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WaveFormRevealer: An analytical framework and predictive tool for the simulation of multi-modal guided wave propagation and interaction with damage

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Abstract
This article presents the WaveFormRevealer—an analytical framework and predictive tool for the simulation of guided Lamb wave propagation and interaction with damage. The theory of inserting damage effects into the analytical model is addressed, including wave transmission, reflection, mode conversion, and nonlinear higher harmonics. The analytical model is coded into MATLAB, and a graphical user interface (WaveFormRevealer graphical user interface) is developed to obtain real-time predictive waveforms for various combinations of sensors, structural properties, and damage. In this article, the main functions of WaveFormRevealer are introduced. Case studies of selective Lamb mode linear and non-linear interaction with damage are presented. Experimental verifications are carried out. The article finishes with summary and conclusions followed by recommendations for further work.

Keywords
Guided waves, structural health monitoring, damage detection, piezoelectric wafer active sensors, analytical model, nonlinear ultrasonics

Introduction
Guided waves retain a central function in the development of structural health monitoring (SHM) systems using piezoelectric wafer active sensor (PWAS) principles. The modeling of Lamb waves is challenging, because Lamb waves propagate in structures with multi-mode dispersive characteristics. At a certain value of the plate thickness-frequency product, several Lamb modes may exist simultaneously, and their phase velocities vary with frequency. When Lamb waves interact with damage, they will be transmitted, reflected, scattered, and mode converted. Nonlinear interaction with damage may also exist, and this will introduce distinctive features like higher harmonics. These aspects give rise to the complexity of modeling the interaction between Lamb waves and damage. To solve such complicated problems, numerical methods like finite element method (FEM) and boundary element method (BEM) are usually adopted. However, to ensure the accuracy of simulating high-frequency waves of short wavelengths, the transient analysis requires considerably small time step and very fine mesh ($T/\Delta t, \lambda/l_{FEM} \geq 20 \sim 30$), which is expensive both in computational time and computer resources. Analytical model provides an alternative approach to attack the same problem with much less cost. PWAS transducers are a convenient way of transmitting and receiving guided waves in structures for SHM applications. The analytical model of PWAS-generated Lamb waves and its tuning effect has been investigated, and a close-form solution for straight crested guided Lamb wave was derived by Giurgiutiu. Extension of tuning concepts to 2D analytical models of Lamb waves generated by finite-dimensional piezoelectric transducers was given in Raghavan and Cesnik. These analytical developments facilitate the understanding of PWAS-coupled Lamb waves for SHM applications.
However, these analytical solutions only applied to guided wave propagation in pristine structures, whereas the use of Lamb waves in SHM applications requires that their interaction with damage also be studied. After interacting with damage, Lamb waves will carry damage information resulting in waveforms with special characteristics (phase change, new wave packets generation through mode conversion, higher harmonic components, etc.), which need to be investigated for damage detection.

Several investigators have studied the interaction between guided waves and damage analytically using normal-mode expansion and boundary-condition matching. Damage interaction coefficients were derived to quantify the guided wave transmission, reflection, mode conversion, and scatter at the damage site. Due to their mathematical complexity, these analytical solutions are restricted to simple damage geometries: notches, holes, and partially through holes. Extension to more complicated damage geometries has been attempted through series expansion of the rugged damage contour. In the generic case of arbitrary-shape damage, the numerical approaches using space discretization (FEM, BEM) are used due to their convenience, but on the expense of orders of magnitude increase in computational time and/or computer resources.

The design of a SHM system requires computationally efficient predictive tools that permit the exploration of a wide parameter space to identify the optimal combination between the transducers type, size, number, and guided wave characteristics (mode type, frequency, and wavelength) to achieve best detection and quantification of a certain damage type. Such parameter space exploration desiderate can be best achieved with analytical tools which are fast and efficient.

In this article, we describe an analytical approach based on the one-dimensional (1D) (straight crested) guided wave propagation analysis. In our study, we inserted the damage effect into the analytical model by considering wave transmission, reflection, mode conversion, and higher harmonics components described through damage interaction coefficients at the damage site. We do not attempt to derive these damage interaction coefficients here, but assume that they are available either from literature or from FEM, BEM analysis performed separately in a separate computational module. This analytical approach was coded into MATLAB and the WaveFormRevealer (WFR) graphical user interface (GUI) was developed. The WFR can generate fast predictions of waveforms resulting from Lamb wave interaction with damage for arbitrary positioning of PWAS transmitters and receivers with respect to damage and with respect to each other. The users may choose their own excitation signal, PWAS size, structural parameters, and damage description. The current version of the WFR code is limited to 1D (straight crested) guided wave propagation; extension of this approach to two-dimensional (2D) (circular crested) guided wave propagation will be attempted in the future.

