Detection and Characterization of Cracks Using Lamb Wave Propagation

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ABSTRACT

The goal of a structural health monitoring system is to implement processes to detect and characterize damages in engineering structures. These systems may use many different physical phenomena to detect and characterize damages. One of these phenomena is elastic wave propagation in thin plate like structures, known as Lamb wave propagation. To develop a SHM (structural health monitoring) system based on Lamb wave propagation we first need to understand how Lamb waves interact with different damage types. The aim of this study is to develop an understanding of how Lamb waves interact with different types of damages using analytical modeling and FEM. The goal is to identify and characterize a surface breaking crack by understanding its effect on Lamb waves scattered from it.

INTRODUCTION

In the field of non-destructive evaluation (NDE) and structural health monitoring (SHM), methods based on ultrasonic guided waves are most popular for their ability to inspect large structures. Due to this popularity, studying interaction of guided waves with damage has been a major preoccupation amongst NDE and SHM communities. Therefore, the scattering problem of plate guided waves like Rayleigh-Lamb waves is important in the field of NDE and SHM, where elastic waves are used to evaluate material properties as well as to locate and measure defects in critical structures such as tubing in plants, pipelines in chemical processing facilities, and so on. Guided-waves like Lamb waves are being used to find tiny defects over large distances, in structures with insulations and coatings.

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One of the prevalent damages in thin walled structures in general is fatigue cracks. When Lamb wave encounters a crack it results in scatter field. This scatter field carries the information of the crack. The aim of this study is to develop an understanding of how to identify and characterize a surface breaking crack by understanding its effect on Lamb waves scattered from it by using analytical models and FEM.

Numerous studies have been done to solve this problem [1]–[8]. In most of these studies finite element based approach compared with experimental results were used to understand the scattered field. In other studies [3] modal decomposition method was used to predict the scattered wave field. But the continuous boundary condition was scaled down to discrete points across the boundary to satisfy the boundary conditions as it is very challenging to satisfy the continuous boundary condition. In 1982, Gregory et-al [9], [10] introduced a method called ‘projection method’ to satisfy the continuous boundary conditions. This method was also capable of predicting the singularity in stresses in case of a geometric discontinuity [4]. A modified form of projection method was used by Grahn in 2003 [6]. Although this scalar projection method was simpler and able to predict the scattered wave field, the convergence was slow [7]. In 2012 Moreau et-al [7], [11] used a vector projection method which had faster convergence and showed promising results. Recently Poddar et-al [12], [13] have proposed a vector projection method called complex modes expansion and vector projection (CMEP) to deal with scattering problem from damage in general.

To develop our understanding we started building 2D crack models using CMEP [12], [13].

**CMEP MODEL OF A CRACK**

The idea is that an inclined open crack can be approximated as an inclined notch with vanishing width. Therefore, let there be an inclined notch of width \(L = 2b\), inclined at an angle \(\theta\) to the vertical, along the width of an infinitely long and infinitely wide plate of thickness \(h_1\) at a location \(x = x_0\) with the thickness of the plate at the inclined notch being \(h_2\) (Figure 1). Also, let us imagine that there is a wave field represented by \((\Phi_0, H_0)\), travelling in \(+ve\) \(x\) direction in the Region 1. Let us define the reflected wave field as \((\Phi_1, H_1)\) and the transmitted wave field in Region 3 as \((\Phi_3, H_3)\). Let us also define the wave field inside the inclined notch in Region 2 as \((\Phi_2, H_2)\).

These must satisfy the zero-stress boundary condition at the top and bottom of the plate surfaces and horizontal notch surface,

\[
\sigma_{yy} = 0, \tau_{xy} = 0
\]

We expand these reflected and transmitted wave fields in terms of all possible complex Lamb wave modes corresponding to the complex roots of Rayleigh-Lamb frequency equation. Therefore, the scattered wave field is expressed as
where, for the displacement vectors, subscripts $x$ and $y$ indicate the directions of the displacement. For stresses, the subscript $xx$ stands for the normal stress in $x$ direction and subscript $xy$ stands for the shear stress and subscript 0 stands for incident waves in Region 1, 1 stands for reflected waves in Region 1, 2 stands for the trapped waves in Region 2 and 3 stands for transmitted waves in Region 3. The notations $C_{n_i}^B$, $C_{n_2}^F$, $C_{n_2}^B$ and $C_{n_3}^F$ are the unknown amplitudes of the $n_1$th, $n_2$th and $n_3$th complex Lamb wave modes in Regions 1, 2 and 3, respectively with the superscripts $F$ and $B$ stands for forward and backward propagating waves. In the same way, the incident wave field uses subscript 0, i.e.,

$$
\tilde{u}_0 = \begin{bmatrix}
0 u_x \\
0 u_y
\end{bmatrix} \\
\sigma_0 = \begin{bmatrix}
0 \sigma_{xx} & 0 \tau_{xx} \\
0 \tau_{xy} & 0 \sigma_{yy}
\end{bmatrix}
$$

