Effect of structural damping on the tuning between piezoelectric wafer active sensors and Lamb waves

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Abstract
Piezoelectric wafer active sensors have been widely used for Lamb-wave generation and acquisition. For selective preferential excitation of a certain Lamb-wave mode and rejection of other modes, the piezoelectric wafer active sensor size and the excitation frequency should be tuned. However, structural damping depends on the structure material and the excitation frequency and it will affect the amplitude response of piezoelectric wafer active sensor–excited Lamb waves in the structure, that is, tuning curves. Its influence on the piezoelectric wafer active sensor tuning reflects the effect of structural health monitoring configuration considered in the excitation. Therefore, it is important to have knowledge about the effect of structural damping on the tuning between piezoelectric wafer active sensor and Lamb waves. In this article, the analytical tuning solution of undamped media is extended to damped materials using the Kelvin–Voigt damping model, in which a complex Young’s modulus is utilized to include the effect of structural damping as an improvement over existing models. This extension is particularly relevant for the structural health monitoring applications on high-loss materials, such as metallic materials with viscoelastic coatings and fiber-reinforced polymer composites. The effects of structural damping on the piezoelectric wafer active sensor tuning are successfully captured by the improved model, with experimental validations on an aluminum plate with adhesive films on both sides and a quasi-isotropic woven composite plate using circular piezoelectric wafer active sensor transducers.

Keywords
Structural health monitoring, Lamb waves, piezoelectric wafer active sensors, tuning, structural damping, Kelvin–Voigt damping model

Introduction
Structural health monitoring (SHM) is an emerging interdisciplinary research area, which has shown great potential in aerospace, civil, mechanical, and naval structures (Boller et al., 2009; He and Yuan, 2017; Qiu et al., 2013, 2014; Raghavan and Cesnik, 2007; Sohn, 2007; Su et al., 2006; Yuan et al., 2016). Among the SHM technologies, Lamb wave, a type of ultrasonic guided waves propagating between two parallel surfaces without much energy loss, is suitable for the inspection of large areas of complicated structures (Achenbach, 1973; Graff, 1975; Rose, 1999). It has gained increasing attention from SHM communities in recent decades (Mitra and Gopalakrishnan, 2016).

Piezoelectric wafer active sensors (PWAS) are convenient enablers for generating and receiving Lamb waves in structures for SHM applications (Giurgiutiu et al., 2002). Compared with conventional transducers, PWAS are low profile, lightweight, low-cost, and unobtrusive to structures (Giurgiutiu, 2014). They can be permanently bonded on the host structures in large quantities and achieve real-time monitoring of the structural health status. PWAS can actively interrogate structures using a variety of Lamb-wave methods such as pitch-catch, pulse-echo, phased arrays, and electromechanical (E/M) impedance technique (Giurgiutiu, 2014).

Tuning between PWAS and Lamb waves in a plate
Under electric excitation, PWAS undergo oscillatory contractions and expansions that are transferred to the structure through the bonding layer to excite Lamb waves in structures. In this process, several factors...
influence the behavior of the excited Lamb waves: thickness of the bonding layer, geometry of the PWAS, and thickness and material of the structure. The result of the influence of all these factors is the tuning between PWAS and Lamb waves (Giurgiutiu, 2005). The tuning is especially beneficial when dealing with multimode waves, such as Lamb waves. The gist of the concept is that manipulation of the PWAS size and the excitation frequency allows for the selective preferential excitation of a certain Lamb-wave mode and the rejection of other modes, as needed by particular Lamb-wave-based SHM applications.

The analytical model of PWAS-generated Lamb waves and its tuning effect on isotropic metallic plates has been well investigated (Giurgiutiu, 2005; Raghavan and Cesnik, 2005; Santoni-Bottai et al., 2007; Santoni-Bottai and Giurgiutiu, 2012; Sohn and Lee, 2009). Giurgiutiu (2005) first developed the theory of the interaction of a rectangular PWAS with one-dimensional (1D) propagation, that is, straight crested Lamb waves, and presented a closed-form solution based on trigonometric functions. Raghavan and Cesnik (2005) extended tuning concepts to the case of a circular transducer coupled with two-dimensional (2D) propagation, that is, circular crested Lamb waves, and proposed the corresponding tuning prediction formula based on Bessel functions. Sohn and Lee (2009) proposed a calibration technique to address the discrepancy between experimental and theoretical tuning curves of Lamb waves by taking into account the bonding layer and the energy distribution between various modes. Santoni-Bottai et al. (2007) performed theoretical studies and experimental measurements of the Lamb-wave tuning and found that the capability to excite only one desired wave mode based on the tuning is crucial for the embedded ultrasonic structural radar. To improve the model of tuning, an exact shear-lag solution of Lamb wave tuning with PWAS was derived from first principles using the normal mode expansion (NME) method (Santoni-Bottai and Giurgiutiu, 2012). A similar tuning effect is also possible in anisotropic composite plates. However, the theoretical analysis is more complicated due to the anisotropic characteristics of the wave propagation in composite materials. Giurgiutiu and Santoni-Bottai (2011) derived the theoretical predictions of tuning using NME and the dispersion curves of Lamb waves in composite plates. These analytical developments of tuning facilitate the understanding of PWAS-coupled Lamb waves for SHM applications. However, these solutions only apply to the tuning between PWAS and Lamb waves without considering the effect of structural damping.

