A New Ultrasonic Immersion Technique to Retrieve Anisotropic Stiffness Matrix for Dispersion Curves Algorithms

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ABSTRACT

Dispersion curve algorithms require the material properties of the anisotropic medium in which the ultrasonic guided-waves (GW) propagate as input. These material properties are currently obtained through ASTM standard mechanical testing procedures. Practice shows that researchers limit themselves to determining the $E_{11}$, $E_{22}$, $G_{12}$ and $\nu_{12}$, while the other engineering constants are then based on assumptions such as: $E_{33} = E_{22}$. The engineering constants are subsequently converted to the stiffness matrix using the relation between the elastic constants. The calculated stiffness matrix is used as input in the dispersion curve algorithm, however, the predicted results may vary significantly from those obtained experimentally due to the assumption. A more accurate method of determining the stiffness matrix is desired.

In this research, the stiffness matrix components are determined non-destructively using a modified through-transmission ultrasonic immersion technique. Based on the existence of planes of symmetry within an orthotropic material, the stiffness matrix retrieval process was divided into parts to reduce the complexity of the process and increase the accuracy of the solution.

As last, The group velocity dispersion curves are calculated analytically with the material properties from both the ASTM standards and the ultrasonic immersion technique. Both predicted results were compared to experimentally obtained velocities reported in the literature.

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INTRODUCTION

To determine the stiffness matrix components for anisotropic materials it is common to follow the ASTM standards. These standards, however, require destructive mechanical testing and multiple specimens; this is expensive both in time and cost. Methods to determine the stiffness matrix components non-destructively are desired. In particular, focus has been set on techniques based on the analysis of the propagation of bulk waves due to the direct correlation between the material’s stiffness matrix components and the characteristics of the bulk waves [1–4]. One specimen can be used to determine all the stiffness matrix components by orientating the specimen in different directions, measuring the time-of-flight (ToF) experimentally and deriving the phase velocity [4–8].

In 1970 Markham [5] introduced a method to determine the elastic constants for composite laminates using ultrasonics. By measuring the ultrasonic wave velocities in multiple directions Markham determined the elastic constants of the laminate. Smith [6] applied Markham’s method and determined five elastic constants from the measurements. The number of elastic constants determined by Markham’s method was later increased to nine by Gieske and Allred in 1972 [8].

At the same time Gieske and Allred in 1972 [8] correctly observed that the ToF measurements used in Markham’s method corresponds to the group velocity of the propagating wave, while the phase velocity is required to determine the elastic constants. However, Pearson and Murri [9] showed that for transversely isotropic material the group velocity and phase velocity can be interchanged, obtaining the correct elastic properties.

Rokhlin and Wang [10] investigated Pearson and Murri [9] findings in more details and derived the following equation (valid for generally anisotropic materials) for the phase velocity based on the experimentally measured ToF.

\[ v_p(\theta_i) = \left[ \frac{1}{v_f^2} \frac{2\Delta t \cos \theta_i}{hv_f} + \frac{\Delta t^2}{h^2} \right]^{-\frac{1}{2}} \]  

(1)

where

\[ \Delta t = t_0 - t(\theta_i) \]

(2)

where \( t_0 \) is the ToF without the presence of a specimen, \( t(\theta_i) \) is the ToF with the presence of a specimen at an incident angle of \( \theta_i \), \( v_f \) is the velocity of sound in the immersion fluid (water in this dissertation) and \( h \) is the thickness of the specimen.

Hosten et al. [4] and Castaings et al. [7] retrieved the stiffness matrix components including the transversal shear and out-of-plane properties using the through-transmission technique. It is important to note that the transversal shear and out-of-plane properties are difficult to determine experimentally especially for thin laminates.

EXPERIMENTAL METHODOLOGY

To retrieve the stiffness matrix components non-destructively the ultrasonic immersion technique discussed in [1–5,7,8] was utilized. The through-transmission ultrasonic immersion technique is based on the transmission of ultrasonic plane waves through a specimen and receiving the wave field on the other side. Based on the experimentally obtained ToF, the phase velocity in the composite specimen is determined using Rokhlin
and Wang [10] formulation given in Eq. (1) and solving the inverse problem; the phase velocities are a given and the unknowns are the stiffness matrix components.

