Acoustic emission source modeling in a plate using buried moment tensors

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ABSTRACT

Acoustic emission is a widely used and efficient method for structural damage monitoring. Analytical modeling of wave propagation due to dislocations by considering the source as a self-equilibrating moment tensor is a commonly used approach in seismology. In this paper the acoustic emission source definition using moment tensor approach is studied and tried to implement it in thin plates with micro crack acoustic emission source. Depending upon the characteristic of micro crack formed the moment tensor excitation also changes. A study has done to identify the moment tensor components for mod 1, mode 2 or mode 3 like fracture micro crack formation from the classical definition of moment tensor.

Keywords: Acoustic emission, moment tensor, acoustic emission source definition

1. INTRODUCTION

Acoustic emission (AE) has proved to be an efficient technique for health monitoring of structures[1-4]. The sources for acoustic emission may include a wide range of phenomena such as micro cracks, friction, dislocations, phase transformations, friction etc. The elastodynamic waves generated in the structures due to these sources are used to determine the integrity of a structure. AE monitoring is a passive structural health monitoring (SHM) system[5] and it has some potential advantages over conventional active SHM system[6]. One of the main advantages is it requires less number of sensors than an active SHM. However, an AE system requires to sense very smaller signals from any potential state change in a structure. It requires very fundamental understanding of AE source to identify the correct AE signal from the associated noise of the environment[7,8].

The understanding of AE sources lies in very fundamental analysis of elastodynamics vector fields[9,10]. Numerous studies have been done to solve forward problem of acoustic emission by utilizing numerical models[5,11-13]. Majority of these studies used finite element based approach to understand the forward problem in acoustic emission. Several studies have been done by considering the acoustic emission source as monopole and dipole point source mechanisms[11,14]. Analytical modeling of acoustic emission source has also been studied[15]. Many analytical models are considering the acoustic emission source as a of moment tensor[16]. However, these studies focused on half-space application like earthquake, seismology.

The source of acoustic emission is considered as a self-equilibrating moment tensor which acts at a point. Through this study the acoustic emission due to formation of a buried micro crack in a plate is tried to formulate and simulate. The acoustic emission source is considered as a point source moment tensor acting at the center of gravity of the micro crack. The analytical formulation for displacement field due to a harmonic point load in a plate was developed by Achenbach and Xu[2].

The application of moment tensor concept in plate guided wave is the object of this article. Some components of moment tensor are responsible for particular modes of crack formation in the plate. The corresponding AE source for different modes of fracture is represented by the moment tensor expressions. Finally, the analytical expressions for the general solution of out-of-plane displacement field are developed for a particular type of fracture mode.

2. THEORY

Figure 1 represents a volume \( V \) with external surface \( S \) and two internal surfaces labelled \( \Sigma^+ \) and \( \Sigma^- \), which are the opposing faces of an internal discontinuity. The equation of motion is satisfied throughout the interior of the surfaces \( S + \Sigma^+ \Sigma^- \). The displacement field due to internal surface excitation and body force can be written as the following. A detailed description of the theory is given by Aki and Richards[16].
Where \( \eta \) is a general position within \( V \), and \( \xi \) is the general position on \( \Sigma \). The body force acting on the bulk volume is represented by \( "f" \). \( "G" \) represents the elastodynamic green function and the traction is represented by \( T \). \( C_{ijpq} \) represents the stiffness of the material. The displacements on the \( \Sigma^+ \) side may differ from the displacement on the \( \Sigma^- \) side. The displacement discontinuity is denoted by \[ \{u(\xi, \tau)\} \] for \( \xi \) on \( \Sigma^+ \), with square brackets referring to the difference \( u(\xi, \tau)_{\Sigma^+} - u(\xi, \tau)_{\Sigma^-} \). For a spontaneous rupture, \[ [T(u, \nu)] = 0. \] The unit vector \( \nu \) is the normal to \( \Sigma \) pointing from \( \Sigma^- \) to \( \Sigma^+ \). This assumes \( \Sigma \) as an artificial surface which is formed by a spontaneous rupture and across.

\[ u_{\nu}(x, t) = \int_{-\infty}^{\infty} dt \int_{\Sigma} \{u(\xi, \tau)\} C_{ijpq} \nu_j G_{ijpq}(\xi, t; \tau, x, 0) d\Sigma \]

Figure 1. A solid medium with an internal surface discontinuity. The internal surface discontinuity is represented by the surfaces \( \Sigma^+ \) and \( \Sigma^- \). \( V \) and \( S \) represent the bulk body volume and boundary surface.