Effect of structural damping on PWAS tuning

In composite materials, the structural damping plays a major role that cannot be ignored. Gresil and Giurgiutiu (2015) performed PWAS tuning experiments on composite plates and found that higher frequency tuning seems virtually impossible because the Lamb waves strongly diminish above approximately 600 kHz for the quasi-50 mode and approximately 200 kHz for the quasi-40 mode. The structural damping depends on the structure material and the excitation frequency and it will affect the amplitude response of PWAS-excited Lamb waves in structures, that is, Lamb-wave tuning curves. Its influence on the tuning reflects the effect of SHM configuration considered in the excitation. Therefore, it is important to have knowledge about the effect of structural damping on the tuning between PWAS and Lamb waves.

To model structural damping, a damping coefficient is usually considered in a complex formulation of the material Young’s modulus, where the real part (storage modulus) corresponds to the elastic behavior and the imaginary part (loss modulus) corresponds to the dissipative behavior (Christensen, 1982; Graesser and Wong, 1992). This is known as the correspondence principle (Auld, 1973). Several researchers have studied the damping models to predict the attenuation behavior of Lamb waves in composite plates, including hysteretic damping model (Bartoli et al., 2006; Neau, 2003), Kelvin–Voigt damping model (Bartoli et al., 2006; Neau, 2003; Shen and Cesnik, 2016), and Rayleigh damping model (Gresil and Giurgiutiu, 2015; Ramadas et al., 2011). All these research studies indicate that damping models have the capability to capture the effect of structural damping.

Overview of current paper

The main novelty of this article is to investigate the effect of structural damping on the tuning between PWAS and Lamb waves. This study extends the analytical solutions of tuning to damped materials using the Kelvin–Voigt damping model, in which a complex Young’s modulus is utilized to include the effect of structural damping. This extension is particularly relevant for SHM applications on high-loss materials, such as metallic materials with viscoelastic coatings (Mu and Rose, 2008) and fiber-reinforced polymer composites (Lin et al., 2016; Sreekumar et al., 2015). The proposed method is validated on an aluminum plate with adhesive films on both sides and a quasi-isotropic woven composite plate using circular PWAS transducers. It will be shown in this article that structural damping can modify the tuning effect considerably, both through peak shifts and through drastic diminishing of the amplitude at higher frequencies. This article contains analytical developments and experimental measurements. The concepts of this article are applicable to both metals (e.g. metallic plates with viscoelastic coatings) and composites.
Mathematical framework for straight and circular crested Lamb-wave tuning with PWAS transducers

**PWAS basis**

PWAS are the enabling technology for active SHM systems (Giurgiutiu, 2014). PWAS couple the electrical and mechanical effects (mechanical strain, $S_{ij}$; mechanical stress, $T_{kl}$; electrical field, $E_k$; and electrical displacement, $D_j$) through the following tensorial piezoelectric constitutive equations

\[ S_{ij} = s^{E}_{ijkl} T_{kl} + d_{ijkl} E_k \]  
\[ D_j = d_{ijkl} T_{kl} + \varepsilon_{ijk} E_k \]

where $s^{E}_{ijkl}$ is the mechanical compliance of the material measured at zero electric field ($E = 0$), $\varepsilon_{ijk}$ is the dielectric permittivity measured at zero mechanical stress ($T = 0$), and $d_{ijkl}$ represents the piezoelectric coupling effect. PWAS utilize the $d_{31}$ coupling between in-plane strains, $S_1$, $S_2$, and transverse electric field, $E_3$. Just like conventional ultrasonic transducers, PWAS utilize the piezoelectric effect to generate and receive ultrasonic waves. However, PWAS are different from conventional ultrasonic transducers in several aspects (Giurgiutiu, 2014): PWAS are firmly coupled with the host structure through an adhesive bonding, whereas conventional ultrasonic transducers are weakly coupled through the water, gel, or air. PWAS are non-resonant devices that can be tuned selectively into several Lamb-wave modes, whereas conventional ultrasonic transducers are resonant narrow-band devices. PWAS are inexpensive and can be deployed in large quantities on structures, whereas conventional ultrasonic transducers are expensive and used one at a time.

