(b)).

The SH-PWAS installed in configuration-2 showed SH response, where the beam length is perpendicular to the
poling direction. The electromechanical impedance of the bonded transducer to the structure (figure 8) shows peaks at 22 kHz, 47 kHz, 77 kHz, 107 kHz, and 136 kHz. Figure 9 shows the finite element simulation of displacement in the z-direction, which is a shear horizontal response.

3. Guided wave excitation by SH-PWAS

3.1. Analytical review

Consider SH-PWAS bonded to the structure shown in figure 3(a). The structure half thickness is \( d \), and \( \mu \) is the shear modulus of the structure. SH PWAS dimensions are: length \( l \), width \( b \), and thickness \( h \). Shear horizontal waves have a shear-type particle motion contained in the horizontal plane. Cartesian coordinates are defined such that the \( x \)-axis is placed along the wave propagation direction, whereas the \( z \)-axis is the direction of particle motion, and \( y \) is along the plate thickness. The poling direction of the piezoelectric transducer is in \( x \) direction (coincides with global \( z \)-axis coordinate of the structure). An approximated 1D analytical model with a \( z \)-invariant assumption is well-developed in many previous studies [19–21]. The analytical model only predicts SH wave motion of particle oscillation along \( z \) direction and propagating in \( x \) direction. We use the analytical model to predict dispersion wave speeds of SH waves. The displacement is assumed to be harmonic

\[
 u(x, y, t) = U_z(y)e^{-i\xi(x-ct)}
\]  

(7)

where \( \xi \) is the wave number in \( x \) direction. Guided SH waves in plates (similar to guided Lamb waves) are multimodal in nature; as the frequency of excitation increases, new modes are excited in the plate. The frequencies at which new modes appear are called cut-off frequencies. The cut-off frequency can be determined by solving the characteristic equation

\[
 \eta^2 \sin(\eta d) \cos(\eta d) = 0 \text{ for } \eta d \text{ values and by substituting in } \eta \text{ defined from } \eta \omega \xi = \frac{c}{s} 2^2 2^2 , \text{ and } c_s \text{ is the shear wave speed. We define cut-off frequency in units of Hz or normalized frequency } \eta d/c_s . \text{ The } n \text{th symmetric mode displacement is}
\]

\[
 u_s^n(x, y, t) = B_n \cos \left( \eta_s^n y \right) e^{-i\xi_s^n z} e^{i\omega t} 
\]  

(9)

and the \( n \)th antisymmetric mode displacement is

\[
 u_a^n(x, y, t) = A_n \sin \left( \eta_a^n y \right) e^{-i\xi_a^n z} e^{i\omega t} .
\]  

(10)

The total displacement is

\[
 u(x, y, t) = \left[ A_n \sin \left( \eta_a^n y \right) e^{-i\xi_a^n z} + B_n \cos \left( \eta_s^n y \right) e^{-i\xi_s^n z} \right] e^{i\omega t} .
\]  

(11)

The amplitudes \( A_n, B_n \) are normalized with respect to power flow and found to be [22]

\[
 B_n = \sqrt{\frac{1}{\alpha p \xi_{a,n}^2 d}} , \quad A_n = \sqrt{\frac{1}{\alpha p \xi_{s,n}^2 d}} .
\]  

(12)

Solving the characteristic equation

\[
 \sin(\eta d) \cos(\eta d) = 0 \text{ results in finding wave speeds and group velocities.}
\]

Analytical evaluation for shear horizontal wave speeds and group velocities is presented in figure 10 for a 1 mm thick aluminium plate. Wave speeds are normalized with respect to shear (transverse) wave speed, which equals 3129 m s\(^{-1}\) for our case study aluminium 2024-T3 alloy.
The predicted SH wave modes were three modes in the 4000 kHz frequency window (corresponds to \( fd/c = 0.64 \)). The first SH mode is SH0; it is a symmetric mode of vibration and has a constant propagation speed at any excitation frequency. The second SH mode is SH1; it is an antisymmetric mode with a cut-off frequency \( \approx 0.25(1565 \text{ kHz}) \). The third mode in our simulation results is SH3; it is symmetric like SH0. However, it is dispersive, i.e., does not have a constant propagation speed. The cut-off frequency of SH3 is \( \approx 0.5(3\text{1}30 \text{ kHz}) \).

