Numerical and experimental investigation of damage severity estimation using Lamb wave–based imaging methods

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Abstract
In this article, estimation of crack size, shape, and orientation was investigated numerically and experimentally using Lamb waves. A hybrid global–local approach was used in conjunction with the imaging methods for the numerical simulation. The hybrid global–local approach allowed fast and efficient prediction of scattering wave signals for Lamb wave interaction with crack from various incident directions. The simulation results showed the directionality effect of the scattering wave signals and suggested an optimum transmitter–sensor configuration. Two imaging methods were used: one involves the synthetic time reversal concept and the other involves Gaussian distribution function. Both imaging methods show very good agreement during simulations. Experiments were designed and conducted based on the simulated results. A network of eight piezoelectric wafer active sensors was used to capture the scattering waves from the crack. Both the pitch-catch and pulse-echo experimental modes were used. The directionality effect of incident Lamb waves on the imaging results was studied. The effect of summation, multiplication, and combined algorithms for each imaging method was studied. It was found that both methods can successfully predict the crack size and orientation. An attempt was made to use these imaging methods for detecting and sizing smaller sized damage (1- to 3-mm-diameter hole). It was found that these methods can successfully localize the hole, but size estimation was a bit challenging because of the smaller dimensions. The scattering waves for various hole sizes were studied.

Keywords
Structural health monitoring, wave propagation, synthetic time reversal, crack sizing, scattering waves

Introduction
State of the art
In recent years, the damage quantification using Lamb waves has become one of the topical research areas. Lamb waves are suitable for fast damage detection, because they propagate at very high speeds and may complete the inspection in a very short period of time. They can propagate a long distance with very little energy loss, so they enable the inspection of large areas of structures. The analyses of Lamb wave interaction could potentially detect, localize, and estimate the size of various kinds of damage in structures (Chang et al., 2007; Masserey and Fromme, 2015; Saravanan et al., 2015). An experimental study was performed to obtain an appropriate Lamb wave mode to detect structural defects (Ghosh et al., 1998).

Attempts have been made for quantifying structural defects using Lamb wave scattering field (Baghalian et al., 2017; He et al., 2016). A network of sensors was used for defect visualization in the pitch-catch experimental mode (Ihn and Chang, 2008). Damage index was calculated for each sensing path to characterize the defect. Crack orientation was quantitatively determined using scattered Lamb wave and by evaluating the different amplitudes of energy peaks in the Hilbert spectra (Lu et al., 2007). A system of an electromagnetic acoustic transducer (EMAT) array was designed for detecting artificial defects in large metallic structures using symmetric Lamb wave mode (Wilcox et al., 2005). The phased array filter approach was developed for damage

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defect imaging in a large isotropic plate using Lamb wave (Kwon and Kim, 2016; Purekar et al., 2004). A synthetic time reversal method was developed for imaging the structural damage using a network of sensors (Wang et al., 2004). Defects with larger dimensions were used for imaging and a threshold value was set for defect imaging.

Local wavenumber analysis approach was used for damage quantification in isotropic or composite plates (Mesnil et al., 2014; Tian et al., 2013). A transverse crack in a metallic beam was quantified by a signal processing algorithm in the time–frequency domain based on the wavelet transform technique (Su et al., 2003). This technique suppresses the diverse broadband interferences and effectively extracts useful damage information. However, damage quantification in a one-dimensional structure was considered. Numerical methods are used for impact damage localization using several networks of sensors. The results showed that the errors of impact damage localization decreased with increasing the number of sensors (Migot and Giurgiuțiu, 2017). Manufacturing defect in a composite was identified using wavefield imaging (Juarez and Leckey, 2016). It had been shown that the time of flight (TOF) of a Lamb wave pulse can be used for damage detection in a composite (Kessler et al., 2002). An imaging method based on correlation analysis called “RAPID” (reconstruction algorithm for probabilistic inspection of defects) was used to inspect corrosion in a wing skin (Hettler et al., 2015). A network of lead zirconium titanate (PZT) sensors and the integration of the RAPID algorithm were used to detect and localize damages in thermoplastic matrix composite plates (Azuara et al., 2018). An integrated methodology has been developed to detect an impact event on a composite plate using a coarse network of sensors. Impact localization and impactor feature estimation were performed based on the dynamic response signals (Theodosiou et al., 2018).