Depending on application types, PWAS transducers can serve several purposes (Giurgiutiu, 2014): (a) active sensing of far-field damage using pulse-echo, pitch-catch, and phased-array methods; (b) active sensing of near-field damage using high-frequency electromechanical impedance spectroscopy (EMIS) and thickness gage mode; and (c) passive sensing of damage-generating events through detection of low-velocity impacts and acoustic emission at the tip of advancing cracks. Figure 1 shows the schematic of PWAS application modes.

**Straight and circular crested Lamb-wave tuning without damping**

The analytical model of PWAS-generated Lamb waves and its tuning effect in undamped isotropic plates have been well understood. In this section, straight and circular crested Lamb-wave tuning without damping was discussed.

**Straight crested Lamb-wave tuning without damping**

The straight crested Lamb-wave tuning without damping was solved by applying the Fourier transform and the symmetric and antisymmetric boundary conditions (Giurgiutiu, 2005). In the case of ideal bonding, the...
shear stress in the bonding layer is concentrated at the ends and the pin-force model is used. The closed form is given by Equation (108) of Giurgiutiu (2014: 595), which gives the in-plane wave strain at the plate surface as

\[ e_a(x, t)_{z-d} = -i \frac{\alpha \tau_a}{\mu} \sum_{j-0}^j \left( \sin \xi_j a \right) \frac{N_j(\xi_j)}{D_j(\xi_j)} e^{i(\xi_j x - \omega t)} \]

\[ -i \frac{\alpha \tau_a}{\mu} \sum_{j-0}^j \left( \sin \xi_j a \right) \frac{N_j(\xi_j)}{D_j(\xi_j)} e^{i(\xi_j x - \omega t)} \]

(3)

where

\[ N_j(\xi) = \xi \beta (\xi^2 + \beta^2) \cos \alpha d \cos \beta d; \]

\[ D_j = (\xi^2 - \beta^2)^2 \cos \alpha d \sin \beta d + 4 \xi^2 \alpha \beta \sin \alpha d \cos \beta d \]

\[ N_A(\xi) = -\xi \beta (\xi^2 + \beta^2) \sin \alpha d \sin \beta d; \]

\[ D_A = (\xi^2 - \beta^2)^2 \sin \alpha d \cos \beta d + 4 \xi^2 \alpha \beta \cos \alpha d \sin \beta d \]

\[ \alpha = \frac{\omega^2}{c_p^2} - \xi^2; \beta = \frac{\omega^2}{c_s^2} - \xi^2; \]

\[ c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}}; \lambda = \frac{\nu}{(1+\nu)(1-2\nu)} E; \mu = \frac{1}{2(1+\nu)} E \]

where \( a \) is the half length of the PWAS size; \( d \) is the half thickness of the plate; \( c_p \) and \( c_s \) are the longitudinal (pressure) and transverse (shear) wave speeds, respectively; \( \lambda \) and \( \mu \) are Lamé constants of the structure material; and \( E, \nu, \) and \( \rho \) are Young’s modulus, Poisson’s ratio, and material density of the structure, respectively. The terms \( \sin(\xi_j a) \) and \( \sin(\xi_j a) \) control the tuning between the PWAS and Lamb waves. Superscripts or subscripts \( S \) and \( A \) signify the symmetric and antisymmetric Lamb-wave modes, respectively. The expressions \( D_S(\xi) \) and \( D_A(\xi) \) are derivatives of \( D_S \) and \( D_A \) with respect to \( \xi \) evaluated at the corresponding \( \xi^2 \) and \( \xi^4 \) poles. The wavenumber \( \xi \) of a specific Lamb-wave mode at a given angular frequency \( \omega \) is obtained from the solutions of the Rayleigh–Lamb equation

\[ \frac{\tan \beta d}{\tan \alpha d} = \left( \frac{-4 \alpha \beta \xi^2}{(\xi^2 - \beta^2)^2} \right)^{\pm 1} \]

(4)

where \( +1 \) exponent corresponds to symmetric Lamb-wave modes and \( -1 \) exponent corresponds to antisymmetric Lamb-wave modes.