### 3.2. Experimental studies

#### 3.2.1. Proof of concept

Three sets of experiments are performed. The first set of experiments was a proof of concept that was performed on 3.4 mm thick aluminium 7075 T6 alloy plate (figure 11). The SH-PWAS was 15 mm \( \times \) 15 mm \( \times \) 1 mm, and its material was APC850. SH-PWAS poling direction was along the z-direction (figure 11(c)). The distance between the two SH-PWAS was 150 mm, and the excitation was a 3-count tone burst signal with 10 V amplitude. The excitation frequencies used were 30, 45, 60, 75, and 90 kHz, as shown in the waveforms (figure 12). It was noticed that the received signals in figure 12 waveforms were non-dispersive, i.e., they had shown the same shape as the excitation signal, \( \approx 3 \) count tone burst), especially at frequencies 60, 75, and 90 kHz; this implies that the wave packet speed does not change with frequency, and that this is the intrinsic property of SH0 (the first shear horizontal guided wave in isotropic materials).
In addition, it was observed that no waves propagate along (path 2) in figure 11(c). The actuation mechanism of SH-PWAS that is shown from free transducer mode shapes (figure 2) implies that the SH-PWAS resonates in the \( z \)-direction, and the generated waves propagate along the \( x \)-direction. Comparison between analytical and experimental results is shown in figure 13.

3.2.2. Pitch–catch experiments between combinations of SH-PWAS and in-plane PWAS transducers. The second set of experiments were a rigorous combination of pitch–catch experiments between (a) SH-PWAS transducers with different orientations (to study effect of poling direction) and (b) pitch–catch experiments between SH-PWAS transducers and regular PWAS. SH-PWAS materials was APC850, and the dimensions were 15 mm \( \times \) 15 mm \( \times \) 1 mm. The regular PWAS material was APC850, and a circular PWAS of diameter 15 mm and 0.2 mm thickness was used. A detailed set up is shown in figure 14(b). The aim behind those combinations of experiments was to have a better understanding of the following cases:

1. Does the SH-PWAS transmit only SH waves to another SH-PWAS?
2. Can regular PWAS receive SH waves transmitted by SH-PWAS?
3. How does SH-PWAS behave when excited by waves coming from regular PWAS? (The opposite situation of question 2.)
4. How does SH-PWAS behave if oriented 90 degrees; does it transmit SH waves in this case? Does it receive SH waves?

Seven experiments were performed on a 1 mm thick aluminium 2024-T3 square plate 1220 \( \times \) 1220 mm with a frequency sweep up to 300 kHz. Table 1 summarizes the experiments and captured waves in each case.

The experiment between two SH-PWAS transducers showed the generation of shear horizontal waves, providing that both transducers are installed such that their polarization vectors are parallel to each other (experiment #1), (figure 15(a)).

However, for two SH-PWAS transducers installed such that their polarization directions are perpendicular to each other, the signals that SH-PWAS4 received from SH-PWAS6 had the speeds of \( S_0 \) for the first symmetric Lamb wave modes, (experiment #4) (figure 15(c)).

Experiment #6 (the reverse situation of experiment #4) showed identical results to experiment #4. This was done to verify reciprocity and lack of nonlinear effects. The exciter SH-PWAS6 was oriented in the correct direction to send SH waves towards the receiver SH-PWAS4. SH-PWAS4 was the one oriented with 90°. In such a situation, we expected that transmitter SH-PWAS6 was sending out SH waves; however, receiver SH-PWAS4 neither responded nor picked SH waves, but rather picked \( S_0 \) waves (figure 15(c)). This suggests that the transmitter excites the \( S_0 \) wave in the measured direction. This observation is further explained in the discussion section of guided wave propagation results.

Another feature was observed: when SH-PWAS5 excites SH waves, the regular extensional mode PWAS2 picked up two types of guided waves: a Lamb wave antisymmetric \( A_0 \) mode as well as a \( SH_0 \) wave (experiment #2) (figure 15(b)). It was not expected that extensional type PWAS transducers resonate in shear mode and convert shear-mode waves to output voltage. This observation is further discussed in the discussion of guided wave propagation results section.