The Lamb wave scattering from a simulated damage in a thick steel beam was considered using both the pitch-catch and pulse-echo experimental modes (Sun et al., 2009). The interaction of Lamb waves with a rivet hole crack was studied both theoretically and experimentally (Bhuian et al., 2017; Fromme and Sayir, 2002). The studies showed that the crack in the hole significantly changes the scattered wavefield. Fatigue crack and defects in adhesive bonding were identified by analyzing the scattering Lamb wave (He et al., 2017; Tashakori et al., 2018). Crack length estimation in rivet holes was attempted using training datasets (He et al., 2013). However, only a few number of features such as correlation coefficient, signal’s amplitude, and phase were considered. A network of PZT sensors and wavenumber analysis were used for crack identification and crack imaging (Lu et al., 2006; Yu et al., 2015). Ultrasonic imaging approach was used for predicting the envelope of scattering waves (Ebrahimkhanlou et al., 2016). The damage was simulated by placing magnets. The tomographic imaging technique was used for quantifying the corrosion damage in a hole (Wang et al., 2004). The iterative algebraic reconstruction technique was utilized for generating the imaging results.

The scope of this study

This article presents the numerical and experimental investigations to estimate the crack size using Lamb wave–based imaging methods. A hybrid global–local approach was used for fast and efficient numerical simulations. Two imaging methods were used: (1) a synthetic time reversal concept and (2) a Gaussian distribution function. Each imaging method was investigated with the scattering waves from the pitch-catch and pulse-echo experimental modes. The effect of summation, multiplication, and combined algorithms was studied. These imaging methods were also attempted to perform localization and size estimation for various small size holes (1 to 3 mm diameter).

Principles of the imaging methods

Two imaging methods were implemented using the scattered wave signals. The first method involved synthetic time reversal to determine the field values of each pixel using the scattering signal’s amplitude. This method is hereafter referred to as “method A.” The flowchart of method A is illustrated in Figure 1. The concept of the synthetic time reversal method was first presented by Wang et al. (2004). In this study, summation, multiplication, and combined algorithms were used to visualize the damage in conjunction with the synthetic time reversal concept.

The second method involved the Gaussian distribution function and scattering signals. This method is hereafter referred to as “method B.” The flowchart of method B is illustrated in Figure 2. In this method, at first, the scattering wave signals were determined using the pitch-catch and/or pulse-echo experimental mode/s. The interested area was divided into pixels. The TOF of every pixel was determined using the equation of ellipse/circle. The pitch-catch experimental mode involved elliptical orbit and the TOF can be determined using equation (1)

$$t_{ij} = \frac{\sqrt{(x_T - x_i)^2 + (y_T - y_i)^2}}{v_g} + \frac{\sqrt{(x_i - x_R)^2 + (y_i - y_R)^2}}{v_g}$$

where $t_{ij}$ is the TOF of the scattered signal; $x_T$, $y_T$, $x_R$, $y_R$, and $x_i$, $y_i$ are the coordinates of the transmitter sensor, receiver sensor, and pixel, respectively; and $v_g$ is the group velocities of Lamb wave.
mode. When the pixels lay on the damage orbit of a particular sensing path, then \( t_{ij} = t_d \) (\( t_d \) is the TOF for damage; Figure 3).

The pulse-echo experimental mode involved the circular orbit and the TOF can be determined using equation (2). This orbit has one sensor in its center which works as both a transmitter and a receiver at the same time

\[
t_{ij} = \frac{2\sqrt{(x_i - x_{TR})^2 + (y_j - y_{TR})^2}}{v_g}
\]  