The function \( G \) and its derivatives are continuous, so that \( G \) satisfies the equation of motion inside the volume and on the surface \( \Sigma \). In absence of body force it gives the representation

\[ u_{\nu}(x, t) = \int_{-\infty}^{\infty} dt \int_{\Sigma} \{u(\xi, \tau)\} C_{ijpq} \nu_j G_{ijpq}(\xi, t; \tau, x, 0) d\Sigma \]

This expression gives the displacement field in a bulk body due to an internal surface excitation. From the above expression we can conclude that the excitation due to an internal surface motion can be considered as

\[ \{u(\xi, \tau)\} C_{ijpq} \nu_j \]

This excitation is a second order tensor. It has the unit of moment per unit area and it is commonly called “moment density tensor.” An integral of this moment density tensor over a finite area is called “moment tensor.” The tensor represents a point excitation acting at the center of gravity of the finite area. So the excitation due to an internal surface motion that produces wave propagation can be approximated by a moment tensor which is acting at a point. Moment tensor has diagonal “dipole force” excitations and off diagonal “couple force” excitations.

### 3. MOMENT TENSOR FOR DIFFERENT MODES OF FRACTURE

We have the expression for moment density tensor due to the discontinuity as

\[ m_{pq} = [u_{\nu}] C_{ipq} \]

Where, \( C_{ipq} = \lambda \delta_{ip} \delta_{pq} + 2 \mu (\delta_{ip} \delta_{pq} + \delta_{iq} \delta_{pj}) \) the stiffness matrix and \( \lambda, \mu \) are the lame moduli. Substituting the expression for stiffness matrix in to the moment density tensor equation will give the expression for moment density tensor as given below.

\[ m_{pq} = \lambda [u_{\nu}] \delta_{pq} + 2 \mu \left[ [u_{p}] \nu_q + [u_q] \nu_p \right] \]

In the following sub sections the moment density tensor for different modes of fracture at a finite area will be discussed.
3.1 Mode 1 fracture

Figure 2. Mode 1 fracture in a body

Figure 2 above shows the mode 1 crack formation in an elastic body. From the figure we can observe that the crack surface Σ is formed in the 1-2 plane. 'ν' the normal to Σ which is pointing from Σ→ to Σ+ is a unit vector in the direction of 3. Which means ν₁=0, ν₂=0, ν₃=1. The displacement discontinuities in 1, 2, and 3 direction is represented as [u₁]=0; [u₂]=0; [u₃] = u₃|Σ⁺− u₃|Σ⁻, respectively. Only [u₃] exists in the displacement discontinuity, because the displacement discontinuity happens in 1-2 plane in the direction of 3 for a pure mode 1 fracture. When we substitute these characteristics of mode 1 fracture to equation for moment density tensor, we get the moment density tensor for a pure mode 1 fracture as following:

\[
m = \begin{pmatrix}
\lambda [u_1] & 0 & 0 \\
0 & \lambda [u_1] & 0 \\
0 & 0 & (\lambda + 2\mu) [u_3]
\end{pmatrix}
\]  

(mode 1 fracture)  

(5)  

We observe that the moment density tensor for mode 1 fracture has three diagonal components \(m_{11} = \lambda [u_1] \), \(m_2 = \lambda [u_3] \), and \(m_3 = (\lambda + 2\mu) [u_3] \).

3.2 Mode 2 fracture

Figure 3. Mode 2 fracture in a body

For mode 2 fracture, the characteristics of fracture can be identified as [u₁] = u₁|Σ⁺− u₁|Σ⁻ ; [u₂] =0; [u₃] =0. For discontinuity happening in 1-2 plane, ν₁=0, ν₂=0, ν₃=1. Hence, the moment density tensor for a pure mode 2 fracture can be expressed as the following:

\[
m = \begin{pmatrix}
0 & 0 & \mu [u_1] \\
0 & 0 & 0 \\
\mu [u_1] & 0 & 0
\end{pmatrix}
\]  

(mode 2 fracture)  

(6)  