The expressions in equation (3) can also be used for less-than-ideal bonding conditions in which the shear stress \( \tau \) varies with \( x \) as \( \tau(x) \) by replacing \( \tau_a \) and \( a \) with their effective values \( \tau_e \) and \( a_e \). At low frequencies, only two propagating Lamb-wave modes exist, \( S_0 \) and \( A_0 \), and general solutions of equation (3) have only two terms, that is

\[ e_a(x, t)_{z-d} = -i \frac{\alpha \tau_a}{\mu} \left( \sin \xi_0^s a \right) \frac{N_0(\xi_0^s)}{D_0(\xi_0^s)} e^{i(\xi_0^s x - \omega t)} \]

\[ -i \frac{\alpha \tau_a}{\mu} \left( \sin \xi_0^g a \right) \frac{N_0(\xi_0^g)}{D_0(\xi_0^g)} e^{i(\xi_0^g x - \omega t)} \]

Circular crested Lamb-wave tuning without damping. The circular crested Lamb-wave tuning without damping was derived based on Bessel functions. In the case of ideal bonding of a circular PWAS transducer that expands and contracts radially, the shear stress in the bonding layer is concentrated on the PWAS outer contour in the form of radially acting horizontal tractions. The closed form of the in-plane wave strain at the plate surface is given by Equation (166) of Giurgiutiu (2014: 607)

\[ e_r(r, t)_{z-d} = -i \frac{\alpha \tau_a}{\mu} \left( \sin \xi_0^r a \right) \frac{N_0(\xi_0^r)}{D_0(\xi_0^r)} \left( \frac{\xi_0^r H_0^1(\xi_0^r r) - H_1^1(\xi_0^r r)}{r} \right) \]

(6)

where \( a \) is the radius of the circular PWAS transducer and \( r \) is the distance between the point of interest and the transmitter PWAS. \( \tau_a \) represents shear stress between the transducer and the host structure. \( \lambda \) and \( \mu \) are Lamé constants of the structure material. \( J_1 \) is the Bessel function of order one, which captures the tuning effect between PWAS and the host structure. \( H_0^1 \) and \( H_1^1 \) are the first-kind Hankel function of order one and zero, respectively, which represent an outward propagating 2D wave field. \( \xi \) is the frequency-dependent wavenumber calculated from the Rayleigh–Lamb equation in equation (4). The solutions in equation (6) can also be used for less-than-ideal bonding conditions, in which the shear stress \( \tau \) varies with \( r \) as \( \tau(r) \) by replacing \( \tau_a \) and \( a \) with their effective values \( \tau_e \) and \( a_e \). At low frequencies, only two propagating Lamb-wave modes exist, \( S_0 \) and \( A_0 \), and the solutions of equation (6) have only two terms, that is

\[ e_a(r, t)_{z-d} = -i \frac{\alpha \tau_a}{\mu} \left( \sin \xi_0^s a \right) \frac{N_0(\xi_0^s)}{D_0(\xi_0^s)} \left( \frac{\xi_0^s H_0^1(\xi_0^s r) - H_1^1(\xi_0^s r)}{r} \right) \]

\[ -i \frac{\alpha \tau_a}{\mu} \left( \sin \xi_0^g a \right) \frac{N_0(\xi_0^g)}{D_0(\xi_0^g)} \left( \frac{\xi_0^g H_0^1(\xi_0^g r) - H_1^1(\xi_0^g r)}{r} \right) \]
Damping effects on Lamb-wave tuning with PWAS transducers

This section discussed damping effects on Lamb-wave tuning with PWAS transducers. Structural damping depends on the structure material and the excitation frequency and it will affect the amplitude response of PWAS-excited Lamb waves in structures, that is, tuning curves. Its effect on the Lamb-wave tuning with PWAS transducers reflects the influence of the SHM configuration considered in the excitation. Hence, it is important to have an understanding of the effect of structural damping.

Damping models for Lamb-wave propagation

The mathematical approach to predict the structural damping that attempts to model the deformation and flow of the viscoelastic material is known as the science of rheology. There are two commonly used models: Maxwell damping model, where a spring is in series with a dashpot, and Kelvin–Voigt damping model, where a spring is in parallel with a dashpot (Graesser and Wong, 1992). In this article, the Kelvin–Voigt damping model based on the frequency-dependent damping coefficient, which is more applicable to the frequency-dependent attenuation of Lamb waves (Sreekumar et al., 2015), is adopted to investigate the effect of structural damping on the tuning between PWAS and Lamb waves.

The Kelvin–Voigt damping model is presented in Figure 2, in which the stress acting on the body is assumed to be proportional to the strain and its time derivative (the strain rate)

\[ \sigma = E \varepsilon + \dot{\gamma} \varepsilon \]  

For wave propagation, the time-dependent behavior of stress and strain is assumed to be harmonic \( e^{-i\omega t} \), given by the expressions

\[ \sigma = \sigma_0 e^{i\xi x - i\omega t}; \varepsilon = \varepsilon_0 e^{i\xi x - i\omega t} \]  

where \( \xi \) is the wavenumber and \( \omega \) is the angular frequency. Substituting equation (9) into equation (8) yields

\[ \sigma_0 = (E - i\gamma \omega)\varepsilon_0 = E^* \varepsilon_0 \]  

where \( E^* \) is referred to the complex Young’s modulus

\[ E^* = E - i\gamma \omega \]  

Define the storage modulus, \( E' \), corresponding to the elastic behavior and the loss modulus, \( E'' \), corresponding to the dissipative behavior as

\[ E' = E; \quad E'' = \gamma \omega \]  

Figure 2. The Kelvin–Voigt damping model.