Finally, regular PWAS2 was excited, and the signal was caught by SH-PWAS5 (experiment #3). Similarly, PWAS2 was excited, and the signal was caught by SH-PWAS4 (experiment #7).

Experiment #3 was identical to experiment #2, where the SH-PWAS5 picked up SH waves (exactly like figure 15(b)). In experiment #7, where PWAS2 was excited, and the signal...
was caught by SH-PWAS4, the received waveforms corresponded to guided Lamb waves only (figure 15(d)).

3.2.3. Directivity of SH-PWAS. The third set of experiments involves a similar setup to that of set #2, but with added transducers at 30° and 60° degree angles. The complete setup is shown in figure 14(a). Experiment #1 (SH-PWAS5 → SH-PWAS6) indicates the zero angle direction pitch–catch, experiment #1(30) indicates the 30° pitch–catch, and experiment #1(60) indicates the 60° pitch–catch.

Receiver SH-PWAS6 transducers in experiment #1(30) and (60) are no longer having parallel poling direction to transmitter SH-PWAS5.

Similarly, experiments #2 and #4 are performed at different angles: 0°, 30°, and 60°. Figure 16 shows the

Figure 12. Waveforms associated with pitch–catch SH waves experiment on 3.4 mm thick aluminium.

Figure 13. Experimental versus analytical wave group velocity curves (SH-PWAS experiment on 3.4 mm thick aluminium plate).
directivity patterns for received wave amplitudes at different experiments.

Figure 16(a) shows SH wave amplitudes for a pitch–catch experiment between two SH-PWAS transducers. Starting from parallel poling directions (at 0°), the SH wave amplitude is the maximum (e.g., at 60 kHz). At 30°, the SH wave amplitude decreases and then it further decreases at 60°. This is not observed with all the frequencies. On the other hand, A0 wave amplitudes received at SH-PWAS for the same experiment show an increase in amplitude as the angle increases from 0° to 30° to 60°. This agrees with the previous results of exciting an axial-flexural response along the poling direction. As the angle of the pitch–catch experiment changes towards 60°, a stronger A0 mode is obtained.

Experiment #2 (SH-PWAS5 → regular PWAS2) showed similar patterns to experiment #1. However, the received signals at 60° were noisy. Figure 16(c) shows amplitudes of SH waves received by PWAS2 and generated by SH-PWAS5 for experiment #2, 2(30), and 2(60). Figure 16(d) shows amplitudes of received A0 waves. Experiments #3 are the opposites of experiments #2. Those are not performed in this study.

Experiment #4 (SH-PWAS4 → SH-PWAS6) involves the pitch–catch experiments between two SH-PWAS transducers having poling directions perpendicular to each other (for 0° case). Figure 16(e), f are for the same received S0 wave amplitudes but at different frequencies. They are plotted on two polar plots because of considerable change in amplitude values in [mV] between 45 kHz, 75 kHz, and 255, 300 kHz. It is observed that the S0 amplitudes are much less at lower frequencies. Also, it is observed that the perpendicular poling directions—experiment #4(0)—cause the least S0 wave amplitudes. S0 wave amplitudes are much higher at 30° and 60° angles between poling directions of the two transducers.

Table 1. Description of experiments showing excitation and receiver PWAS transducers for each experiment and the possible paths of wave propagation.

<table>
<thead>
<tr>
<th>Experiment No. and description of pitch–catch configuration</th>
<th>Captured waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment (#1) SH5 → SH6</td>
<td>SH0, A0</td>
</tr>
<tr>
<td>Experiment (#2) SH5 → PWAS2</td>
<td>SH0, A0</td>
</tr>
<tr>
<td>Experiment (#3) PWAS2 → SH5</td>
<td>SH0, A0</td>
</tr>
<tr>
<td>Experiment (#4) SH4 → SH6</td>
<td>S0</td>
</tr>
<tr>
<td>Experiment (#6) SH6 → SH4</td>
<td>S0</td>
</tr>
<tr>
<td>Experiment (#7) PWAS1 → SH4</td>
<td>A0, S0</td>
</tr>
</tbody>
</table>