We observe that the moment density tensor for mode 2 fracture has only two self-equilibrating off-diagonal components \(m_{13} = \mu [u_1] \) and \(m_{31} = \mu [u_1] \).
3.3 Mode 3 fracture

![Figure 4. Mode 3 fracture in a body](image)

For mode 3 fracture, the characteristics of fracture can be identified as $[u_1] = 0; [u_2] = u_2|_{S^+} - u_2|_{S^-}; [u_3] = 0$. For discontinuity happening in the 1-2 plane, $\nu_1 = 0, \nu_2 = 0, \nu_3 = 1$. Hence, the moment density tensor for a pure mode 3 fracture can be expressed as the following:

$$m = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \mu[u_2] \\ 0 & \mu[u_2] & 0 \end{bmatrix} \quad \text{(mode 3 fracture)} \quad (7)$$

We observe that the moment density tensor for mode 3 fracture has only two self-equilibrating off-diagonal components $m_{23} = \mu[u_2]$ and $m_{32} = \mu[u_2]$.

3.4 Moment tensor

If the formation of the crack surface is uniform across the finite cross section on which it is being formed, we can integrate the moment density tensor across the uniformly formed crack surface and obtain the moment tensor, $M$. Moment tensor is the excitation acting at a point equivalent to the excitation due to the formation of crack. Hence we can write

$$\text{Moment tensor } M = \int m \, dA \quad \text{(8)}$$

If the newly formed uniform increment in the crack surface is $\Delta A$, approximately we can write the moment tensor in terms of moment density tensor as

$$M = m \Delta A \quad \text{(9)}$$

4. DISPLACEMENT FIELD DUE TO MOMENT TENSOR $M_{ij}$

Suppose a couple moment is generated due to a force vector "$Q$" applied at position $\xi(\alpha, \beta)$ and another force vector "$Q$" applied at $\xi'(\alpha, \beta, \Delta X_j)$ in the opposite direction that of the force applied at $\xi'(\alpha, \beta, \Delta X_j)$, where $\Delta X_j$ is a small distance in the $X_j$ direction as shown in Figure 5.
Figure 5. Couple force applied for generating the moment. As we limit the separation of the forces to zero the couple will become a concentrated moment at $X$.

The displacement field at an arbitrary point $'x'$ due to a point force $Q$ at an arbitrary point $\xi(\alpha, \beta)$ and $Q$ at $\xi(\alpha, \beta + \Delta X_j)$ as shown in Figure 5 is denoted as $u_i^x$ and $u_i^{x'}$ respectively. If the material is linear elastic and the displacement field has a linear relation with the force applied, we can write

$$u_i^x = QG((\alpha, \beta), x)$$

$$u_i^{x'} = QG((\alpha, \beta + \Delta X_j), x)$$

(10)  (11)

The displacement field at $'x'$ due to a couple with moment “$M_{ij} = Q\Delta X_j$” can be written from Eqs (10) and (11) as following.

$$u_i^{M_{ij}} = u_i^x - u_i^{x'} = Q(G((\alpha, \beta), x) - G((\alpha, \beta + \Delta X_j), x))$$

$$= Q\Delta X_j \frac{(G((\alpha, \beta), x) - G((\alpha, \beta + \Delta X_j), x))}{\Delta X_j}$$

(12)  (13)

For an infinitesimally small distance $\Delta X_j$, we apply the limits to get

$$u_i^{M_{ij}} = \lim_{\Delta X_j \to 0} \frac{Q\Delta X_j}{Q, \Delta X_j \to M_{ij}} \frac{(G((\alpha, \beta), x) - G((\alpha, \beta + \Delta X_j), x))}{\Delta X_j}$$

(14)

Using first order differential formula, Eq (14) can be written as

$$u_i^{M_{ij}} = \lim_{\Delta X_j \to 0} \frac{M_{ij}}{Q, \Delta X_j \to M_{ij}} \frac{\partial}{\partial X_j} G((\alpha, \beta), x)$$

(15)

So the displacement field for a moment tensor component with forces acting in $i$ direction and a separated in the $j$ direction is given as

$$u_i^{M_{ij}} = M_{ij} \frac{\partial}{\partial X_j} G((\alpha, \beta), x)$$

(16)