**PWAS Fundamentals**

PWAS couple the electrical and mechanical effects (mechanical strain, $S_{ij}$; mechanical stress, $T_{ij}$; electrical field, $E_i$; and electrical displacement, $D_j$) through the tensorial piezoelectric constitutive equations

$$S_{ij} = s^p_{ijkl}T_{kl} + d_{ijkl}E_k$$

$$D_j = d_{ijkl}T_{kl} + e^T_{jk}E_k$$

where $s^p_{ijkl}$ is the mechanical compliance of the material measured at zero electric field ($E = 0$), $e^T_{jk}$ is the dielectric permittivity measured at zero mechanical stress ($T = 0$), and $d_{ijkl}$ represents the piezoelectric coupling effect. PWAS utilize the $d_{11}$ coupling between in-plane strains, $S_{11}$, $S_{22}$, and transverse electric field $E_3$.

PWAS transducers can be used as both transmitters and receivers. Their modes of operation are shown in Figure 1. PWAS can serve several purposes: (a) high-bandwidth strain sensors, (b) high-bandwidth wave exciters and receivers, (c) resonators, and (d) embedded modal sensors with the electromechanical (E/M) impedence method. By application types, PWAS transducers can be used for (a) active sensing of far-field damage using pulse-echo, pitch-catch, and phased-array methods, (b) active sensing of near-field damage using high-frequency E/M impedence method and thickness gage mode, and (c) passive sensing of damage-generating events through detection of low-velocity impacts and acoustic emission at the tip of advancing cracks (Figure 1). The main advantage of PWAS over conventional ultrasonic probes is in their small size, lightweight, low profile, and small cost. In spite of their small size, PWAS are able to replicate many of the functions performed by conventional ultrasonic probes.

**Analytical modeling of Lamb waves interacting with damage**

**Analytical modeling of guided Lamb waves propagation in a pristine structure**

One aspect of the difficulties in modeling Lamb wave propagation is due to their multi-mode feature. WFR is capable of modeling multi-mode Lamb wave propagation in structures. From Rayleigh–Lamb equation, it is found that the existence of certain Lamb mode depends on the plate thickness-frequency product. The fundamental S0 and A0 modes will always exist, but...
the higher modes will only appear beyond the cutoff frequencies. This section describes how an electrical tone burst applied to a transmitter PWAS (T-PWAS) transducer propagates through a structural waveguide to the receiver PWAS (R-PWAS) transducer in pitch-catch mode (Figure 2).

The propagation takes place through ultrasonic guided Lamb waves which are generated at the T-PWAS through piezoelectric transduction and then captured and converted back into electric signal at the R-PWAS. Since several Lamb wave modes traveling with different wave speeds exist simultaneously, the electrical tone-burst applied on the T-PWAS will generate several wave packets. These wave packets will travel independently through the waveguide and will arrive at different times at the R-PWAS where they are converted back into electric signals through piezoelectric transduction. The predictive analytical model for Lamb wave propagation between the T-PWAS and R-PWAS is constructed in frequency domain in the following steps (Figure 3(a)).

**Step 1.** Perform Fourier transform of the time-domain excitation signal $V_T(t)$ to obtain the frequency-domain excitation spectrum, $\tilde{V}_T(\omega)$. For a tone burst, the signal $V_T(t)$ and its Fourier transform $\tilde{V}_T(\omega)$ are shown in Figure 4.

**Step 2.** Calculate the frequency-domain structural transfer function $G(x_r, \omega)$ from T-PWAS to R-PWAS. The structure transfer function $G(x_r, \omega)$ is given in equation (99) in the study by Giurgiutiu, page 327, which gives the in-plane wave strain at the plate surface as

$$
\varepsilon_x(x, t) = -i \frac{\alpha \tau_0}{\mu} \left\{ \sum_{\xi} (\sin \xi a) \frac{N_S(\xi)}{D_S(\xi)} e^{-i(\xi a - \omega t)} + \sum_{\xi'} (\sin \xi' a) \frac{N_A(\xi')}{D_A(\xi')} e^{-i(\xi'a - \omega t)} \right\}
$$