Figure 14. Numbering and directions of pitch–catch experiments on aluminium plate: (a) directivity experiment, (b) separated experiments for combination of SH-PWAS–regular PWAS pitch–catch configurations.
3.3. Finite element simulations

The models in section 2.3 predict SH-PWAS effects at 0° and 90°, separately. Also, it is hard to combine (axial-flexural) and (shear horizontal) separate responses of figure 3(b) into a 2D analytical model. Hence, 2D multiphysics FEM simulations are constructed to better understand the possible excited waves by SH-PWAS and to verify directivity experiments. Shear horizontal SH0, symmetric S0 and antisymmetric A0 Lamb waves were picked by FEM simulations.

The finite element model was constructed for the bonded SH-PWAS to the structure. The SH-PWAS dimensions were $15 \text{ mm} \times 15 \text{ mm} \times 1 \text{ mm}$, while in-plane PWAS dimensions were $7 \text{ mm} \times 7 \text{ mm} \times 0.2 \text{ mm}$. The mesh size of SH-PWAS elements was 0.5 mm and 4 elements, per the 1 mm thickness. A 1 mm aluminium 2024 alloy plate was used in our simulations. The plate was a 450 mm square plate. The structure maximum element size was set to 4 mm and 2 elements through the 1 mm thick aluminium plate. The plate was modelled with free BC, and the SH-PWAS was perfectly bonded from the bottom surface and free from the upper surface.

Excitation signal was 3-count tone burst with center frequency 60 kHz and voltage amplitude of 10 V. The time step selected was 0.5 $\mu$s, and simulation time was 200 $\mu$s. Figure 17 shows the results of the simulations. Figure 17(a) shows the displacement field in the $z$-direction, i.e., the direction of shear horizontal particle oscillation. SH0 waves had a strong oscillation in the $z$-direction and propagated in the $x$-direction between the transmitter and receiver SH PWAS transducers. Antisymmetric A0 and symmetric S0 modes were observed propagating in the $z$-direction. For comparison, the waves excited by in-plane PWAS (figure 17(b)) are reported; only A0 and S0 existed. The simulations in figure 17 are both captured at a simulation time that equals 77 $\mu$s. The displayed parameter in figure 17(b) is $e_Z$ the out of plane strain; it was selected rather than the displacement fields to be able to show S0 and A0 modes together.

When FEM simulation was repeated between the two SH-PWAS transducers, but with the transmitter SH-PWAS oriented by 90 degrees (figure 18), the waves that propagated toward the receiver SH-PWAS were S0 and a noisy A0. This was in good agreement with the observed results from experiment #4 (figures 14(b) and 15(c)). Figure 18 shows the displacement fields in the $x$-direction at 40 $\mu$s. Particle motion in the $x$-direction was selected because, for such a configuration, SH waves had a particle oscillation in the $x$-direction and propagated in the $z$-direction. Besides, the S0 Lamb wave was propagating in the $x$-direction—with dominant particle motion in the $x$-direction.

3.4. Discussion of guided wave propagation results

The FEM simulations of SH wave propagation between two SH-PWAS transducers (figure 17(a)) validate the transducer
actuation mechanism of exciting SH waves in the direction perpendicular to the poling direction. SH wave amplitude decreases as the direction of the measured response changes from 0° towards 90°; this agrees with figure 16(a) at excitation frequency 60 kHz. Recalling experiment #6 in the pitch–catch experiments, i.e., the opposite of experiment #4 (figure 14(b)), the receiving of the S0 waves seem to contradict with the results of figure 15(a), where SH0 and A0 were only captured along the direction perpendicular to the poling direction of transmitter SH-PWAS. Referring to figure 17(a), a very weak S0 mode appears along 45 degrees from the x-direction (and almost vanishes along the x-direction). Hence, one can conclude that the SH-PWAS actually excites S0 waves in the same direction of exciting the SH

![Experiment-1](image1)
![Experiment-2](image2)
![Experiment-4](image3)

**Figure 16.** Amplitudes of different waves at different angles of pitch–catch experiments, associated with directivity experiment.
waves; and this is due to 2D effects and to the fact that structure particle vibrations at one side of the transducer definitely affect vibrating particles at the other sides. The considerably reduced S0 wave amplitudes are proven from figure 16(e), f along the 0° direction.