(2)
The field value of each pixel was determined using the Gaussian distribution function (Su et al., 2009). The equation of the Gaussian distribution function is shown below

\[
f_{ij}^k(x, y) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-x_i)^2 + (y-y_j)^2}{2\sigma^2}}
\]

where \(f_{ij}^k(x, y)\) is the field value of every pixel \(P(x, y)\) of the image of a particular sensing path \(T_m - R_n\) (transmitter \(T_m\) and receiver \(R_n\)), \(x\) represents the TOF at each pixel point \(t_{ij}\) for the sensing path \(T_m - R_n\), and \(\mu\) represents the TOF of the scattering wave packet for the sensing path \(T_m - R_n\) which can be determined experimentally. The standard deviation, \(\sigma\), describes the variability or dispersion of a data set, which was taken as half the time range of the wave packet. \(m\) and \(n\) represent the indexes of the transmitters and receivers, respectively, which vary from 1 to the total numbers of transmitters \((N_T)\) and receivers \((N_R)\), such as \(m = 1, 2, 3, \ldots, N_T\) and \(n = 1, 2, 3, \ldots, N_R\).

To fuse all the images of different sensing paths, summation and/or multiplication algorithms were used (Michaels and Michaels, 2007; Wang et al., 2004) following the equations below

\[
P_{\text{sum}}(x, y) = \sum_{k=1}^{N} f_{ij}^k(x, y)
\]

\[
P_{\text{mult}}(x, y) = \prod_{k=1}^{N} f_{ij}^k(x, y)
\]

where \(P_{\text{sum}}(x, y)\) and \(P_{\text{mult}}(x, y)\) are the total field values of each pixel point using the summation or multiplication algorithm and \(N\) represents the total number of sensing paths, that is

\[
N = \frac{N_T!}{2!(N_T - 2)!}
\]

**Numerical investigation of crack size estimation**

In the past, many efforts have been made to develop analytical or numerical models of wave propagation and interaction with damages (Moreau et al., 2012; Wang and Chang, 2005). Numerical and experimental studies were adopted to analyze a crack at the interface of a structure subjected to pure shear loading and ultrasonic loading. The results show the scattered waves generated by the interaction of elastic waves with the crack (Hafezi and Kundu, 2017). The local interaction simulation approach (LISA) was developed to study the nonlinear interaction of guided waves with fatigue cracks. The results show the nonlinear higher harmonics and mode conversion phenomena during the wave interaction with crack (Shen and Cesnik, 2017; Shen et al., 2018). In this article, a hybrid global–local (HGL) simulation approach (Shen and Giurgiutiu, 2016) has been used for the numerical study. A schematic illustration of the HGL simulation is shown in Figure 4. A set of eight piezoelectric wafer active sensor (PWAS) transducers were used numbered from 1 to 8. Each of them was located at an equal distance of 150 mm from the center of the crack. The crack length was 18 mm. The Lamb waves were generated from the transmitter PWAS. The transmitter PWASs (T1, T2, T3) were located at 0°, 45°, and 90° to the crack. For each transmitter, the rest of the PWAS transducers acted as receivers. The directionality of incident Lamb wave and scattered wave was studied.

The HGL approach involved local finite element method (FEM) simulation of the damage site accompanied with an exact analytical solution outside the damage region. The dimensions of the local FEM models were 100 mm × 100 mm × 1.6 mm. A nonreflective boundary (NRB) was modeled using spring-damper elements surrounding the plate edges as in Shen and Giurgiutiu (2015) to minimize the edge-reflected waves. The overview of the entire HGL process is illustrated in Figure 5(a) following Shen and Giurgiutiu (2016). The wave damage interaction coefficients (WDICs) were obtained from the local FEM analysis and then inserted into the global analytical framework. The WDICs were obtained for three separate FEM models (0°, 45°, and 90° crack orientation) as shown in Figure 5(b) to (d). The FEM model parameters were chosen based on our previous work as in Bhuiyan et al. (2017).

The WDIC is the measure of Lamb wave interaction with the crack and is a complex-valued parameter. The WDIC polar plots for all three incident directions of Lamb waves are shown in Figure 6. Both amplitude and phase of WDIC show that the scattered waves from the crack have the directionality effect. For example,
when Lamb waves were incident at 45° to the crack, the maximum scattering amplitude occurred at 135° to the crack. This means that a receiver sensor would pick up stronger scattering signals when it would be installed in this direction. It can also be noticed that the maximum WDIC occurs when the Lamb waves were incident perpendicular (90°) to the crack. For the 0° Lamb wave incident, weak WDIC occurred. This study of directionality effect suggested using the optimum transmitter and receiver combination for the imaging method as discussed later.