(2)
where \( \xi \) is the frequency-dependent wavenumber of each Lamb wave mode and the superscripts \( S \) and \( A \) refer to symmetric and antisymmetric Lamb wave modes. If only the two fundamental modes, \( S_0 \) and \( A_0 \), are present, then \( G(x, \omega) \) can be written as

\[
G(x, \omega) = S(\omega)e^{-i\xi^S x} + A(\omega)e^{-i\xi^A x}
\]

\[
S(\omega) = \kappa_{PWAS} \sin \xi^S \frac{N_0(\xi^S)}{D_0(\xi^S)}
\]

\[
A(\omega) = \kappa_{PWAS} \sin \xi^A \frac{N_0(\xi^A)}{D_0(\xi^A)}
\]

\[
(3)
\]
where

\[ N_s(\xi) = \xi \beta \left( \xi^2 + \beta^2 \right) \cos \alpha d \cos \beta d; \]
\[ D_s = (\xi^2 - \beta^2)^2 \cos \alpha d \sin \beta d + 4 \xi^2 \alpha \beta \sin \alpha d \cos \beta d; \]
\[ N_A(\xi) = -\xi \beta \left( \xi^2 + \beta^2 \right) \sin \alpha d \sin \beta d; \]
\[ D_A = (\xi^2 - \beta^2)^2 \sin \alpha d \cos \beta d + 4 \xi^2 \alpha \beta \cos \alpha d \sin \beta d; \]

\[ \alpha^2 = \frac{\omega^2}{c_p^2} - \xi^2; \quad \beta^2 = \frac{\omega^2}{c_t^2} - \xi^2; \quad c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}; \]
\[ c_t = \sqrt{\frac{\mu}{\rho}}; \quad \kappa_{PWAS} = -i \frac{\alpha \tau_0}{\mu}; \]

(4)

\[ \kappa_{PWAS} \] is the complex transduction coefficient that converts applied voltage into guided wave strain at the T-PWAS, \( a \) is half length of PWAS size, and \( d \) is plate half thickness. The modal participation functions \( S(\omega) \) and \( A(\omega) \) determine the amplitudes of the \( S_0 \) and \( A_0 \) wave modes. The terms \( \sin(\xi^2 a) \) and \( \sin(\xi^4 a) \) control the tuning between the PWAS transducer and the Lamb waves. \( \lambda \) and \( \mu \) are Lamé’s constants of the structural material, and \( \rho \) is the material density. The wavenumber \( \xi \) of a specific mode for certain frequency \( \omega \) is calculated from Rayleigh–Lamb equation

\[ \tan \beta d = \left[ \frac{-4 \alpha \beta \xi^2}{(\xi^2 - \beta^2)^2} \right]^{\frac{1}{2}} \]

(5)

where \( +1 \) exponent corresponds to symmetric Lamb wave modes and \( -1 \) exponent corresponds to antisymmetric Lamb wave modes.

**Step 3.** Multiply the structural transfer function by frequency-domain excitation signal (Figure 4(b)) to obtain the frequency-domain signal at the R-PWAS, that is, \( \tilde{V}_R(x_r, \omega) = G(x_r, \omega) \cdot \tilde{V}_T(\omega) \). Hence, the wave arriving at the R-PWAS location is

\[ \tilde{V}_R(x_r, \omega) = S(\omega) \tilde{V}_T(\omega) e^{-i \xi x_r} + A(\omega) \tilde{V}_T(\omega) e^{-i \xi x_r}; \]

(6)

This signal in equation (6) can be decomposed into symmetric and antisymmetric components

\[ \tilde{V}_R^S(x_r, \omega) = S(\omega) \tilde{V}_T(\omega) e^{-i \xi x_r} \]

(7)

\[ \tilde{V}_R^A(x_r, \omega) = A(\omega) \tilde{V}_T(\omega) e^{-i \xi x_r} \]

(8)

**Step 4.** Perform the inverse Fourier transform to obtain the time-domain wave signal at the R-PWAS, that is

\[ V_R(x_r, t) = \text{IFT}\{ \tilde{V}_R(x_r, \omega) \} \]

(9)

Due to the multi-mode character of guided Lamb wave propagation, the received signal has at least two separate wave packets, \( S_0 \) and \( A_0 \) (Figure 5).

This analysis can be extended to include higher guided wave modes (\( S_1, A_1 \), etc.), that is

\[ V_R(x_r, t) = \text{IFT}\{ \tilde{V}_R(x_r, \omega) \} \]

(9)

**Figure 4.** Tone burst signal: (a) time domain and (b) frequency domain (From Giurgiutiu, p. 153).

**Figure 5.** (a) T-PWAS signal and (b) R-PWAS signal. T-PWAS: transmitter piezoelectric wafer active sensor; R-PWAS: receiver piezoelectric wafer active sensor.
At generation, we consider the damage as a new wave source picked up by the R-PWAS transducer at carry the damage information with them, and are structure transfer function up to the damage location, Step 2.

This step is identical to step 1 of the pristine case. Perform Fourier transform of the time-domain excitation signal $V_T(t)$ to obtain the frequency-domain excitation spectrum, $\tilde{V}_T(\omega)$.