The feature observed in experiment #2 in section 3.2.2 was that the regular in-plane PWAS was able to pick up SH waves. This means that it resonates in its extensional-contraction mechanism when the shear wave front hits the transducer. Two dimensional effects can be the reason; that is, SH waves excited by SH-PWAS (with structure particles vibrations in the z-direction) arrive at regular PWAS with the z-direction vibrations; and, due to 2D effects, z-direction oscillations are actually considered extension-contraction oscillations (if viewed from another diameter of the receiver PWAS). In addition to the 2D effects, SH waves can be mode converted at the receiver PWAS because the transducer itself is considered an inhomogeneity in the wave field. A similar observation in [11] suggested that the S0 wave mode converts to SH0. SH0 can be mode converted (at the time of flight of receiving SH0) to a mode that regular PWAS interacts with.

4. Power and energy transduction with SH-PWAS

The study of power and energy transduction between PWAS and a bonded structure has been presented in [23], where exact guided Lamb waves’ power and energy are studied. Energy transfers from electrical to mechanical in the transducer; then, the mechanical energy causes the wave to propagate. This paper presents an analytical model for the SH waves’ power and energy based on the normal mode expansion (NME) technique. The solution assumes straight crested harmonic waves and that no evanescent (i.e., non-propagating) waves exist. Mode amplitudes are normalized with respect to power flow; and the actual amplitudes can be determined from equation (12).

Considering that only SH waves are propagating; the surviving strains are

\[ 2S_{xz} = \frac{\partial u_x}{\partial y}, \quad 2S_{yz} = \frac{\partial u_y}{\partial x}. \]  

(13)

Strains and stresses can be evaluated given the total displacement, equation (11); however, as we will show later, the symmetric and the antisymmetric displacements can be in separate solutions because the orthogonality condition cancels the terms involving multiplications between cosine and sine terms from symmetric and antisymmetric modes. Hence, we can proceed with a separate analysis. This can be useful to separate wave energy and power and to quantify the partition of symmetric modes as well as antisymmetric ones. Following the method presented in [22], modal participation factors are
found to be

$$ a^s_n(x) = \begin{cases} 
\frac{v^n_s(x)}{4P_{mn}} e^{-i\frac{\omega}{c} t(x)} \int_{-\frac{b}{2}}^{\frac{b}{2}} e^{i\omega x} t(x) dx & \text{for } -\frac{b}{2} < x < \frac{b}{2} \\
0 & \text{for } x < -\frac{b}{2}.
\end{cases} \tag{14} $$

Noting that equation (13) is valid for the forward propagating mode only, $b$ is the width of SH-PWAS, and it is the conjugate of velocity field in $z$ direction for the mode $n$, and $t_c$ is the PWAS traction or shear stress. We denote $a^s_n(x)$ by a $+\sin$ sign to show that it is for the forward propagating mode.

Modal participation factor is an extra term to be multiplied by the wave amplitudes. It is a function of the distance $x$ and also accounts for the transducer dimension $b$. We define $a^s_n$ as the modal participation factor for nth symmetric mode, and similarly $a^s_n$ for the nth antisymmetric mode. Equations (11), (14) yield the strains and stresses as

$$ 2S_{xc} = \left[ A_n i \eta_n^A \cos \eta_n^S \sin \xi_n^S e^{-i \xi_n^S} - B_n \eta_n^S \sin \eta_n^S \sin \xi_n^S e^{-i \xi_n^S} \right] e^{i\omega t} \tag{15} $$

The total strain response (due to symmetric and antisymmetric waves) and the conjugate values of the strain are

$$ 2S_{xc} = a^s_n(-i A_n \left( \xi_n^S \sin \eta_n^A \right) e^{-i \xi_n^S} + a^s_n(i B_n \left( \xi_n^S \cos \eta_n^A \right) e^{-i \xi_n^S}) $$

and one single antisymmetric mode, taking into account the modal participation factors.