The boundary conditions at the inclined surface of the notch are traction free. Therefore, using equations (2), (3) yields
\[
\begin{bmatrix}
\sigma_0 + \sigma_1
\end{bmatrix}
\begin{bmatrix}
\cos \theta \\
\sin \theta
\end{bmatrix} = 0 \quad \text{and} \quad \sigma_3
\begin{bmatrix}
\cos \theta \\
-\sin \theta
\end{bmatrix} = 0 \quad \text{at the notch surfaces} \quad (4)
\]

\[
\begin{bmatrix}
\sigma_0 + \sigma_1
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix} = \sigma_2
\begin{bmatrix}
1 \\
0
\end{bmatrix} \quad \text{and} \quad \tilde{u}_0 + \tilde{u}_1 = \tilde{u}_2; \quad x = x_0 - b, \quad -h_1/2 \leq y \leq h_2 - h_1/2 \quad (5)
\]

\[
\sigma_3
\begin{bmatrix}
1 \\
0
\end{bmatrix} = \sigma_2
\begin{bmatrix}
1 \\
0
\end{bmatrix} \quad \text{and} \quad \tilde{u}_3 = \tilde{u}_2; \quad x = x_0 + b, \quad -h_1/2 \leq y \leq h_2 - h_1/2 \quad (6)
\]

Equations (4), (5) and (6) are dependent on \( y \). To make them independent of \( y \), we project them on appropriate complete orthogonal vector space [6]. To take advantage of the orthogonality of the Lamb wave modes [14] we project the stress boundary conditions, in equations (4), (5) and (6) onto the conjugate of the displacement vector space of the complex Lamb wave modes in Region 1. By the same token, we project the displacement boundary conditions, equations (5) and (6), onto the conjugate of the stress vector space of the complex Lamb wave modes in Region 2. This approach, following the general principles of the complex reciprocity theorem outlined in references [14], [15], [16], yield the following results:

The projection vector space for stress equations (4), (5) and (6) is,

\[
\tilde{\mathbf{u}}_1 = \text{conj} \begin{bmatrix}
\mathbf{u}_x \\
\mathbf{u}_y
\end{bmatrix}_\pi = \begin{bmatrix}
\tilde{\mathbf{u}}_x \\
\tilde{\mathbf{u}}_y
\end{bmatrix}_\pi, \quad \mathbf{n}_1 = 1, 2, 3, ...
\quad (7)
\]

After projecting equation (4) and (5) onto equation (7), the stress boundary conditions in equations (4) and (5), take the form,

\[
\int_{h_2 - h_1/2}^{h_2/2} \left( \begin{bmatrix}
\sigma_0 + \sigma_1
\end{bmatrix}
\begin{bmatrix}
\cos \theta \\
-\sin \theta
\end{bmatrix} \right) \cdot \tilde{\mathbf{u}}_1 dy + \int_{-h_1/2}^{h_1/2} \left( \begin{bmatrix}
\sigma_0 + \sigma_1
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix} \right) \cdot \tilde{\mathbf{u}}_1 dy
\]

\[
= \int_{-h_1/2}^{h_1/2} \left( \begin{bmatrix}
\sigma_2
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix} \right) \cdot \tilde{\mathbf{u}}_1 dy
\quad (8)
\]

where, \( \int_a^b P \cdot Q dy = \langle P, Q \rangle_a^b \) represents the inner product. Upon doing the same operation on equation (4) and (6), the stress boundary conditions in equation (4) and (6) take the form,

\[
\int_{h_2 - h_1/2}^{h_2/2} \left( \begin{bmatrix}
\sigma_3
\end{bmatrix}
\begin{bmatrix}
\cos \theta \\
-\sin \theta
\end{bmatrix} \right) \cdot \tilde{\mathbf{u}}_1 dy + \int_{-h_1/2}^{h_1/2} \left( \begin{bmatrix}
\sigma_3
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix} \right) \cdot \tilde{\mathbf{u}}_1 dy = \int_{-h_1/2}^{h_1/2} \left( \begin{bmatrix}
\sigma_2
\end{bmatrix}
\begin{bmatrix}
1 \\
0
\end{bmatrix} \right) \cdot \tilde{\mathbf{u}}_1 dy
\quad (9)
\]