Substitution of equation (12) into equation (11) yields

\[ E^* = E' - iE'' \]  

The damping coefficient \( \eta \) can be introduced as

\[ \eta = \frac{E''}{E'} \]  

Hence, the complex Young’s modulus of equation (13) can be rewritten as

\[ E^* = E' (1 - i\eta) \]  

From the derivation of the complex Young’s modulus for the Kelvin–Voigt damping model, it can be found that the damping coefficient \( \eta \) is linearly dependent on frequency. In the literature (Castaings and Hosten, 2000; Neau, 2003), the reference damping coefficient \( \eta_0 \) was measured at a reference frequency \( f_0 \) corresponding to the frequency of the transducer used in conventional ultrasonic interferometry. In our study, the reference frequency \( f_0 \) is initially chosen at the upper limit of the frequency range (\( f_{0\,initial} = 2 \text{ MHz} \)) and the damping coefficient \( \eta_0 \) is initially taken as 5\% (\( \eta_{0\,initial} = 5\% \)). Then, the reference frequency \( f_0 \) and the damping coefficient \( \eta_0 \) are modified by repeated updating of the model to achieve an acceptable match with experimental tuning curves. Finally, the complex Young’s modulus \( E^* \) at a generic frequency, \( f \), can be obtained by scaling the damping coefficient at the reference frequency \( f_0 \)

\[ E^* = E' \left(1 - i\eta_0 \frac{f}{f_0}\right) \]  

The Kelvin–Voigt damping model results in a small attenuation below \( f_0 \) and a larger attenuation above \( f_0 \). The complex Young’s modulus in equation (16) can be substituted in equations (5) and (7) to modify the analytical solutions of tuning by introducing the effect of structural damping.

1D tuning of straight crested Lamb waves with damping

In this study, we focus our attention on fundamental Lamb-wave modes (\( S_0 \) and \( A_0 \)), which find the widest
application in Lamb-wave-based SHM applications. However, higher modes (S1, A1, S2, A2, etc.) in the practical application may also exist. The analytical framework can be easily extended to consider these modes under the same principle. Equation (5) has been coded into MATLAB, which provides the tuning curves of the structures for a given PWAS length, material properties, material thickness, and frequency range.

A quantitative study of the damping effect on Lamb-wave tuning using three various damping coefficients $\eta_0 (0, 1\%, 10\%)$ was conducted. In the simulation, the reference frequency $f_0$ is fixed to 2 MHz, which is the upper limit of the frequency range. First, 1D tuning curves without damping ($\eta_0 = 0$) of a 9 mm PWAS and a 1.016 mm 2024-T3 aluminum alloy plate are shown in Figure 3. It can be noted that the normalized strain curves follow the general pattern of a sine function. As indicated in Figure 3, the high-frequency response has a relatively large amplitude, compared with the amplitude at low frequencies. However, it is not true for the materials with damping. To improve the prediction by including damping effect, the Kelvin–Voigt damping model is utilized to investigate damping effects using the complex Young’s modulus with various damping coefficients.

Figure 4 presents 1D tuning curves with small damping ($\eta_0 = 1\%$) using the Kelvin–Voigt damping model. It can be noticed that the amplitudes at higher frequencies are relatively diminished by introducing the structural damping, whereas the low-frequency response almost remains the same, which is in agreement with theoretical predictions of the Kelvin–Voigt damping model. To investigate the larger damping effect, the damping coefficient is increased from 1% to 10%. The corresponding 1D tuning curves with large damping ($\eta_0 = 10\%$) are shown in Figure 5. It should be noted that the amplitudes at higher frequencies are significantly reduced compared with the results of small damping. At the same time, the low-frequency response still has a large amplitude. It can be concluded that the structural damping can considerably modify 1D tuning curves at higher frequencies, and the damping model can capture the effect of structural damping.

The attenuation phenomenon of Lamb waves not only depends on the damping coefficient but also depends on the propagation distance. A numerical study was performed to investigate the effect of propagation distance on Lamb-wave tuning. The 1D tuning curves at three various propagation distances (100, 120, and 140 mm) were calculated using our proposed model. Figure 6 shows the corresponding 1D tuning curves with the damping coefficient of 10% and reference frequency of 2 MHz. It can be noted that the amplitude of the tuning curves decreases slightly with the propagation distance.