From the displacement equation (11), we obtain the velocity and the conjugate velocity as

$$ v_c = a^s_n(i a_n A_n \left( \sin \eta_n^A \right) e^{-i \xi_n^S} + a^s_n(i B_n B_n \left( \cos \eta_n^A \right) e^{-i \xi_n^S}) $$

$$ \bar{v}_c = a^s_n(-i A_n \left( \cos \eta_n^A \right) e^{i \xi_n^S} + a^s_n(-i B_n B_n \left( \sin \eta_n^A \right) e^{i \xi_n^S}). \tag{17} $$

The time averaged power is defined as

$$ \langle p \rangle = -\frac{1}{2} \int_A \left( \bar{v}_c v_c \right) dA = -\frac{b}{2} \int_{-d}^{d} \mu \left\{ 2 S_{xc} \nu_c \right\} dy. \tag{18} $$

Substituting equations (16), (17) in equation (18) and simplifying yields the time averaged power as

$$ \langle p \rangle = -\frac{b}{2} \int_{-d}^{d} \mu \left\{ 2 S_{xc} \nu_c \right\} dy $$

$$ \langle p \rangle = -\frac{b}{2} \int_{-d}^{d} \mu \left\{ a^s_n A_n \left( \sin \eta_n^A \right) e^{-i \xi_n^S} \right\} \int_{-d}^{d} \sin \eta_n^A \nu_c dy $$

$$ \langle p \rangle = -\frac{b}{2} \int_{-d}^{d} \mu \left\{ a^s_n A_n \left( \sin \eta_n^A \right) e^{-i \xi_n^S} \right\} \int_{-d}^{d} \sin \eta_n^A \nu_c dy $$

First, terms with multiplied sine and cosine functions from the symmetric mode and the antisymmetric mode are cancelled; for the characteristic equations of the symmetric and the antisymmetric modes, either sine or cosine terms will be zero at a time. Hence, there is no dependency between symmetric and antisymmetric modes. Second, terms with $\sin(2\eta_n d)$ appearing with the analysis of single types of waves are also crossed out because $\sin(2\eta_n d) = 2 \sin(\eta_n d) \cos(\eta_n d)$, and for our characteristic equations for symmetric and antisymmetric, either sine or cosine terms will be zero at a time. The final result for wave power takes the form

$$ \langle p \rangle = \frac{\mu a^s_n}{2} \int_{-d}^{d} \left\{ \sum_n a^s_n B_n \left( \sin \eta_n^A \right) e^{-i \xi_n^S} \right\} \left\{ \sum_n a^s_n A_n \left( \sin \eta_n^A \right) e^{i \xi_n^S} \right\}. \tag{19} $$

The time-averaged power varies at different $x$ values, as the $x$ dependency comes from the modal participation factors.
All the following numerical illustrations are shown at the top surface of the structure \((y = d)\) and at the edge of the transducer, where \((x = b/2)\).

With similar analysis, we define time-averaged kinetic energy

\[
\langle k_1 \rangle = \frac{1}{2} \rho \int_A \frac{1}{2} \bar{v} \cdot \bar{v} dA
\]

and the final analytical form will be

\[
\langle k_1 \rangle = \frac{b}{4} \int_d \{ \rho \bar{v} \cdot \bar{v} \} dy = \frac{b \rho \bar{v}^2}{4} \left[ \sum_n \left[ a_n^2 B_n \right] + \sum_n \left[ A_n \right] \right].
\]

Time averaged potential energy is defined as

\[
\langle v_1 \rangle = \frac{1}{4} \int_A \left\{ \left( \mu \right) 2 S_{\xi \xi} 2 \bar{S}_{\xi \xi} + \left( \mu \right) 2 S_{\eta \eta} 2 \bar{S}_{\eta \eta} \right\} dA.
\]