Figure 1: Elastic constants, reference axes and planes of propagation of ultrasonic waves for orthotropic material elaborated in the form of thin plate [11]

The inverse problem for an orthotropic material requires the optimization of nine independent stiffness matrix components \((C_{11}, C_{22}, C_{33}, C_{44}, C_{55}, C_{66}, C_{12}, C_{13}, C_{23})\) simultaneously. To reduce the complexity and increase the accuracy of the solution, the optimization process is divided into three parts, reducing the number of independent components that must be optimized at a time. The division into three parts was obtained by utilizing planes of symmetry, which reduces the required number of stiffness matrix components in order to calculate the phase velocity. The stiffness matrix components required for each plane of symmetry and the remaining components in the non-symmetric plane are listed below and illustrated in Fig. 1 [11]. It is important to note that \(C_{33}\) is determined from the phase velocity at \(\theta_i = 0^\circ\) in the \((x_1, x_3)\)-plane, while \(C_{11}, C_{55}\) and \(C_{13}\) are determined based on \(\theta_i \neq 0^\circ\).

- Propagation without a specimen
  - Determine the ToF \(t_0\).

- Propagation with a specimen in combination with the previously obtained \(t_0\)
  - Propagation in the \((x_1, x_3)\)-plane of symmetry
    * At an incident angle of \(\theta_i = 0^\circ\) determine \(C_{33}\)
    * At an incident angle of \(\theta_i \neq 0^\circ\) determine \(C_{11}, C_{55}\) and \(C_{13}\)
  - Propagation in the \((x_2, x_3)\)-plane of symmetry
    * At an incident angle of \(\theta_i \neq 0^\circ\) determine \(C_{22}, C_{44}\) and \(C_{23}\)
  - Propagation in a non-symmetry plane
    * At an incident angle of \(\theta_i \neq 0^\circ\) determine \(C_{66}\) and \(C_{12}\).

After each part, the optimized stiffness component is stored and the initial guess is updated to be used in the next step. At the end, all the stiffness components are optimized and the full matrix is reconstructed.

The through-transmission technique, as discussed in the literature, has the disadvantage of requiring an experiment without a specimen to determine \(t_0\). To perform an experiment without specimen, the specimen has to be removed, which requires manual
intervention that can act as a potential source of errors (minor adjustments in the fixture would yield a difference in ToF). To avoid the need of no-specimen experiment, the through-transmission technique was combined with the pulse-echo technique into a new method: the LAMSS approach.

In the LAMSS approach the data capturing was automated such that a range of incident angles between $\pm 60^\circ$ was obtained for the specimen orientated in both (x1,x3)- and (x1,x3)-plane of symmetry. From the collected data the critical angles (the incident angle at which a wave no longer propagates through the specimen but on its surface) in the both (x1,x3)- and (x1,x3)-plane of symmetry were determined. The critical angles allow the direct retrieval of several stiffness matrix components. Using the critical angle and Snell’s law the velocity in the fiber- (1-) or transverse (2)-direction were determined, which in turn yielded the $C_{11}$ and $C_{22}$ stiffness matrix components directly; thereby reducing the complexity of the optimization problem. In addition, it is important to notice that when the material is assumed to be transversely isotropic several simplifications can be applied to the steps listed above. Due to the transversely isotropic conditions $C_{66} = C_{55}$, $C_{12} = C_{13}$ and $C_{44} = (C_{22} - C_{23})/2$: (i) no experiment in a non-symmetry plane is required; and (ii) in the (x2,x3)-plane of symmetry only $C_{23}$ had to be determined.

**LAMSS approach**

- **Pulse-echo measurement**
  - The difference in ToF between the reflection from the front surface (first reflection) and the back surface (second reflection) is used in Eq. (1) to determine the $C_{33}$ directly.
  - The pulse-echo ToF is used to determine the $t_0$ in the previous method.

- **Through-transmission measurements**
  - Propagation in the (x1,x3)-plane of symmetry
    * At an incident angle of $\theta_i = 0^\circ$ and with the determined $C_{33}$ component, the ToF without the presence of a specimen ($t_0$) can was determined;
    * At an incident angle of $\theta_i \neq 0^\circ$ and using the previously determined $t_0$ determine $C_{11}$, $C_{55}$ and $C_{13}$.
  - Propagation in the (x1,x3)-plane of symmetry-plane of symmetry
    * At an incident angle of $\theta_i \neq 0^\circ$ and using the previously determined $t_0$ determine $C_{22}$, $C_{44}$ and $C_{23}$.
  - Propagation in a non-symmetry plane
    * At an incident angle of $\theta_i \neq 0^\circ$ and using the previously determined $t_0$ determine $C_{66}$ and $C_{12}$.
EXPERIMENTAL RESULTS

The stiffness matrix components were determined for a composite specimen provided by NASA. The engineering constants and guided wave group velocity, based on the determined stiffness matrix components, were compared with those reported by Leckey et al. [12].