### 2D tuning of circular crested Lamb waves with damping

In this section, fundamental Lamb-wave modes (S0 and A0) of the circular PWAS were employed to investigate
2D tuning of circular crested Lamb waves with damping. Equation (7) has been coded into MATLAB, which provides the tuning curves of the structures for a given PWAS length, material properties, material thickness, and frequency range.

First, 2D tuning curves without damping of a 9-mm-diameter circular PWAS and a 1.016 mm 2024-T3 aluminum alloy plate are presented in Figure 7. It can be noted that the normalized strain curves follow a similar pattern in 1D case, and the amplitudes at higher frequencies do not reduce significantly, which is not true for the materials with damping.

To investigate the damping effect, the Kelvin–Voigt damping model is utilized to calculate the 2D tuning curves of damped materials using different damping coefficients. Figure 8 presents 2D tuning curves with small damping (η₀ = 1%) of a 9-mm-diameter circular PWAS and a 1.016-mm 2024-T3 aluminum alloy plate. It should be noticed that the high-frequency response is reduced, to some extent, by the structural damping, whereas the amplitude at lower frequencies almost remains the same.

Similarly, the damping coefficient is increased from 1% to 10% to study the larger damping effect on Lamb-wave tuning. The corresponding 2D tuning curves with large damping (η₀ = 10%) are shown in Figure 9. It can be observed that the response at higher frequencies is strongly reduced compared with the results of small damping, whereas the low-frequency
response remains a large amplitude. It can be concluded that the structural damping can significantly modify the amplitudes at higher frequencies, and its effect on 2D tuning curves can be predicted using the Kelvin–Voigt damping model.

To investigate the effect of the propagation distance, 2D tuning curves at three various propagation distances (100, 120, and 140 mm) were calculated using our proposed method, as shown in Figure 10. It can be observed that the amplitude decreases obviously with the propagation distance. This is because the wave energy contained in the wavefront is distributed over a large radius and the wave amplitude decreases. In a plate, the wave amplitude decreases inversely with the square root of the propagation distance.

**Validation of Lamb-wave tuning with PWAS transducers in the presence of structural damping**

In this section, we present and discuss experimental results used to validate the numerical predictions. The study will demonstrate that the Kelvin–Voigt damping model is capable of capturing the effect of structural damping on Lamb-wave tuning with PWAS transducers for metallic plates and composite plates by comparing with experiments.

**Damping effects on Lamb-wave tuning for aluminum plates**

**Experimental setup.** To investigate damping effects on Lamb-wave tuning for metallic plates, pitch-catch Lamb-wave experiments on an aluminum plate were performed, as shown in Figure 11. The specimen is a 2024-T3 aluminum alloy plate with corrosion-resistant coatings and adhesive films on both sides. Figure 11(a) presents the thin polyvinyl chloride (PVC) film, which is utilized to protect the plate surface by the manufacturer. In this experiment, it plays as a viscoelastic material to increase the structural damping of the plate. The plate dimension is 600 × 600 × 1.016 mm³. Two 9-mm-diameter circular PWAS transducers (STEMINC SM412, 9-mm-diameter and 0.5 mm thick)
were installed on the front side of the plate to perform Lamb-wave tuning analysis, as shown in Figure 11(b). The experimental acquisition was performed between the transmitter PWAS (T-PWAS) and the receiver PWAS (R-PWAS). The frequency range from 10 to 2000 kHz in steps of 10 kHz was explored. The distance between T-PWAS and R-PWAS is 100 mm. An HP 33120A function generator was used to generate three-count tone burst excitation signals to the T-PWAS. A Tektronix TDS 5034B digital oscilloscope, synchronized with the function generator, was used to collect the response signals from the R-PWAS. At each frequency, the wave amplitudes of $A_0$ and $S_0$ modes were extracted using the Hilbert transform to generate the experimental tuning curves.

**Evaluation of results.** Figure 12 presents the experimental Lamb-wave tuning curves of $A_0$ and $S_0$ modes in the aluminum plate with PVC adhesive films. It can be observed that the first maximum of the $A_0$ mode happened at around 100 kHz, at which the amplitude of $S_0$ mode is very small. The first minimum of the $A_0$ mode was found around 240 kHz. At this frequency, the $S_0$ mode amplitude is nonzero and placed on an increasing curve. Thus, the $S_0$ signal is dominant. It should be noticed that the $S_0$ maximum happened at 330 kHz, which is the same frequency as that of the second $A_0$ maximum. Moreover, it can be noted that the structural damping can considerably reduce the wave amplitudes at higher frequencies (1000–2000 kHz).