The scattering waveforms for the 0°, 45°, and 90° incident Lamb wave (transmitters T1, T2, T3) are illustrated in Figure 7(a) to (c). The strongest scattering signal was observed for path 2-4 (transmitter, T2, receiver, R4), which corresponds to the 45° incident Lamb wave and scattering wave received at 135°, which is expected based on the WDIC profile (Figure 6(b)). The scattering waveform for path 3-3 (transmitter, T3, receiver, R3) also shows the stronger scattering signals, which corresponds to the 90° incident Lamb wave and scattering wave received at 90°. Note that WDICs of the scattered S0 Lamb wave mode for the incident S0 Lamb wave (S0_S0) are plotted in Figure 7. The scattered shear horizontal (SH0) mode does exist for the incident S0 Lamb wave, but is not shown here.

For the imaging methods, the sensing signals at receivers for two incident directions (45° and 90°) were used. The imaging results of the simulated signals are illustrated in Figure 8. The combined summation and multiplication algorithm was adopted to produce better images of the crack. It can be observed that both imaging methods predict the crack size to be 16 mm, whereas the simulated crack size was 18 mm. The simulated results also show the horizontal orientation of the crack. It should be noted that during the simulation the scattering signals were not affected by any confounding factors (sensor bonding, environmental noise, electrical...
connections, temperature–humidity, etc.) commonly encountered in practice. Hence, an experimental investigation was also performed as discussed later.

**Experimental investigation of crack detection and size estimation**

**Experimental setup**

A large aluminum plate-like specimen with the dimensions of 1220 mm × 1220 mm × 1.6 mm was used. A sub-inspection area of 610 mm × 610 mm was instrumented with an active network of eight PWAS transducers as shown in Figure 9. This sub-inspection area contained a very narrow slit which was manufactured using a 0.25-mm thin dental cutting disk. This is a simulated crack; for convenience, we will call it hereafter as “crack.” The crack length was about 10 mm and oriented horizontally. The zoomed-in view of the crack is shown in the inset of Figure 9. The center of the horizontal crack was located at (303 mm, 300 mm) with the left bottom corner of the sub-area as the origin. The sensor network circumscribed the crack at the center. The PWAS transducers of the network were distributed with equal radial and angular distances. The diameter of the circular network was 300 mm. The diameter of each PWAS transducer was 7 mm and the thickness was 0.2 mm.

A function generator was used to generate the three-count tone burst excitation signal at a center frequency of 450 kHz. This frequency was chosen based on the tuning curve of PWAS. At this frequency–thickness product, the symmetric Lamb wave (S0) was dominant in this plate specimen. The same PWAS transducer can be used as a transmitter and a receiver of Lamb wave signals. An oscilloscope recorded the received Lamb wave signals. The specimen edges of the sub-area were covered with damping clay to minimize the edge-reflected wave signals. Two experimental modes were used: (1) pitch-catch and (2) pulse-echo modes. In pitch-catch modes, each PWAS transducer acted as a transmitter, whereas the rest of them in the network acted as receivers. In the pulse-echo mode, each PWAS transducer served as a transmitter and the same PWAS

*Figure 6. WDIC directivity plot (amplitude and phase) at 450 kHz for the three different directions (0°, 45°, 90°) of the incident Lamb wave (LW) (“S0_S0” means scattered S0 Lamb waves for the incident S0 Lamb wave): (a) 0° LW incidence, (b) 45° LW incidence, and (c) 90° LW incidence.*
acted as a receiver. In both experimental modes, the imaging methods were performed for crack detection and quantification as discussed next.