Once the specimen was loaded into the water tank, measurements were taken in the \((x_1,x_3)\)- and \((x_2,x_3)\)-plane of symmetry. It is important to notice that if it is known that the material is transversely isotropic, only data from the \((x_1,x_3)\)-plane of symmetry is required. In this investigation, transversely isotropic assumption was verified through the condition that \(C_{22}\) is equal to \(C_{33}\), therefore, both the data \((x_1,x_3)\)- and \((x_2,x_3)\)-plane of symmetry were used.

First, using \((x_2,x_3)\)-plane of symmetry both \(C_{22}\) and \(C_{33}\) were determined. The \(C_{33}\) stiffness matrix component was retrieved using the through-transmission data, while the \(C_{22}\) was retrieved using the first critical angle, the critical angle at which the pressure wave disappears and only a slower shear wave propagates through the specimen, 28.5° in this case. The through-transmission experiment yielded a phase velocity of 3075 m/s:

\[
C_{33} = \rho v_p
\]  

a value of 14.76 GPa was obtained for \(C_{33}\). Second, \(C_{22}\) was retrieved using the critical angle in combination with Snell’s law:

\[
v_p = \frac{v_f}{\sin \theta_{cr}}
\]  

the value for \(C_{22}\) was retrieved to equal 15.10 GPa. The difference between the retrieved values for \(C_{22}\) and \(C_{33}\) was 0.34 GPa, this difference was deemed acceptable (\(\epsilon \leq 3\%\)) and within the range due to measurement errors. Due to the small difference it was concluded that \(C_{22}\) equals \(C_{33}\), therefore the material was assumed to be transversely isotropic. For future calculations, the final value for \(C_{22}\) and \(C_{33}\) was set to the average of the two; 14.93 GPa.

Second, to retrieve the \(C_{11}\) stiffness matrix component the first critical angle in the \((x_1,x_3)\)-plane was required, similar to when retrieving the \(C_{22}\) stiffness matrix component. The experimental phase velocity measurements for the range of incident angles beyond ±45° yielded inaccurate results; therefore, only the data up to ±40° was used for further processing. The incident angle in this case was retrieved using Fig. 2a by searching for an amplitude decrease; it is important to note that the sharp decrease in phase velocity (see Fig. 2b) corresponded to the generation of a GW with a larger amplitude than the pressure wave; therefore, the ToF of the GW was retrieved. Therefore, a closer examination of Fig. 2a was required which yielded a second decrease in amplitude at an incident angle of approximately 8°, this incident angle was used to determine the \(C_{11}\) stiffness matrix component. The critical angle of 8.08° yielded a \(C_{11}\) value of 173.77 GPa.

At last, the other \((C_{44}, C_{55}\) and \(C_{13})\) stiffness matrix components were retrieved using the optimization routine with values of 4.96, 6.22 and 3.77 GPa respectively. It is important to recall the transversely isotropic conditions that yielded \(C_{66}=C_{55}\), \(C_{12}=C_{13}\) and finally \(C_{23} = C_{22} - 2C_{44}\) or 5.01 GPa.
Incident angle $\theta^i$ [degree]
-40 -30 -20 -10 0 10 20 30 40
Phase velocity [m/s]
0
500
1000
1500
2000
2500
3000
3500

Figure 2: Data from the $(x_2,x_3)$-plane of symmetry for each incident angle $\theta_i$: (a) maximum absolute peak amplitude in each $y$-location; and (b) phase velocity for the selected range.

After the full stiffness matrix was retrieved, it was converted to the engineering constant and a comparison was made between the retrieved values and those reported by Leckey et al. [12]. The LAMSS approach yielded realistic values for all the engineering constants. Comparing them to those in the literature show a significant difference for $E_{22}$, $E_{33}$, $G_{23}$ and the Poisson ratios. However, the largest difference is between the $G_{23}$ values. It is important to note that the literature value for $G_{23}$ was not considered reliable since it was not directly obtained from testing but assumed. Another noticeable difference are those in the Poisson ratios: first, $\nu_{23}$ was assumed and not determined; therefore, the difference was deemed acceptable. Second, $\nu_{12}$ and $\nu_{13}$ significantly varied as well. The Poisson ratio $\nu_{12}$ was determined through testing (recall $\nu_{13} = \nu_{12}$) and therefore the optimized results was considered to be less accurate. A possible reasoning for this was the difference in strain values between the mechanical testing procedure which strains the material significantly more (several tenths of a percent of strains are possible) than the ultrasonic immersion in which the material properties were retrieved in the microstrain domain. In addition, the engineering constants determined using mechanical testing where based on the literature, therefore no comments can be made on the correctness of the results, unfortunately.