Step 2. Calculate the frequency-domain structural transfer function up to the damage location, $G(x_d, \omega)$. The structure transfer function $G(x_d, \omega)$ is similar to equation (3) of previous section, only that $x = x_d$, that is

$$G(x_d, \omega) = S(\omega)e^{-i\omega x_d} + A(\omega)e^{-i\omega x_d}$$

Step 3. Multiply the structural transfer function by frequency-domain excitation signal to obtain the frequency-domain signal at the damage location, that is, $\tilde{V}_D(x_d, \omega) = G(x_d, \omega) \cdot \tilde{V}_T(\omega)$. Hence, the signal at the damage location is

$$\tilde{V}_D(x_d, \omega) = S(\omega)\tilde{V}_T(\omega)e^{-i\omega x_d} + A(\omega)\tilde{V}_T(\omega)e^{-i\omega x_d}$$

This signal could be decomposed into symmetric and antisymmetric components

$$\tilde{V}_D^S(x_d, \omega) = S(\omega)\tilde{V}_T(\omega)e^{-i\omega x_d}$$

$$\tilde{V}_D^A(x_d, \omega) = A(\omega)\tilde{V}_T(\omega)e^{-i\omega x_d}$$

Step 4. The wave signal at the damage location takes the damage information by considering transmission, reflection, mode conversion, and higher harmonics. Each of these addition phenomena is modeled as a new wave source at the damage location using damage interaction coefficients (Figure 7). We distinguish two damage interaction types: (a) linear and (b) nonlinear, as discussed next.

**Linear damage interaction.** Wave transmission, reflection, and mode conversion are realized by using complex-amplitude damage interaction coefficients. Our notations are as follows: we use three letters to describe the interaction phenomena, with the first letter denoting the incident wave type, the second letter standing for resulting wave type, and the third letter meaning propagation direction (transmission/reflection). For instance, symmetric-symmetric-transmission (SST) means the incident symmetric waves transmitted as symmetric waves, while symmetric-antisymmetric-transmission (SAT) means incident symmetric waves transmitted and mode converted to antisymmetric waves. Thus, the complex-amplitude damage interaction coefficient $C_{SST} \cdot e^{i\phi_{SST}}$ denotes the transmitted symmetric mode generated by incident symmetric mode with magnitude.

**Figure 6.** A pitch-catch configuration between a transmitter PWAS and a receiver PWAS. PWAS: receiver piezoelectric wafer active sensor:
Similarly, \( C_{\text{SST}} \) and phase \( \varphi_{\text{SST}} \) represent the transmitted antisymmetric mode generated by incident symmetric mode with magnitude \( C_{\text{SAT}} \) and phase \( \varphi_{\text{SAT}} \). These coefficients are determined by the features of the damage and are to be imported into the WFR model.

**Nonlinear damage interaction.** The center frequency of waves arriving at the damage location can be obtained from equations (13) and (14) as \( \omega_c \). The second and third higher harmonics act as wave sources with center frequencies of \( 2\omega_c \) and \( 3\omega_c \), respectively. Modeling of higher harmonics is achieved by moving the frequency-domain signal at the damage location to the right-hand side of the frequency axis by \( \omega_c \) and \( 2\omega_c \), that is, \( \tilde{V}_2D(x_d, \omega) = \tilde{V}_D(x_d, \omega - \omega_c) \) and \( \tilde{V}_3D(x_d, \omega) = \tilde{V}_D(x_d, \omega - 2\omega_c) \) represent the second and third higher harmonics nonlinear wave source.

The nonlinear damage interaction coefficients are defined in the same way as the linear ones. For instance, the complex-amplitude damage interaction coefficient \( C_{\text{SST}}^M \cdot e^{-i\varphi_{\text{SST}}^M} \) denotes the \( M \)th higher harmonics transmitted symmetric mode generated by incident symmetric mode with magnitude \( C_{\text{SST}}^M \) and phase \( \varphi_{\text{SST}}^M \).

**Step 5.** The guided waves from the new wave sources created at the damage location propagate through the rest of the structure and arrive at the R-PWAS. The received wave signal is calculated in frequency domain as

\[
\tilde{V}_R(x_d, x_r, \omega) = \sum_{M=1}^M \left[ C_{\text{SST}}^M e^{-i\varphi_{\text{SST}}^M} \cdot \tilde{V}_D(x_d, \omega) + C_{\text{SAT}}^M e^{-i\varphi_{\text{SAT}}^M} \cdot \tilde{V}_D(x_d, \omega) \right] e^{-i\omega_c^M (x_r - x_d)}
\]

where \( M \) is the number of higher harmonics considered. For linear interaction with damage, \( M \) equals to one.