Using similar analysis to the one we followed in power and kinetic energy, then cancelling \(\sin 2 \eta_1 \cdot d\) terms, results in

\[
\langle v_1 \rangle = \frac{b}{4} \int_d \left\{ \left( \mu \right) 2 S_{\xi \xi} 2 \bar{S}_{\xi \xi} + \left( \mu \right) 2 S_{\eta \eta} 2 \bar{S}_{\eta \eta} \right\} dy = \frac{b}{4} \left\{ \sum_n \mu d \left[ a_n^2 B_n \right] \left[ \left( \xi_1 \right)^2 + \left( \eta_1 \right)^2 \right] \right\} + \sum_n \mu d \left[ a_n^2 A_n \right] \left[ \left( \xi_1 \right)^2 + \left( \eta_1 \right)^2 \right].
\]

Using the relation \(\eta_1^2 = \omega^2 / \bar{c}^2 - \bar{\xi}^2\), we can then prove that time-averaged potential energy equals time-averaged kinetic energy

\[
\langle v_1 \rangle = \frac{b}{4} \int_d \left\{ \left( \mu \right) 2 S_{\xi \xi} 2 \bar{S}_{\xi \xi} + \left( \mu \right) 2 S_{\eta \eta} 2 \bar{S}_{\eta \eta} \right\} dy = \frac{b}{4} \left\{ \sum_n \mu d \left[ a_n^2 B_n \right] \left[ \frac{\alpha_1^2}{\bar{c}_1^2} \right] + \sum_n \mu d \left[ a_n^2 A_n \right] \left[ \frac{\psi_1^2}{\bar{c}_1^2} \right] \right\}.
\]
4.1. Discussion of power and energy model results

Numerical simulations for developed analytical models were shown in figures 19 and 20. Figure 19 shows simulation results of our analytical model for wave power. The SH0 power flow oscillates as a function of frequency with constant amplitude (because of having constant dispersion wave speed). However, the peaks and valley response are due to SH-PWAS finite dimension effect (what is commonly referred to as tuning of the transducer). SH1 (antisymmetric shear horizontal mode) kicked off at 1560 kHz (figures 19(b), (d)). SH2 (symmetric mode) started at 3150 kHz (figures 19(a), (c)). Both SH1 and SH2 are dispersive modes with variable power consumption at different frequencies (because their wave speeds are not constant along the frequency spectrum). Similar conclusions are drawn from simulated results of wave energies (figure 20).

5. Summary and conclusions

The paper discussed the excitation and reception of SH waves using SH-PWAS; that is, piezoelectric wafer active sensor poled in shear direction. The paper also presented predictive models for power and energy of multimodal SH waves.

Excitation of SH waves was analyzed by finite element simulations and experiments. SH0 non-dispersive waves were captured in aluminium plates. Multiple experiments were performed to show the SH waves excitation and receiving capabilities of both SH-PWAS and regular in-plane PWAS transducers. It was shown that positioning and orientation of SH-PWAS affects the generation of SH waves: (1) SH-PWAS excites SH waves in the direction perpendicular to poling direction, and (2) Regular in-plane PWAS can sense SH waves. Additionally, (3) SH-PWAS transducers can sense A0 and S0 Lamb waves. Directivity analysis showed that excited SH wave amplitude gradually decreases as the measuring direction deviates from the maximum received amplitude direction.

A predictive model for a guided SH wave’s power and energy was analytically developed based on a normal mode expansion technique. The model assumed that (a) waves are of straight crested harmonic type, (b) evanescent non-propagating waves are ignored, and (c) the modes are of orthogonal functions. The amplitudes of each mode were normalized with respect to the power flow, and modal participation factors were determined. Modal participation factors are a function of transducer dimension. The wave power, kinetic energy, and potential energy were modelled, and numerical results were presented. As expected, the kinetic energy equals the potential energy in total and for separate modes as well, due to the fact that modes are orthogonal. SH0 mode wave power and wave energy oscillate with frequency but have a constant amplitude due to the constant wave propagation.
speed of SH0 in isotropic materials. SH1 and SH2 modes are dispersive shear horizontal modes.

Future work will include investigation of SH waves excitation in composite materials, a predictive analytical model for SH-PWAS electromechanical impedance for the free transducer and when bonded onto the structure, and further studies on modelling SH wave excitability by the SH-PWAS.

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