Scattering waveforms for the pitch-catch mode

First, the waveform results of the pitch-catch experiment are presented. For demonstration, only one set of signals is shown in Figure 10. In this set, PWAS #3 was the transmitter and the rest of the seven PWAS transducers were the receivers. Figure 10(a) shows a set of baseline signals which were recorded in the beginning without any crack in the circular network. Figure 10(b) shows a set of measured signals with a 10-mm crack in the circular network. In these signals, the first wave packets are S0 Lamb waves. These S0 wave packets are stronger than the A0 wave packet as expected at 450 kHz frequency. The path as indicated in Figure 10 represents a pair of transmitter and receiver. For each transmitter, there are seven possible paths for which the wave signals can be received by the receivers.

There are some additional scattered wave packets due to the crack in the specimen. These scattered wave packets are plotted in Figure 11. The scattered wave signals (scattered S0) were determined by the subtraction of baseline signals from the signals with 10-mm crack. This entire process was repeated for all possible sets of transmitter and receivers. The scattered signals from all sets of transmitter and receiver were used for imaging methods to quantify the crack size.

Imaging results for the pitch-catch mode

The direction of incident Lamb waves with respect to the crack significantly affects the imaging results. The crack localization imaging results of method A (with summation algorithm) for one transmitter are illustrated in Figure 12. The pixel with the highest field value (the brightest pixel) indicates the crack location. Three incident directions were considered. For 0° incidence of the S0 Lamb wave, the imaging result is very poor since the brightest pixel appeared near the transmitter (Figure 12(a)). This is because the scattering signals are very weak for 0° incidence. For 45° incidence of the S0 Lamb wave, the imaging result (Figure 12(b)) is fair since the scattering signals are relatively stronger. For 90° incidence of the S0 Lamb wave (the incident wave was perpendicular to the crack orientation), the imaging result (Figure 12(c)) is good since the scattered signals are the strongest of these three cases. Similar results were obtained for method B; they are not repeated here for the sake of brevity. One transmitter may not be sufficient to estimate the crack size, but it may predict the location of the crack when it is placed in the best location with respect to the crack.

For estimating the crack size, method A (with summation algorithm) imaging was used with a set of two transmitters (for each transmitter, the rest of the seven PWAS transducers were the receivers). The imaging results are illustrated in Figure 13. The crack tips can be identified using the sensing paths of two transmitters. Two dots with maximum field value (index value) can be obtained which represent the two crack tips. Figure 13(a) shows the imaging result after setting a threshold value (about 80% of maximum field value). This image was obtained using the scattering wave signals of all the sensing paths of the transmitter set PWAS #2 and #3. From the zoomed-in image, two strong spots can be identified. The distance between the two spots is 9 mm which is close to the crack length. Also, it can be noted that the crack is oriented horizontally as seen from the imaging result.

The similar procedure was repeated for the transmitter set PWAS #2 and #6. In this case, both transmitters...
are oriented at 45° to the crack. Based on the imaging results shown in Figure 13(b), the crack length can be estimated to be 8 mm. This predicted crack length deviates from the actual crack length since the scattering wave signals were relatively weaker for the 45° incident Lamb wave. When PWAS #2 and #7 were used as the transmitters, the predicted crack length was 12 mm as shown in Figure 13(c).

The crack size can also be determined using a combined summation and multiplication algorithm for determining the total field values of pixels. The advantage of this method is that no threshold setting is required. This combined algorithm was used with both method A and method B. The imaging results of both methods are illustrated in Figure 14. A set of two transmitters (PWAS #2 and #6) were used for this purpose. All seven sensing paths were used for each transmitter.

First, the summation algorithm was applied to generate the image for an individual transmitter. Then, the multiplication algorithm was applied to these images to obtain the final image as shown in Figure 14. Hence, two images were fused providing a clean image by this combined algorithm.

Figure 14(a) shows the imaging results of method A which predicts the crack length of 8 mm. Figure 14(b) shows the imaging results of method B which predicts the crack length of 12 mm. Both methods show that the crack is oriented horizontally.