**Step 6.** Perform inverse Fourier transform to obtain the time-domain receiver sensing signal

\[
V_R(x_d, x_r, t) = \text{IFFT}\{\tilde{V}_R(x_d, x_r, \omega)\}
\]

It should be noted that the above analysis only considers S0 and A0 modes. But the principle could be easily extended to higher modes (S1, A1, etc.). The difficulty with extending to higher modes will be on defining the increasing number of transmission, reflection, and mode conversion coefficients. For each excited Lamb mode, the interaction with damage may result in more mode conversion possibilities. In this study, the WFR has been designed to simulate (a) multi-mode (S0, A0, S1, A1) Lamb waves propagation in pristine plates and (b) fundamental modes (S0 and A0) Lamb wave interaction with damage.

**WFR interface and main functions**

The analytical representation of this process was coded in MATLAB and resulted in the GUI called WFR shown in Figure 8.
WFR allows users to control several parameters: structure material properties, PWAS size, location of sensors, location of damage, damage type (linear/non-linear damage of various severities), and excitation signal (frequencies, count numbers, signal mode excitation, arbitrary waveform type, etc.). Dual display of waveforms allows for the sensing signals to be shown at two different sensor locations. For instance, Figure 8 shows two receiver waveforms at locations \( x_1 = 0 \) mm and \( x_2 = 500 \) mm as measured from the transmitter (in this case \( x_1 = 0 \) mm means that R-PWAS-1 collocated with the T-PWAS). Thus, PWAS-1 shows the reflections from damage, and PWAS-2 shows the signal modified after passing through the damage. Users are able to conduct fast parametric studies with WFR. It may take several hours for commercial finite element software to obtain an acceptable-accuracy solution for high-frequency, long distance propagating waves, but it takes only several seconds to obtain the same solution with WFR. Besides analytical waveform solutions, the WFR can also provide users with wave speed dispersion curves, tuning curves, frequency components of received wave packets, structure transfer function, and so on. All the calculated results are fully available to the user, and could be saved in Excel files by clicking the “SAVE” button. Figure 9 shows a case study for Lamb wave propagation of a 100 kHz tone burst in a 1-mm-thick aluminum plate. Figure 9(a) shows the dispersion curves; Figure 9(b) shows the excitation spectrum overlap with the S0 and A0 tuning curves. Figure 9(c) shows the spectra of the S0 and A0 packets displaying frequency shifts to the right and to the left, respectively, due to the interaction between excitation spectrum and the tuning curves. Figure 9(d) shows the structure transfer function \( G(x, \omega) \).

Besides the main interface, WFR has two subinterfaces shown in Figure 10: (a) damage information platform and (b) guided wave spatial propagation solver. The damage information platform allows users to input the damage location and damage interaction coefficients. For example, SST represents the magnitude of transmitted S0 mode generated by an incoming
S0 mode, whereas SAT and phi-SAT represent the magnitude and phase of the transmitted A0 mode resulting from the mode conversion of an incoming S0 mode. The values of these damage interaction coefficients are not calculated by the WFR. This gives the users the freedom to define their own specific problem. For instance, a particular type of damage (plastic zone, fatigue, cracks) with certain degree of severity will have different interaction characteristics with the interrogating guided waves. These coefficients may be determined experimentally or calculated through other methods (analytical, FEM, BEM, etc.). Among all the above methods, FEM approach shows good results for obtaining the interaction coefficients of arbitrary shaped damage. Successful examples and details can be found in Velichko and Wilcox20,21 and Moreau et al.22 In an example presented later in this study, we used a trial-and-error approach to tune the WFR coefficients to the data obtained from experiments and finite element simulations.

The spatial propagation solver is like a B-scan. Using the analytical procedure, we obtain the time-domain waveform solution at various locations along the structure. Thus, the time-domain waveform solutions of a sequence of points along the wave propagation path are obtained. If we select the sequence of solution points fine enough, a time-spatial domain solution of the wave field is obtained. The spatial solution of wave field at a particular instance in time is available as shown in Figure 10(b). After the time-spatial solution of wave field is obtained, we can do the frequency–wavenumber analysis23 to see the wave components of the signal (Figure 11). These will be illustrated in the case studies discussed later in this article.

Case studies

Linear interaction with damage of selective Lamb wave modes

WFR allows users to select single mode (S0 and A0) or multi-mode (S0 and A0) to be excited into the structure. Three test cases were conducted: (a) incident S0 wave linear interaction with damage, (b) incident A0 wave linear interaction with damage, and (c) combined S0 and A0 waves linear interaction with damage. The test case setup is shown in Figure 12. The T-PWAS and R-PWAS are placed 600 mm away from each other on a 1-mm-thick aluminum 2024-T3 plate. The damage is
placed 200 mm from the T-PWAS. A 5-count Hanning window modulated tone burst centered at 100 kHz is used as the excitation. The time-domain and the time–frequency domain signals of the test cases are shown in Figure 13.