5. OUT-OF-PLANE DISPLACEMENT FIELD DUE TO MOMENT TENSOR

This section describes the theoretical equations for the out-of-plane displacement fields in a plate due to different components of moment tensor. The displacement field due to a point load excitation is converted to displacement fields due to moments by using the limiting process explained in the previous section. For a harmonic point load the non-axisymmetric displacement field in a plate is formulated by Achenbach and Xu. We use these solutions to obtain the out-of-plane displacement field due to moment tensor in a plate.
Consider a plate as shown in Figure 6. For harmonic point force, $Q$ acting in $X_1$ direction the out-of-plane displacement field is given as

$$u_{z}^{0,S} = Q \sum_{n=0}^{\infty} A_{n}^{s}(z_{0}) W_{n}^{s}(z) H_{1}^{(2)}(k_{r}r) \cos \theta = Q G_{z}^{0,s}$$  \hspace{1cm} (17)$$

$$u_{z}^{0,A} = Q \sum_{n=0}^{\infty} A_{n}^{a}(z_{0}) W_{n}^{a}(z) H_{1}^{(2)}(k_{r}r) \cos \theta = Q G_{z}^{0,a}$$  \hspace{1cm} (18)$$

For harmonic force acting in $X_2$ direction the out-of-plane displacement field is given as

$$u_{z}^{0,S} = Q \sum_{n=0}^{\infty} A_{n}^{s}(z_{0}) W_{n}^{s}(z) H_{0}^{(2)}(k_{r}r) \sin \theta = Q G_{z}^{0,s}$$  \hspace{1cm} (19)$$

$$u_{z}^{0,A} = Q \sum_{n=0}^{\infty} A_{n}^{a}(z_{0}) W_{n}^{a}(z) H_{0}^{(2)}(k_{r}r) \sin \theta = Q G_{z}^{0,a}$$  \hspace{1cm} (20)$$

For harmonic force acting in $Z$ direction the out-of-plane displacement field is given as

$$u_{z}^{0,S} = -Q \sum_{n=0}^{\infty} C_{n}^{s}(z_{0}) W_{n}^{s}(z) H_{1}^{(2)}(k_{r}r) = Q G_{z}^{0,s}$$  \hspace{1cm} (21)$$

$$u_{z}^{0,A} = Q \sum_{n=0}^{\infty} C_{n}^{a}(z_{0}) W_{n}^{a}(z) H_{0}^{(2)}(k_{r}r) = Q G_{z}^{0,a}$$  \hspace{1cm} (22)$$

Where $S$ denotes symmetric displacement, $A$ denotes antisymmetric displacement. $A_{n}^{s}(z_{0})$, $A_{n}^{a}(z_{0})$, $C_{n}^{s}(z_{0})$ and $C_{n}^{a}(z_{0})$ are the coefficients of modal expansion which are functions of frequency $\omega$, shear wave speed $c_{s}$, pressure wave speed $c_{p}$, wave number $k_{r}$, half thickness $h$ of the plate, location of the force applied $z_{0}$ and magnitude of harmonic force applied $Q$.

The thickness-wise modeshapes are given by

$$V_{s}^{n} = s_{1} \cos(pz) + s_{2} \cos(qz)$$  \hspace{1cm} (23)$$

$$W_{s}^{n} = s_{3} \sin(pz) + s_{4} \sin(qz)$$  \hspace{1cm} (24)$$

$$V_{a}^{n}(z) = a_{1} \sin(pz) + a_{2} \sin(qz)$$  \hspace{1cm} (25)$$

$$W_{a}^{n}(z) = a_{3} \cos(pz) + a_{4} \cos(qz)$$  \hspace{1cm} (26)$$
Where $s_1, s_2, s_3, s_4$ and $a_1, a_2, a_3, a_4$ are expressed as

\begin{align*}
  s_1 &= 2 \cos(qh) \\
  s_2 &= -(k_n^2 - q^2) / k_n^2 \cos(ph) \\
  s_3 &= -2(p / k_n) \cos(qh) \\
  s_4 &= -(k_n^2 - q^2) / qk_n \cos(ph) \\
  a_1 &= 2 \sin(qh) \\
  a_2 &= -(k_n^2 - q^2) / k_n^2 \sin(ph) \\
  a_2 &= -(k_n^2 - q^2) / k_n^2 \sin(ph) \\
  a_2 &= -(k_n^2 - q^2) / k_n^2 \sin(ph)
\end{align*}

(27)

