POWER AND ENERGY TRANSDUCTION IN PIEZOELECTRIC WAFER ACTIVE SENSORS FOR STRUCTURAL HEALTH MONITORING: MODELING AND APPLICATIONS

by

Bin Lin

Bachelor of Science
Nanjing University, China, 2000

Master of Science
Nanjing University, China, 2003

Master of Industrial Statistics
University of South Carolina, 2008

Submitted in Partial Fulfillment of the Requirements
For the Degree of Doctor of Philosophy in
Mechanical Engineering
College of Engineering & Computing
University of South Carolina

2010

Accepted by:
Victor Giurgiutiu, PhD, Major Professor; Chairman, Examining Committee
Yuh J. Chao, PhD, Committee Member
Xiaodong Li, PhD, Committee Member
Timir Datta, PhD, Committee Member
James Buggy, PhD, Dean of the Graduate School
ACKNOWLEDGMENTS

Over the years I’ve been working on my degree, I have received considerable assistance and encouragement. I would like to express my great gratitude to my academic advisor, Dr. Victor Giurgiutiu, for introducing me to the field of smart materials and structures. His kind patience, immense support and technical knowledge have been a constant source of encouragement for me. I am greatly appreciative of the time he has dedicated to guiding me throughout the course of my Ph. D. studies. Working under his supervision has been a truly rewarding experience, which I will carry with me throughout my life.

I would like to thank the other members of my committee: Dr. Yuh Chao, Dr. Xiaodong Li, and Dr. Timir Datta, for their advice and encouragement. The committee members have given me so many valuable suggestions and insightful feedback during my dissertation writing and revision process.

I appreciate the help and support of my colleagues at the Laboratory for Active Materials and Smart Structures (LAMSS). I am thankful to Lingyu Yu, Buli Xu, Jingjing Bao, Radu Pomirleanu, Weiping Liu, Giola Santoni-Bottai, Adrian Cuc, Christopher Jenkins, James Kendall, James Doane, Patrick Pollock, Thomas Behling, Gregory Crachilo, Joel Bost, Thomas Ball, Sinan Meric for their help and encouragement.
In addition, I would like to express my thankfulness to National Science Foundation, National Institute of Technology, and Air Force Office of Scientific Research, for their financial support.

Finally, I would like to thank my family members. To my dad Zhian Lin and mum Sizhu Xu, thank you for your support and encouragement all the way through my Ph.D. study. To my dear wife Fan Zhu, thank you for supporting me all these years. Through good and bad times, you love and support never changed. To Heidi, my lovely daughter, you have brought me endless joy and happiness.
ABSTRACT

The goal of this research is to develop the scientific and engineering basis for piezoelectric wafer active sensors (PWAS) in structural health monitoring (SHM). PWAS are studied in both theoretical and practical aspects. The research focuses on three areas: (I) modeling of PWAS power and energy transduction; (II) PWAS installation and durability study; (III) novel miniaturized PWAS approaches for SHM.

In part I, the dissertation presents a systematic investigation of power and energy transduction of PWAS. The PWAS transmitter model, receiver model, and pitch-catch model are developed. Frequency response functions are developed for voltage, current, complex power, active power, etc. The power and energy transduction with PWAS attached to a structure are investigated and the following aspects are discussed: (i) the electrical power of transmitter and receiver; (ii) the mechanical power of transmitter and receiver; (iii) wave power and energy in structures. A parametric study with respect to PWAS size, PWAS impedance, and external electrical load was performed to provide a PWAS design tool for PWAS actuating, sensing and power harvesting applications. In addition, the analytical PWAS modeling is compared with coupled-field finite element results.

In part II, the dissertation presents an experimental study in which issues of PWAS fabrication, installation, and testing were investigated. The installation procedure of
PWAS on metallic structures was examined statistically using design of experiments method (DOE) with pre-manufactured piezoelectric wafers adhesively bonded to the structural surface. The durability and survivability of PWAS on metallic structures under various exposures (temperature cycling, freeze-thaw, outdoor environment, operational fluids, large strains, fatigue load cycling) were tested thoroughly. Both installation DOE study and durability tests showed that the bonding layer is the weak link that may lead in-service failure due to loss of contact with the structural substrate.

In part III, the dissertation explores several novel PWAS configurations that may overcome the shortcomings of conventional PWAS. The study of novel PWAS configurations includes: composite-PWAS, PVDF-PWAS, and nano-PWAS. Methods for in-situ fabrications of composite and PVDF-PWAS on curved and/or complicated structural surfaces were explored. A novel nano-PWAS concept was also investigated. The nano-PWAS requires much less power and can be fabricated directly on the structure. In collaboration with University of Texas at San Antonio, University of Texas Arlington, Army Research Laboratory, several deposition methods, materials, and substrates were investigated for thin-film nano-PWAS fabrication. As a result, nano-scale BaTiO$_3$ thin films with enhanced piezoelectric properties were developed on Ni and Ti substrates.