Figure 13 shows that new wave packets appear due to the interaction between interrogation Lamb waves and damage. Incident S0 wave will generate A0 wave from mode conversion at the damage, whereas incident A0 wave will generate S0 wave from mode conversion at the damage. However, from the time-frequency analysis, it could be observed that after linear interaction, the frequency spectrum of the waves still center around the excitation frequency 100 kHz.

Nonlinear interaction with damage of selective Lamb wave modes

As test cases for nonlinear interaction between Lamb waves and damage, three simulations were carried out:

Figure 10. User interfaces: (a) damage information platform and (b) guided wave spatial propagation solver.

Figure 11. Frequency–wavenumber display window.
(a) incident S0 wave nonlinear interaction with damage, (b) incident A0 wave nonlinear interaction with damage, and (c) combined S0 and A0 waves nonlinear interaction with damage. The test case setup is the same as shown in Figure 12, only the interaction with damage is nonlinear. The time signals and the time-frequency analysis of the test cases are shown in Figure 14.

It can be observed in Figure 14 that after nonlinear interaction with the damage, the waveforms become distorted and contain distinctive nonlinear higher harmonics. For S0 waves which are less dispersive at the given frequency range, the nonlinear higher harmonics stay inside the wave packet. However, for A0 waves which are dispersive at the given frequency range, the higher harmonic components travel faster, leading the way and may escape from the fundamental wave packet.

**Experimental verifications**

**Multi-mode Lamb wave propagation in a pristine plate**

In our study, two PWAS transducers were mounted on a 3.17-mm-thick aluminum 7075-T6 plate. Figure 15 shows the experiment setup. The T-PWAS sends out ultrasonic guided waves into the structure. The guided waves, that is, Lamb waves propagate in the plate,
undergoing dispersion and are picked up by the R-PWAS. The Lamb waves are multi-modal, hence several wave packets appear in the received signal. Agilent 33120A Arbitrary Waveform Generator is used to generate 3-count Hann window modulated tone burst excitations. A Tektronix Digital Oscilloscope is used to record the experimental waveforms. The excitation frequency is increased from 300 to 600 kHz.

Corresponding plate material, thickness, PWAS size, and sensing location information is input into the WFR. The analytical waveforms of various frequencies are obtained. Figure 16 shows the comparison between analytical solution from WFR and experimental data. It can be observed that at 300 kHz, only S0 and A0 modes exist. The WFR solution matches well with experimental data. At 450 kHz, S0 mode becomes more dispersive; besides S0 and A0 modes, A1 mode starts to pick up with highly dispersive feature. At 600 kHz, S0, A0, and A1 modes exist simultaneously. The simulation results and the experimental data have slight differences due to the fact that 1D analytical formulas and pin force excitation assumptions are used in this study. To further validate WFR predictions, we also conducted 2D FEM simulation with pin force excitation (1D Lamb wave propagation simulation). Figure 17 shows the comparison between WFR and FEM simulations. It can be observed that the 300 and 450 kHz waveforms match very well between WFR and FEM. Signals of 600 kHz also have reasonably good agreement. It should be noted, even for 1D Lamb wave propagation simulation, that the 600 kHz wave computation requires considerably small element size and time marching step. The FEM simulation for such high-frequency, short-wavelength situation is becoming prohibitive due to the heavy consumption of computation time and computer resources. On the contrary, WFR only requires several seconds to obtain the same results due to its highly efficient analytical formulation.

The guided wave spatial propagation solver in WFR is used to obtain the time–space wave field (B-scan) as shown in Figure 18(a). The frequency–wavenumber analysis is conducted next, as shown in Figure 18(b).
The 600 kHz case is used as an example. From the B-scan, S0, A0, and A1 wave components can be observed. Frequency–wavenumber analysis gives very clear information on the wave mode components of the wave field. Transmitted S0 wave (S0-T), A0 wave (A0-T), and A1 wave (A1-T) are clearly noticed in Figure 18(b).

**Linear interaction between Lamb waves and damage**

**Pitch-catch mode.** Figure 19 shows the experimental specimen (3.17-mm-thick Aluminum-7075-T6 plate), with PWAS #3 used as the transmitter (T-PWAS) and PWAS #4 used as the receiver (R-PWAS). A notch \((h_1 = 2.5 \text{ mm}, d_1 = 0.25 \text{ mm})\) is machined on the plate, 143.5 mm from the T-PWAS. The wave propagation path from T-PWAS to R-PWAS is 303 mm. The 3-count Hanning window modulated tone burst signals with center frequencies varying from 150 to 300 kHz are used as the excitation.