The constants $A_n^s, A_n^a, C_n^s, C_n^a$ are given by

\begin{align*}
  A_n^s &= k_n V_n^s(z_o) / 4i I_{on}^s \\
  A_n^a &= k_n V_n^a(z_o) / 4i I_{on}^a \\
  C_n^s &= k_n W_n^s(z_o) / 4i I_{on}^s \\
  C_n^a &= k_n W_n^a(z_o) / 4i I_{on}^a
\end{align*}

(29) (30) (31) (32)

\begin{align*}
  I_{on}^s &= \mu [c_1^s \cos^2(ph) + c_2^s \cos^2(qh)] \\
  I_{on}^a &= \mu [c_1^a \sin^2(ph) + c_2^a \sin^2(qh)]
\end{align*}

(33) (34)

where $c_1^s, c_2^s, c_1^a, c_2^a$ is given as

\begin{align*}
  c_1^s &= \frac{(k_n^2 - q^2)(k_n^2 + q^2)}{2q^2k_n^3} \left[ 2qh(k_n^2 - q^2) + (k_n^2 + 7q^2) \sin(2qh) \right] \\
  c_2^s &= \frac{(k_n^2 + q^2)}{pk_n^3} \left[ 4phk_n^2 - 2(k_n^2 - 2p^2) \sin(2ph) \right] \\
  c_1^a &= \frac{(k_n^2 - q^2)(k_n^2 + q^2)}{2q^2k_n^3} \left[ 2qh(k_n^2 - q^2) + (k_n^2 + 7q^2) \sin(2qh) \right]
\end{align*}

(35) (36) (37)
\[ c_2^n = \frac{(k_n^2 + q^2)}{p k_n^2} \left[ 4 \phi k_n^2 - 2(k_n^2 - 2p^2) \sin(2ph) \right] \]  

(38)

\( r \) denotes the radial distance from the point of application of the load to an arbitrary point in the plate where the displacement fields is to be measured. \( H \) represents Hankel function.

By using the limiting process which is explained in the previous section, we can write the symmetric and antisymmetric displacement field due to a dipole \( M_{11} \) in the moment tensor as,

\[ u^{M_{11S}}_\varepsilon = M_{11} \frac{\partial}{\partial x_1} G_{\varepsilon}^{0,S} = M_{11} \frac{\partial}{\partial x_1} \sum_{n=0}^{\infty} A_n^S(z_n) W_s^n(z) H_1^{(2)}(k_n r) \cos \theta \]  

(39)

\[ u^{M_{11A}}_\varepsilon = M_{11} \frac{\partial}{\partial x_1} G_{\varepsilon}^{0,A} = M_{11} \frac{\partial}{\partial x_1} \sum_{n=0}^{\infty} A_n^A(z_n) W_A^n(z) H_1^{(2)}(k_n r) \cos \theta \]  

(40)

Performing differentiation with respect to \( X_1 \)

\[ u^{M_{11S}}_\varepsilon = M_{11} \sum_{n=0}^{\infty} A_n^S W_s^n(z) \left[ \frac{1}{r} \left[ -H_1^{(2)}(k_n r)(\cos^2 \theta - 1) \right] + \frac{k_n}{2} \left( H_2^{(2)}(k_n r) - H_0^{(2)}(k_n r) \right) \cos^2 \theta \right] \]  

(41)

\[ u^{M_{11A}}_\varepsilon = M_{11} \sum_{n=0}^{\infty} A_n^A W_A^n(z) \left[ \frac{1}{r} \left[ -H_1^{(2)}(k_n r)(\cos^2 \theta - 1) \right] + \frac{k_n}{2} \left( H_2^{(2)}(k_n r) - H_0^{(2)}(k_n r) \right) \cos^2 \theta \right] \]  

(42)

Following the same procedure the out-of-plane displacement field due to different components of moment tensor can be derived as follows

Due to \( M_{22} \)

\[ u^{M_{22S}}_\varepsilon = M_{22} \frac{\partial}{\partial x_2} G_{\varepsilon}^{0,S} = -M_{22} \sum_{n=0}^{\infty} A_n^S W_s^n(z) \left[ \frac{H_1^{(2)}(k_n r) \sin 2\theta}{2r} \right] + \frac{k_n}{4} \left( H_0^{(2)}(k_n r) - H_2^{(2)}(k_n r) \right) \sin 2\theta \]  