The projection vector space for displacement equations (5) and (6) is,
After projecting equation (5) onto equation (10), the displacement boundary conditions in equation (5), takes the form,

\[ \int_{-h/2}^{h/2} [\bar{u}_0 + \bar{u}_1] \cdot \bar{\sigma}_2 \, dy = \int_{-h/2}^{h/2} \bar{u}_2 \cdot \bar{\sigma}_2 \, dy \]  

(11)

Upon doing the same operation on equation (6), the stress boundary conditions in equation (6), take the form,

\[ \int_{-h/2}^{h/2} \bar{u}_3 \cdot \bar{\sigma}_2 \, dy = \int_{-h/2}^{h/2} \bar{u}_2 \cdot \bar{\sigma}_2 \, dy \]  

(12)

For numerical calculation we consider finite values for the indices \( n_1, n_2, n_3, \bar{n}_1, \bar{n}_2 \). We assume, \( n_1 = 1, 2, 3, ..., N_1 \), \( n_2 = 1, 2, 3, ..., N_2 \), \( n_3 = 1, 2, 3, ..., N_3 \), \( \bar{n}_1 = 1, 2, 3, ..., \bar{N}_1 \), \( \bar{n}_2 = 1, 2, 3, ..., \bar{N}_2 \). By assuming \( N_1 = N_2 = N_3 = \bar{N}_1 = \bar{N}_2 = N \), we get \( 4N \) equations in \( 4N \) unknowns. These equations, upon manipulation, take the form

\[ \Rightarrow [T]_{2N \times 2N} \{C\}_{2N \times 1} = [S]_{2N \times 1} \]  

(13)

with \( C \) being the unknown coefficient matrix for reflected and transmitted waves. In this analytical process if we consider vanishing notch width, \( 2b \to 0 \), we obtain the analytical setup for an inclined crack.

**FEM MODEL OF A CRACK**

To develop an understanding of scattering of Lamb waves from a crack, we made a finite element model using commercial FE software ANSYS. The thickness was chosen to be 5 mm. The frequency range for comparison was chosen to be 25 kHz to 300 kHz to avoid exciting A1 and other higher propagating modes. To achieve accuracy, PLANE183 element with 0.1 mm length was chosen.

To obtain results in frequency domain we did harmonic analysis in ANSYS. To achieve a transient response from a finite dimensional model by harmonic analysis we introduced non reflective boundary (NRB) at both the end of the model to eliminate the boundary reflections and thereby eliminating standing wave. The NRBs were created using COMBIN14 spring damper element. These elements were arranged at the top and bottom surfaces of the NRBs and also at both the ends. The damping coefficients of the elements were varied gradually in a sinusoidal pattern starting from zero (Figure 2). This eliminated any reflection from the edge of the NRB itself. The incident S0 wave was generated by exciting the top and bottom nodes simultaneously.
at the transmission location. The scattered waves were detected at the top and bottom nodes at the sensing locations in Region 1 and Region 3.

![Figure 2. Schematics of the finite element model for harmonic analysis.](image)

A similar model was created without a crack to capture the incident wave field only. Then we subtract the incident wave field from the wave field at Region 1 obtained from the step model to get the reflected wave fields. The transmitted wave field was obtained directly from Region 3. The symmetric and antisymmetric modes were separated by averaging the summation and subtraction of displacements at the top and bottom nodes.

**CONCLUSION**

From Figure 3 we can see that, for vertical cracks the scattered wave amplitudes are directly proportional to the depth of the crack. For incident S0 mode the reflected S0, reflected A0 and transmitted A0 amplitudes increases linearly with the depth of the crack. Also for the same case the transmitted S0 amplitude decreases linearly with the increase of the depth of the crack. This shows us, at least for the forward problem, the depth of the crack can be characterized and predicted by capturing the amplitude of the reflected and transmitted Lamb wave modes.

For inclined cracks with incident S0 mode, we can see from Figure 4 that for the same projected vertical depths the reflected S0 wave mode shows a sudden dip in the amplitude. The dip happens at a frequency depending on the inclination of the crack. From the same figure we can also see that this frequency varies linearly with the inclination angles. A similar trend is visible in the transmitted S0 mode only, instead of a dip there appears a raise. However unlike the case of vertical cracks, for inclined cracks it is difficult to extract the information about inclination from the converted A0 modes that are reflected or transmitted.

Therefore from Figure 3 and Figure 4 we conclude that in the case of Lamb waves scattering from a crack in a plate the information about the crack is carried by the scattered wave modes. For an incident S0 Lamb wave mode the information about the depth and inclination of the crack can be extracted from the reflected and transmitted S0 Lamb wave mode without much difficulty.
Figure 3. Variation of scatter coefficients for vertical cracks on 5mm thick plate for incident S0.

Figure 4. Variation of scatter coefficients for inclined cracks with half thickness vertical projected depths for incident S0.

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