During the experiment, it was noted that the best agreement between experiment and prediction was achieved using the effective PWAS length, a theoretical PWAS length smaller than that of the real PWAS transducer (Santoni-Bottai et al., 2007; Sohn and Lee, 2009). In the development of the theory, it is assumed

![Experimental setup on 2024-T3 aluminum alloy plate with PVC adhesive films: (a) PVC adhesive films on both sides and (b) PWAS on the front side.](image)

**Figure 11.** Experimental setup on 2024-T3 aluminum alloy plate with PVC adhesive films: (a) PVC adhesive films on both sides and (b) PWAS on the front side.

![Experimental Lamb-wave tuning curves with damping (9-mm-diameter circular PWAS and 1.016 mm aluminum plate with PVC adhesive films).](image)

**Figure 12.** Experimental Lamb-wave tuning curves with damping (9-mm-diameter circular PWAS and 1.016 mm aluminum plate with PVC adhesive films).
that there is an ideal bonding between the PWAS and the host structure. This assumption means that the stresses between the PWAS transducers and the plate are fully transferred at the PWAS ends. In reality, the stresses are transferred at a region adjacent to the PWAS ends (Santoni-Bottai and Giurgiuțiu, 2012). Therefore, the effective PWAS length of 8.4 mm for the 9-mm-diameter circular PWAS transducer is used in the improved analytical model of tuning to achieve a good agreement between experiment and prediction.

To predict theoretical 2-D tuning curves with damping and compare it with experimental results, the improved analytical model of tuning using the Kelvin–Voigt damping model was utilized. First, the reference frequency \( f_0 \) was initially chosen at the upper limit of the frequency range \( (f_{\text{initial}} = 2 \text{ MHz}) \) and the damping coefficient \( \eta_0 \) was initially taken as 5% \( (\eta_{\text{initial}} = 5\%) \).

Next, the reference frequency \( f_0 \) and the damping coefficient \( \eta_0 \) were adjusted by repeated updating of the model to achieve an acceptable match with experimental tuning curves. Eventually, we found that the most appropriate values for the damping coefficient \( \eta_0 \) and the reference frequency \( f_0 \) are 8% and 1.5 MHz, respectively.

The comparisons between experimental and analytical 2D tuning curves of \( A_0 \) and \( S_0 \) modes are presented in Figure 13. Theoretical predictions with damping of the two modes agree well with experimental measurements. However, a difference at the frequency range from 350 to 1000 kHz for \( A_0 \) mode can be observed. The error may be attributed to the effect of the bonding layer. As shown in Figure 13, both experimental and theoretical results demonstrate that the damping introduced by PVC adhesive films can considerably modify the tuning curves, both through peak shifts and through drastic diminishing of the amplitude at higher frequencies.

**Damping effects on Lamb-wave tuning for composite plates**

In the case of viscoelastic fiber–reinforced polymer composites, one of the main complexities involved in the use of Lamb waves is the attenuation caused by structural damping. This is essentially due to viscoelastic properties of the fiber and matrix constituents (Asamene et al., 2015; Treviso et al., 2015). In this section, damping effects on Lamb-wave tuning for composite plates were investigated.

**Experimental setup.** The pitch-catch Lamb-wave experiments between T-PWAS and R-PWAS on a woven composite plate were conducted to investigate the damping effect on Lamb-wave tuning. The specimen under investigation is a woven carbon fiber–reinforced polymer composite plate. The plate dimension is \( 350 \times 350 \times 2 \text{ mm}^3 \) and the layup is \([0^\circ, 45^\circ, 45^\circ, 0^\circ]\) s.

A large number of PWAS transducers (STEMINC SM412; 9-mm-diameter and 0.5 mm thick) were installed on the top surface of the plate, as shown in Figure 14. Three-count tone burst excitation signals were applied to the T-PWAS. The frequency range...
from 10 to 2000 kHz in steps of 10 kHz was explored. The response signals were collected from the R-PWAS 1, 2, and 3 in the 0° direction and R-PWAS 4 in the 45° direction. The distances between T-PWAS and R-PWAS 1 and 4 are 100 mm. The interval of the R-PWAS 1, 2, and 3 is 20 mm. Since Lamb waves have a much stronger attenuation in composite plates, a power amplifier (NF Corporation HSA4014) was used to amplify the excitation signal to 140 V peak to peak. At each frequency, the wave amplitudes of $A_0$ and $S_0$ modes were extracted using the Hilbert transform to generate the experimental tuning curves.