S0 and A0 waves are transmitted by the T-PWAS. At the notch, S0 waves will be transmitted as S0 waves and also will be mode converted to transmitted A0 waves. A0 waves will be transmitted as A0 waves and also will be mode converted to transmitted S0 waves. All these transmitted waves will propagate along the rest of the structure and be picked up by the R-PWAS. The damage interaction coefficients are physically determined by the size, severity, type of the damage. In this study, we used a trial-and-error approach to tune the WFR damage interaction coefficients to the data obtained from the experiments. The adjusted damage interaction coefficients which gave best match with experiments for 150 kHz excitation case are shown in Table 1.

Figure 20 shows the WFR simulation results compared with experiments. It can be noticed that the analytical waveforms agree well with experimental data. A new wave packet is generated due to mode conversion at the notch.

**Pulse-echo mode.** Figure 21 shows the experimental setup for pulse-echo active sensing method. The same specimen is used, with an R-PWAS bounded side by
side to the T-PWAS. The 3-count Hanning window modulated tone burst signals with the center frequency of 95.5 kHz is used as the excitation. Guided Lamb waves generated by the T-PWAS will propagate into the structure, reach the notch, and be reflected back as echoes. At the notch, S0 waves will be reflected as S0 waves.
**Figure 19.** Experiment for Lamb wave linear interaction with a notch (pitch-catch mode).
T-PWAS: transmitter piezoelectric wafer active sensor; R-PWAS: receiver piezoelectric wafer active sensor.

**Figure 20.** Comparison between WFR simulations and experiments for Lamb wave interaction with a notch in pitch-catch mode. WFR: WaveFormRevealer.
waves and also will be mode converted to reflected A0 waves. A0 waves will be reflected as A0 waves and also will be mode converted to reflected S0 waves. All the echoes will reach the R-PWAS and be picked up.

The adjusted damage interaction coefficients which gave best match with the experiment are shown in Table 2.

Figure 22 shows the WFR simulation result compared with the experiment. The reflected S0 and A0 wave packets could be observed. The new waves between S0 and A0 wave packets are from mode conversion at the notch. The analytical simulation matches the experiment data. Differences are noticed: first, the direct waves have a phase shift due to the fact that the R-PWAS and T-PWAS are some distance away from each other, while in our analytical model, we consider them to be at the same location; second, the boundary reflections are present and mixed with the weak echoes from the notch in the experiment, but in our model, the boundary reflections are not considered.

Figure 23 shows the results from WFR spatial propagation solver. The wave transmission, reflection, and mode conversion can be clearly noticed in both the B-scan and frequency–wavenumber analysis. It is apparent that the wave field contains transmitted S0 and A0 modes, and reflected S0 and A0 modes.

Nonlinear interaction between Lamb waves and damage

A guided wave pitch-catch method may be used to interrogate a plate with a breathing crack which opens and closes under tension and compression.6,24 The ultrasonic waves generated by the T-PWAS propagate into the structure, interact with the breathing crack, acquire nonlinear features, and are picked up by the R-PWAS. This process is shown in Figure 24. The nonlinear interaction between Lamb waves and the breathing crack will introduce nonlinear higher harmonics into the interrogation waves. A multi-physics transient finite element model was used to simulate the Lamb wave interaction with a nonlinear breathing crack. The damage interaction coefficients obtained from fitting the FEM solution (Table 3) were input into the WFR simulator.

Figure 25 shows the comparison between FEM and the WFR analytical solution. It is noticed that the FEM results and the analytical solution agree very well because the damage interaction coefficients were fitted to the FEM solution. The time-domain waveforms show nonlinear characteristics of noticeable nonlinear

Figure 22. Comparison between WFR simulations and experiments for Lamb wave interaction with a notch in pulse-echo mode. WFR: WaveFormRevealer.
distortion in S0 packet and zigzags in the new packet. The frequency spectrums show distinctive nonlinear higher harmonics (200 and 300 kHz). Since we only consider up to the third higher harmonic in this case study, the frequency domain of analytical solution shows only the first three peaks, while the finite element solution have even higher harmonics. But the solution up to the third higher harmonics is accurate enough to render an acceptable waveform in time domain.

The guided wave spatial propagation solver in WFR was used to obtain the time–space wave field. Figure 26 shows the time–space wave field and frequency–wavenumber analysis of Lamb wave interaction with nonlinear breathing crack.

Table 3. Nonlinear interaction coefficients.