(43)

\[ u^{M_{22A}}_\varepsilon = M_{22} \frac{\partial}{\partial x_2} G_{\varepsilon}^{0,A} = -M_{22} \sum_{n=0}^{\infty} A_n^A W_A^n(z) \left[ \frac{H_1^{(2)}(k_n r) \sin 2\theta}{2r} \right] + \frac{k_n}{4} \left( H_0^{(2)}(k_n r) - H_2^{(2)}(k_n r) \right) \sin 2\theta \]  

(44)

Due to \( M_{33} \)

\[ u^{M_{33S}}_\varepsilon = M_{33} \frac{\partial}{\partial z} G_{\varepsilon}^{0,S} = M_{33} \sum_{n=0}^{\infty} \frac{k_n}{4} \left[ s_n p \cos(pz_n) \right] + \frac{1}{4} H_0^{(2)}(k_n r) \]  

(45)

\[ u^{M_{33A}}_\varepsilon = M_{33} \frac{\partial}{\partial z} G_{\varepsilon}^{0,A} = M_{33} \sum_{n=0}^{\infty} \frac{k_n}{4} \left[ a_n q \sin(qz_n) \right] W_A^n(z) H_0^{(2)}(k_n r) \]  

(46)
6. OUT-OF-PLANE DISPLACEMENT FIELD DUE TO A MODE 1 LIKE MICROCRACK

For a mode 1 fracture the theoretical moment tensor is composed of three diagonal dipoles as shown in Eq.(5). Suppose a sudden micro crack is formed in a structure causing acoustic emission. Let us assume the micro crack formed is mode 1 fracture in nature. The moment tensor excitation due to the sudden formation of a mode 1 like micro crack can be approximated to three independent force dipole excitations as shown below from Eqs. (5) and (9)

\[
M = \begin{pmatrix}
M_{11} & 0 & 0 \\
0 & M_{22} & 0 \\
0 & 0 & M_{33}
\end{pmatrix} \Delta A
\]

From Eq (5), if the strength of dipole \(M_{11}\) is considered to be unity, the unit moment tensor for a mode 1 like micro crack excitation can be written as the following

\[
M = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & (\lambda + 2\mu)/\lambda
\end{pmatrix}
\]

(48)

The total symmetric displacement field due to simultaneous action of three force dipoles will be the sum of displacement field due to individual force dipoles. For a unit moment tensor excitation as shown in Eq (48), we can write the symmetric part of out-of-plane displacement field \(u_z\) as

\[
u_z^S = u_z^{M_{11}S} + u_z^{M_{22}S} + u_z^{M_{33}S}
\]

(49)

\[
u_z^S = M_{11} \sum_{n=0}^{\infty} A_n^S W^n_s(z) \left[ \frac{1}{r} [H^{(1)}_n(k_n r)(\cos^2 \theta - 1)] + \frac{k_n}{2} (H^{(2)}_n(k_n r) - H^{(2)}_n(k_n r) \cos^2 \theta) \right] - M_{22} \sum_{n=0}^{\infty} A_n^S W^n_s(z) \left[ \frac{H^{(2)}_n(k_n r) \sin 2\theta}{r^2} \right] + \frac{k_n}{4} \left( \frac{H^{(2)}_n(k_n r) - H^{(2)}_n(k_n r) \sin 2\theta}{r} \right)
\]

\[
M_{33} \sum_{n=0}^{\infty} \frac{k_n}{4H_{\lambda \mu}} \left[ s_q \cos(q_{\lambda \mu}z) \right] \left[ s_q \cos(q_{\lambda \mu}z) \right]
\]

(50)

Hence,

\[
u_z^S = \sum_{n=0}^{\infty} A_n^S W^n_s(z) \left[ \frac{1}{r} [H^{(1)}_n(k_n r)(\cos^2 \theta - 1)] + \frac{k_n}{2} (H^{(2)}_n(k_n r) - H^{(2)}_n(k_n r) \cos^2 \theta) \right] - \sum_{n=0}^{\infty} A_n^S W^n_s(z) \left[ \frac{H^{(2)}_n(k_n r) \sin 2\theta}{r^2} \right] + \frac{k_n}{4} \left( \frac{H^{(2)}_n(k_n r) - H^{(2)}_n(k_n r) \sin 2\theta}{r} \right)
\]