Scattered waveforms and imaging results for the pulse-echo mode

In the pulse-echo experimental mode, each PWAS transducer acted simultaneously as both a transmitter
and a receiver. The S0 Lamb waves originated from a PWAS, hit the crack, and scattered back to the same PWAS. The pulse-echo signals were recorded by the oscilloscope for all the sensors (PWAS #1, ..., #8). The waveform signals are shown in Figure 15. It can be observed that path 3-3 and path 7-7 have the strongest scattering waveforms since they are directly perpendicular to the crack. The path 2-2, path 4-4, path 6-6, and path 8-8 have relatively weaker signals since they are at an oblique (45°/C176) incidence. These signals appear to have a similar amplitude as expected. However, path 1-1 and path 5-5 have very weak scattering signals since they are parallel to the crack.

To estimate the crack size, the pulse-echo waveform signals were fed into the two imaging methods (methods A and B). In both methods, the summation algorithm without any threshold setting was used to determine the total field value at a pixel. The imaging results using method A and method B are illustrated in Figure 16(a) and (b), respectively. It shows that the sensing paths were circular in shape. The intersecting sensing paths resulted in a horizontal band of bright dots where the field values were concentrated. This is because the scattering waves originated from this region.

From the zoomed-in image, two intensive red spots can be identified which represent the crack tips. According to method A, the crack length can be estimated as 9 mm. According to method B, the crack length can be estimated as 12 mm. In both methods, the crack orientation appears to be horizontal. However, method A shows many pixels with a similar level of field values on both sides of the two highest field values along the crack length (which appear as a band of red pixels along the entire crack length). On the other hand, method B shows concentrated bright pixels at the two crack tips.

In both experimental modes (pitch-catch and pulse-echo), it appears that method A slightly underestimates the actual crack length, whereas method B slightly overestimates the actual crack length.

**Through-thickness hole detection and size estimation**

The Lamb wave–based imaging methods were used to detect through-thickness hole with various diameters. The pitch-catch experiments were conducted in a...
Experimental setup

An aluminum plate-like specimen with the dimensions of 600 mm × 600 mm × 1.6 mm was used in this experiment (Figure 17). A 1-mm through-thickness hole was manufactured at a location with the coordinates (315 mm, 249 mm), while the bottom left corner was the origin. The specimen was instrumented with six PWAS transducers (PWAS #0, ..., #5) as shown in Figure 17. These transducers were bonded at arbitrary locations around the hole. A function generator was used to generate the three-count tone burst signals at a center frequency of 450 kHz and an oscilloscope was used to record the signals. At this frequency–thickness product, the symmetric Lamb wave was dominant in the aluminum plate. The baseline signals were recorded for all possible sensing paths corresponding to the 1-mm hole. The hole size was enlarged gradually: starting from 1 mm to 1.5, 2, 2.5, and 3 mm. The waveform signals were measured for each hole size at all possible sensing paths.

Imaging results for various hole sizes

Method A was used for hole localization and size estimation. Both summation and multiplication algorithms were used. The imaging results for the three hole sizes (2, 2.5, and 3 mm diameter) are presented in Figure 21. The imaging results of the summation algorithm are shown on the left side and those of the multiplication algorithm are shown on the right side.
algorithm are shown on the right side of Figure 21. The results of the multiplication algorithm are shown as a zoomed-in view for clarity. The sensing paths of four transmitters (PWAS #0, #2, #3, #4) were used to

**Figure 12.** Directionality effect on the imaging results of method A: (a) 0° incident (transmitter PWAS #1), (b) 45° incident (transmitter PWAS #2), and (c) 90° incident (transmitter PWAS #3).
Figure 13. Estimation of crack size based on method A imaging results using the sensing paths of two transmitters: (a) PWAS #2 and #3, (b) PWAS #2 and #6, and (c) PWAS #2 and #7. “T” indicates the transmitter. A threshold value (80% of maximum field value) was set to obtain these images.
produce these images. These transmitters were chosen since they produced relatively stronger scattering waves.

From Figure 21, it can be observed that as the hole size increases the imaging result gets better. Because the larger hole size (3mm diameter) produced stronger scattering waves, it has better imaging result. The hole can be localized based on the highest field value of pixels. For a larger hole, the brighter pixels become more concentrated and provide accurate localization. For example, the imaging localization result predicts the hole to be located at (315 mm, 248 mm), whereas the actual location of the hole is at (315 mm, 249 mm). The multiplication algorithm provided similar results as the summation algorithm. The only difference is that the multiplication algorithm provides a cleaner image than the summation algorithm.