**Evaluation of results.** Experimental Lamb-wave tuning curves of $A_0$ and $S_0$ modes at 100 mm in the 0° direction are shown in Figure 15. A similar pattern can be observed from the 2D tuning curves, where the first maximum of the $A_0$ mode happened at around 80 kHz, at which the amplitude of $S_0$ mode is very small. The first minimum of the $A_0$ mode was found around 210 kHz. At this frequency, the $S_0$ mode amplitude placed on an increasing curve, which means that the $S_0$ signal is dominant. It should be noticed that the $S_0$ maximum happened at 300 kHz. Moreover, it can be noted that the amplitudes at higher frequencies (850–2000 kHz) were reduced significantly due to the structural damping of the woven composite plate.

To capture the effect of structural damping, the improved analytical model of 2D tuning was used to predict the theoretical tuning curves. Based on the assumption of quasi-isotropic material for this woven composite plate, the theoretical tuning curves can be calculated using the effective Young’s modulus and Poisson’s ratio (Gresil and Giurgiutiu, 2015). This is because the woven composite with the layup is quasi-isotropic, which possesses the same material properties in all the directions. Gresil and Giurgiutiu (2015) performed pitch-catch experiments to measure group-velocity curves in different propagation directions (0°, 30°, 45°, 60°, and 90°) on the woven composite plate and found that the quantities were the same in all the directions.

To validate the quasi-isotropic assumption of the woven composite plate, experimental 2D tuning curves obtained from R-PWAS 4 in the 45° direction were compared with the results in the 0° direction, as shown in Figure 16. It can be found that the tuning curves in the two various directions are almost the same. It means that the specimen considered in this article is a quasi-isotropic plate, which possesses the same material properties in all the directions.

To determine the reference frequency $f_0$ and the damping coefficient $h_0$ of the Kelvin–Voigt damping model, the same procedure used in section “Evaluation of results” was employed. Eventually, we found that the most appropriate values for the damping coefficient $h_0$ and the reference frequency $f_0$ are 12% and 1.5 MHz, respectively.

The comparisons between experimental and theoretical 2D Lamb-wave tuning curves of $A_0$ and $S_0$ modes are shown in Figure 17. A good match between experimental measurements and theoretical predictions with damping was achieved for the two modes. It should be noted that the theory also predicts that the $S_0$ maximum would happen at 300 kHz, $A_0$ maximum would happen at 80 kHz, and the rejection of the $A_0$ would occur at 210 kHz; these predictions were validated by the experiments. However, it can be observed that the damping coefficient of the woven composite plate ($h_0 = 12\%$) is larger than that of the aluminum plate ($h_0 = 8\%$), which means that the woven composite plate has a much stronger damping effect. It can be concluded that both experiment and prediction demonstrate that the structural damping of the woven composite plate can significantly change the tuning curves, both through peak shifts and through drastic diminishing of the amplitude at higher frequencies.
In addition, a further experimental measurement was conducted by looking at the tuning curves at various propagation distances (100, 120, and 140 mm). The experimental 2D tuning curves were obtained by extracting the wave amplitudes of the received signals from R-PWAS 1, 2, and 3, as shown in Figure 18. Theoretical 2D tuning curves at different propagation distances were calculated by our model and compared with the experiment. Normalized amplitudes using the maximum value of the tuning curves at 100 mm were plotted.

A good match between the experiment and prediction can be observed. Figure 18 (a) shows the comparison of $A_0$ mode at different locations, and two peaks
and one valley were predicted by our model, which were validated by the experiments. Figure 18 (b) presents the comparison of \( S_0 \) mode at various propagation distances, and only one peak around 300 kHz can be observed. Some small differences between the experiment and prediction can also be observed; this may be attributed to the variation in the R-PWAS transducers and the corresponding bonding layers. In general, both experiment and prediction demonstrated that the amplitude of the tuning curves decreases with the propagation distance.

**Summary, conclusions, and future work**

In this article, the analytical solution of Lamb-wave tuning for undamped media is extended to damped materials using the Kelvin–Voigt damping model, in which a complex Young’s modulus is utilized to
account for the effect of structural damping. To validate the proposed method and demonstrate its capabilities, experimental measurements were performed on an aluminum plate with PVC adhesive films on both sides and a quasi-isotropic woven composite plate using 9-mm-diameter circular PWAS transducers. Experimental results show that structural damping can modify the tuning effect considerably, both through peak shifts and through drastic diminishing of the amplitude at higher frequencies. Moreover, the amplitude of the tuning curves would decrease with the propagation distance. It was concluded that the improved model achieved good agreements with experimental measurements and the effects of structural damping were successfully captured. This is an important improvement over the conventional solution of Lamb-wave tuning in the literature. The concepts of this article are applicable to both metals (e.g. metallic plates with viscoelastic coatings) and composites.

For future work, it is desirable that the improved analytical solution should be extended to capture the complicated and direction-dependent damping behaviors in general composite materials.

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