(51)

\[
\left( \lambda + 2\mu \right) \sum_{n=0}^{\infty} \frac{k_n}{4H_{\lambda \mu}} \left[ s_q \cos(q_{\lambda \mu}z) \right] \left[ s_q \cos(q_{\lambda \mu}z) \right]
\]

Similarly the antisymmetric part of out-of-plane displacement field displacement field \(u_z\) due to a unit moment tensor is give as follows

\[
u_z^A = u_z^{M_{11}A} + u_z^{M_{22}A} + u_z^{M_{33}A}
\]

(52)
\[ u^k_z = M_{11} \sum_{n=0}^{\infty} A_n^kW_n^m(z) + \left[ \frac{1}{r}H_1^{(2)}(k_r)(\cos^2 \theta - 1) + \frac{k}{2}(H_0^{(2)}(k_r) - H_0^{(2)}(k_r))\cos^2 \theta \right] - M_{22} \sum_{n=0}^{\infty} A_n^kW_n^m(z) + \left[ \frac{H_1^{(2)}(k_r)\sin 2\theta}{2r} + \frac{k}{4}((H_0^{(2)}(k_r) - H_0^{(2)}(k_r))\sin 2\theta) \right] + \]

\[ M_{33} \sum_{n=0}^{\infty} \frac{k_n}{4iH_m^{(2)}}[a_n \sin(p\pi_0) + a_n q \sin(q\pi_0)]W_n^m(z)H_0^{(2)}(k_r) \]

Hence,

\[ u^k_z = \sum_{n=0}^{\infty} A_n^kW_n^m(z) + \left[ \frac{1}{r}H_1^{(2)}(k_r)(\cos^2 \theta - 1) + \frac{k}{2}(H_0^{(2)}(k_r) - H_0^{(2)}(k_r))\cos^2 \theta \right] - \sum_{n=0}^{\infty} A_n^kW_n^m(z) + \left[ \frac{H_1^{(2)}(k_r)\sin 2\theta}{2r} + \frac{k}{4}((H_0^{(2)}(k_r) - H_0^{(2)}(k_r))\sin 2\theta) \right] + \]

\[ \left( \frac{\lambda + 2\mu}{\lambda} \right) \sum_{n=0}^{\infty} \frac{k_n}{4iH_m^{(2)}}[a_n \sin(p\pi_0) + a_n q \sin(q\pi_0)]W_n^m(z)H_0^{(2)}(k_r) \]

Figure 7. Micro crack formed at quarter the thickness in the z direction from the origin in Aluminum 2024-T3 plate.

A Matlab code was generated for Eq (51) and (54). Consider the case of a mode 1 like fracture micro crack formed in a plate as shown in figure 7. The material is considered as Aluminum 2024-T3 with thickness 1.6 mm. The formation of the micro crack is assumed to be a Gaussian unit moment tensor excitation similar to Eq (48) with 1.25 μs rise time. The normalized out-of-plane displacement field at a distance of 50 mm from the source location is plotted and is shown below.
CONCLUSION

Representing the excitation for wave propagation due to a sudden formation of dislocation by an equivalent moment tensor is a widely used methodology in seismology. The same approach can be implemented in the case of acoustic emission due to micro crack formation in metallic structures. The components of moment tensor changes according to the nature of micro crack. If the micro crack is a mode 1 like micro crack, the excitation will be equivalent to three independent force dipole excitations concentrated at the center of gravity of the micro crack. If the micro crack is a mode 2 like micro crack the equivalent moment tensor will have components in $M_{31}$ and $M_{13}$. For a mode 3 like micro crack $M_{23}$ and $M_{32}$ moment tensor components will construct the equivalent excitation. The mechanism of micro crack formation has a huge influence on the type of excitation and the generated wave fields due to acoustic emission. The crack formation generates acoustic emission (AE) events. The effect of AE events is to create waves that can be sensed at a distance. In thin-wall structures, the AE waves travel in the form of multi-modal guided waves. This paper has shown how the formation these multi-modal AE guided waves can be predicted using the moment tensor concept.

ACKNOWLEDGMENTS

The authors are grateful for the financial support from US Department of Energy (DOE), Office of Nuclear Energy, under grant numbers DE-NE 0000726 and DE-NE 0008400.

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