The multiplication algorithm results in Figure 21 show brighter pixels with a circular shape. These circular brighter pixels suggest that the damage is a point source or circular in shape. However, in this case, the size of the hole is difficult to predict since the hole size is very small. It may need further investigation to determine the smaller size holes. However, the imaging localization result predicts the hole to be located at (315 mm, 248 mm), whereas the actual location of the hole is at (315 mm, 249 mm). The multiplication algorithm provided similar results as the summation algorithm. The only difference is that the multiplication algorithm provides a cleaner image than the summation algorithm.

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Figure 16. Crack size estimation for the pulse-echo experimental mode using (a) method A and (b) method B (both methods used the summation algorithm; no thresholding applied).

Figure 17. An instrumented aluminum plate with a through-thickness hole. The hole size was enlarged gradually: starting from 1 mm to 1.5, 2, 2.5, and 3 mm.
methods could successfully localize the smaller hole. Method B also predicted similar results as method A and is not repeated for the sake of brevity.

**Summary, conclusion, and suggestions for future work**

**Summary**

This article briefly explained the principles of two imaging methods by presenting two flowcharts. Numerical simulations were performed with the HGL approach and imaging methods (methods A and B). Method A involves the synthetic time reversal concept and method B involves Gaussian distribution function. Several local FEM simulations were performed for various Lamb wave incidences (0°, 45°, and 90° to the crack). The directionality effect and imaging results were studied. Two experimental investigations of two types of damage situations were considered: (1) crack and (2) through-thickness hole. The first experiment involved a sparse array of eight PWAS transducers to quantify the crack size using the imaging methods. The pitch-catch and pulse-echo experimental modes were considered for each imaging method. The scattering waveform and imaging results were presented for both methods in each experimental mode. The second experiment involved a network of sparse array of six PWAS transducers.
Figure 21. Imaging results using the summation (left side) and multiplication (right side) algorithms for various hole sizes: (a) 2 mm, (b) 2.5 mm and (c) 3 mm diameter.
transducers to quantify the hole size using the same imaging methods. The pitch-catch experimental mode was used to obtain the scattering signals. The relation between the hole size and the scattering signals was studied.

Conclusion

The HGL approach with imaging methods provided a fast and efficient simulation of crack size estimation. The polar plot of the simulated WDICs revealed the directionality effect of the scattered wave signals emanated from the crack. Both the incident and sensing directions of Lamb waves affect the scattering wave signals which eventually affect the quality of imaging results. The flowcharts of two imaging methods were developed. The simulations and experiments show that both imaging methods successfully quantify the crack size, shape, and orientation within an acceptable level of accuracy. The experiment involved confounding factors that affect the measurement of scattering wave signals. These scattering signals play an important role in crack quantification. Stronger scattering waves result in better imaging results. The optimum sensor configuration results in better scattering signals which eventually give better damage quantification. For the pitch-catch mode, the sensing paths of two transmitters in optimum locations may be enough to quantify the crack size. The synthetic time reversal–based imaging method (method A) can be used with and without threshold setting. The use of the combined summation and multiplication algorithm may eliminate the need of thresholding for better crack imaging. Both the pitch-catch and pulse-echo modes show that the scattering waves are stronger when incident waves are perpendicular to the crack. In both experimental modes, the Gaussian distribution–based imaging method (method B) seems to overestimate the crack size. In an attempt to estimate a smaller (1–3 mm) through-thickness hole size, it was found that a smaller crack produces relatively weaker scattering waves. These scattering waves can be used for hole detection and localization but may not be sufficient to quantify the hole size.

Future work

These imaging methods may be used with passive acoustic emission (AE) signals to quantify the AE source. The experiment involving various hole diameters can be extended further. The diameter of the hole may be increased gradually to a size where one can use these imaging methods to quantify the hole size. These methods may be used for sizing of actual fatigue crack. It would be interesting to study how small fatigue crack can be sized using these methods. The study could be extended for estimating the butterfly crack size in rivet holes.

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