<table>
<thead>
<tr>
<th>Magnitude coefficient</th>
<th>$C_{1S}$</th>
<th>$C_{2S}$</th>
<th>$C_{1A}$</th>
<th>$C_{2A}$</th>
<th>$C_{3S}$</th>
<th>$C_{3A}$</th>
<th>$C_{1T}$</th>
<th>$C_{2T}$</th>
<th>$C_{3T}$</th>
<th>$C_{1R}$</th>
<th>$C_{2R}$</th>
<th>$C_{3R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value (normalized)</td>
<td>0.900</td>
<td>0.420</td>
<td>0.820</td>
<td>0.100</td>
<td>0.082</td>
<td>0.100</td>
<td>0.050</td>
<td>0.110</td>
<td>0.032</td>
<td>0.082</td>
<td>0.100</td>
<td>0.025</td>
</tr>
<tr>
<td>Phase coefficient</td>
<td>$\phi_{1S}$</td>
<td>$\phi_{1A}$</td>
<td>$\phi_{1T}$</td>
<td>$\phi_{1R}$</td>
<td>$\phi_{2S}$</td>
<td>$\phi_{2A}$</td>
<td>$\phi_{2T}$</td>
<td>$\phi_{2R}$</td>
<td>$\phi_{3S}$</td>
<td>$\phi_{3A}$</td>
<td>$\phi_{3T}$</td>
<td>$\phi_{3R}$</td>
</tr>
<tr>
<td>Value ($^\circ$)</td>
<td>0</td>
<td>100</td>
<td>-35</td>
<td>90</td>
<td>0</td>
<td>120</td>
<td>90</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Transmission, reflection, and mode conversion phenomena at the damage can be clearly noticed. The frequency–wavenumber analysis reveals the wave components during the interaction process. The wave field contains transmitted S0 and A0 waves and reflected S0 and A0 waves. Nonlinear higher harmonics can be observed at 200 kHz.

The WFR-guided wave spatial propagation solver can provide the spatial wave pattern at any instance of time. The spatial waveforms at 0, 25, 50, 75, 100, 125, 150, 175, and 200 $\mu$s are displayed in Figure 27. The spatial waveforms shows (a) Lamb waves propagating into the structure at $T = 25 \mu$s, (b) Lamb modes separating into distinct packets at $T = 50 \mu$s, (c) Lamb wave
Figure 25. (a) Comparison between finite element simulation (FEM) and analytical simulation (WFR), (b) frequency spectrum of S0 packet, (c) frequency spectrum of new packet and (d) frequency spectrum of A0 packet.

WFR: WaveFormRevealer; FEM: finite element method.

Figure 26. (a) Time–space wave field (B-scan) and (b) frequency–wavenumber analysis from WFR.

WFR: WaveFormRevealer.
Figure 27. Spatial wave propagation of Lamb wave interaction with breathing crack (calculated using WFR). WFR: WaveFormRevealer.
packets interaction with the damage also at $T = 50 \mu s$, and (d) wave transmission, reflection, mode conversion, and nonlinear distortion of waveforms at various instances ($T = 75, 100, 125, 150$, and $200 \mu s$).

**Summary, conclusions, and future work**

**Summary**

In this study, we presented the WFR—an analytical framework and predictive tool for the simulation of guided Lamb wave interaction with damage. The theory of inserting damage effects into the analytical model was addressed, including wave transmission, reflection, mode conversion, and nonlinear harmonics components. The analytical model was coded into MATLAB, and the WFR GUI was developed to obtain fast predictive waveforms for arbitrary combinations of sensors, structural properties, and damage. Main functions of WFR were introduced, including the calculation for dispersion curves, tuning curves, frequency spectrum of sensing signal, plate transfer function, time–space domain waveforms with damage effects, frequency–wavenumber analysis, and the capability of considering arbitrary user defined excitation signals. Test cases were carried out. Experimental verifications were presented. The predictive solution from WFR agreed well with experiments and finite element simulations. WFR can be downloaded from: http://www.me.sc.edu/Research/lamss/html/software.html.

**Conclusion**

The WFR was capable of calculating dispersion curves, tuning curves, frequency components of wave packets, and structural transfer function. It could be used to obtain time–space domain waveforms with damage effects and frequency–wavenumber analysis. WFR could provide fast predictive solutions for multi-mode Lamb wave propagation and interaction with linear/nonlinear damage. The solutions compared well with experiments and finite element simulations. It was also found that computational time savings of several orders of magnitude are obtained by using the analytical model WFR instead of FEM methods. WFR allowed users to conduct fast parametric studies with their own designed materials, geometries, and excitation.

**Future work**

Rational methods for determining damage interaction coefficient values need to be found (not trial and error). Work should be carried out to extend the analysis to 2D wave propagation (three-dimensional (3D) FEM and 2D WFR). The 2D WFR with damping effect should be built to simulate wave attenuation in waveguides. Boundary reflection and damage effects in 2D wave propagation should be investigated. Attempts for simulating guided wave propagation in composite structures should be made using WFR.

**Declaration of conflicting interests**

The authors declare that there is no conflict of interest.

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**References**