In the literature, a GW experiment was performed to determine the group velocity in the $0^\circ$ direction. Both the literature and the LAMSS engineering constants were used as input in the unified analytic method (UAM) [13–15] formulation to predict the group velocity for the given frequency (200 kHz), and the accuracy of the predictions compared to those reported in the literature. As it can be seen Tab. If the predicted group velocity error has decreased from 14.1% to 5.8% when using the engineering constants obtained through the LAMSS approach. It was therefore concluded that the LAMSS approach yielded more accurate results when the objective was to retrieve accurate dispersion curves.
TABLE I: Material properties M7-8552, engineering constants comparison

<table>
<thead>
<tr>
<th>Property</th>
<th>Leckey et al. [12] [GPa]</th>
<th>LAMSS approach [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$</td>
<td>171.40</td>
<td>173.22</td>
</tr>
<tr>
<td>$E_{22}$</td>
<td>9.08</td>
<td>13.32</td>
</tr>
<tr>
<td>$E_{33}$</td>
<td>9.08</td>
<td>13.32</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>5.29</td>
<td>6.26</td>
</tr>
<tr>
<td>$G_{13}$</td>
<td>5.29</td>
<td>6.26</td>
</tr>
<tr>
<td>$G_{23}$</td>
<td>2.80</td>
<td>5.01</td>
</tr>
<tr>
<td>$\nu_{12}$ [-]</td>
<td>0.320</td>
<td>0.204</td>
</tr>
<tr>
<td>$\nu_{13}$ [-]</td>
<td>0.320</td>
<td>0.204</td>
</tr>
<tr>
<td>$\nu_{23}$ [-]</td>
<td>0.500</td>
<td>0.329</td>
</tr>
</tbody>
</table>

TABLE II: Group velocity [m/s] at 200 kHz in the 0° direction

<table>
<thead>
<tr>
<th></th>
<th>Experimental [12]</th>
<th>Leckey et al. [12]</th>
<th>LAMSS approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group velocity [m/s]</td>
<td>2149 ± 13</td>
<td>1835</td>
<td>2012</td>
</tr>
<tr>
<td>Error $\epsilon$ [%]</td>
<td>-</td>
<td>14.1 - 15.1</td>
<td>5.8 - 6.9</td>
</tr>
</tbody>
</table>

DISCUSSION AND CONCLUSION

This manuscript presented a modified ultrasonic immersion technique to retrieve the stiffness matrix components non-destructively; the LAMSS approach.

The LAMSS approach combined the through-transmission and pulse-echo techniques such that there was no need for an additional experiment without the presence of the specimen to determine the ToF in the coupling medium (water). Furthermore, to reduce the complexity of the inverse problem (determining the stiffness matrix components from experimentally obtained velocities) several stiffness matrix components were retrieved directly based on the critical angle approach. Based on the value for $C_{22}$ and $C_{33}$ it was concluded the specimen was made out of a transversely isotropic material, therefore corresponding conditions ($C_{33} = C_{22}$, $C_{46} = C_{55}$, $C_{13} = C_{12}$ and $C_{44} = C_{22} - C_{23}^2$) were applied in the analysis.

The LAMSS approach was evaluated using a specimen provided by NASA for which its stiffness matrix components were retrieved. The stiffness matrix components were converted to engineering constant such that a direct comparison could be made between the values obtained through the LAMSS approach and the engineering constants presented in the literature.

The comparison revealed that the engineering constants retrieved using the LAMSS approach were different from those listed in the literature. The Poisson ratio $\nu_{23}$, however, was more realistic than the value of 0.50, as the literature suggested. The engineering constants in the literature were determined using ASTM standard procedure, these tests required the specimen to be loaded until failure. The strains experienced by the specimen were orders of magnitude larger when compared to the ultrasonic immersion technique; this is a possible source of the error between the values.

The goal was to retrieve a stiffness matrix to use in a dispersion curve algorithm. Therefore, both sets of the engineering constants were used to predict the group velocity at a frequency of 200 kHz, which in turn were compared to the experimental velocity at 200 kHz (reported in the literature). To predict the group velocity the UAM as discussed in [13–15] was used. The engineering constants obtained through the LAMSS approach
yielded more accurate predictions (5.8% error versus 14.1%). It is therefore concluded that the LAMSS approach can yield a set of engineering constants different from ASTM standards yet yield better dispersion curve predictions.

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