The dissertation ends with conclusions and suggestions for future work.
# TABLE OF CONTENTS

Acknowledgments ............................................................................................................. ii  
Abstract ............................................................................................................................. iv  
List of Tables ...................................................................................................................... x  
List of Figures .................................................................................................................. xii  
List of Symbols and Abbreviations .............................................................................. xxv  

## 1 Introduction ............................................................................................................1  
1.1 Motivation ............................................................................................................1  
1.2 Research Goal, Scope and Objectives .................................................................3  
1.3 Research Areas and Dissertation Organization ....................................................4  

### PART I  PWAS Modeling .............................................................................................5  

## 2 State of the Art of PWAS Modeling .....................................................................6  
2.1 State of the Art of PWAS Transducer Modeling ...................................................6  
2.2 State of the Art of PWAS Energy Transfer Modeling ...........................................11  
2.3 State of the Art of PWAS Finite Element Modeling ............................................12  
2.4 Research Challenges in PWAS Modeling .............................................................13  

## 3 Simplified PWAS Model .....................................................................................14  
3.1 1-D Constitutive Equation .................................................................................14  
3.2 PWAS Pin-force Model ....................................................................................15  
3.3 PWAS Transmitter and Receiver ....................................................................17  
3.4 Pitch-catch Model ............................................................................................24  
3.5 Frequency Response Function .........................................................................26  
3.6 Non-harmonic Excitation .................................................................................26
# Table of Contents

## 4 Single PWAS Analysis Using Wave Propagation Model
- 4.1 Wave Propagation Method .........................................................28
- 4.2 Transmitter Power and Energy ..................................................41
- 4.3 Receiver Power and Energy.........................................................76
- 4.4 Power and Energy under Non-harmonic Voltage Excitation ............87

## 5 Pitch-catch Analysis Using Wave Propagation Model
- 5.1 Pitch-catch Wave Propagation Analysis .........................................91
- 5.2 Pitch-catch Frequency Response Function .......................................100
- 5.3 Pitch-catch Power and Energy Analysis ........................................106
- 5.4 Numerical Simulation ..................................................................123

## 6 Single PWAS Analysis Using Normal Mode Expansion Model
- 6.1 Vibration Analysis using NME ......................................................128
- 6.2 Frequency Response Function using NME .....................................135
- 6.3 Numerical Simulation ..................................................................137

## 7 Pitch-catch Analysis Using Normal Mode Expansion Model
- 7.1 Pitch-catch Vibration Analysis ......................................................139
- 7.2 Pitch-catch Frequency Response Function Using NME ......................144
- 7.3 Numerical Simulation ..................................................................146

## 8 Coupled-Field Finite Element Analysis of PWAS Behavior
- 8.1 Introduction .................................................................................150
- 8.2 FEM Mesh Size Effects ..............................................................152
- 8.3 PWAS Impedance ......................................................................154
- 8.4 PWAS Resonator Mode Shapes ....................................................163
- 8.5 WAS Electric Field ......................................................................166
- 8.6 PWAS Poling Process Analysis .....................................................168
- 8.7 Pitch-catch FEM Analysis ............................................................173

## Part II PWAS Installation and Durability Study

## 9 PWAS Installation and Quality Check
- 9.1 State of the Art ............................................................................179
9.2 Reliable PWAS Installation and PWAS Quality Study ....................... 180
9.3 Quality Check .............................................................................. 184

10 PWAS Installation Evaluation Using Statistical Design
   of Experiment .................................................................................... 193
10.1 Introduction of Design of Experiment ............................................ 194
10.2 DOE Analysis of PWAS Installation ............................................. 198

11 Durability and Survivability of PWAS Transducers ....................... 213
11.1 Overview of PWAS Durability and Survivability Studies .............. 213
11.2 Cryogenic Temperature ............................................................... 215
11.3 High Temperature ....................................................................... 218
11.4 Temperature Cycling ................................................................. 223
11.5 Environmental Outdoors Exposure .............................................. 227
11.6 Submersion Exposure .................................................................. 230
11.7 Large Strains and Fatigue Cyclic Loading .................................... 233

PART III Novel PWAS Configurations .................................................. 236

12 In-situ Fabricated PWAS ................................................................. 237
12.1 Introduction .................................................................................. 237
12.2 State of the Art for In-situ Fabricated Smart Material Active Sensors 239
12.3 Composite PWAS ....................................................................... 247
12.4 PVDF PWAS .............................................................................. 252

13 Thin-Film PWAS ............................................................................ 264
13.1 Introduction To Thin-film Technology .......................................... 264
13.2 State of the Art ............................................................................ 266
13.3 PZT Thin Film Using Sputter Deposition .................................... 280
13.4 BSTO Thin-Films Using Metal-organic Solution Deposition ........ 285
13.5 BTO Thin-Films Using Pulsed Laser Deposition .......................... 297
13.6 nano-PWAS: Structurally Integrated Thin-Film Active Sensors .... 307

14 Conclusions, Major Findings, and Future work ............................. 318
14.1 Conclusions and Major Findings ............................................... 318
14.2 Recommended Future Work ....................................................... 322

References .......................................................................................... 324
Appendix A  Material Properties Input in ANSYS Software.................................333
  A.1  Stiffness/compliance Matrix.............................................................................334
  A.2  Permittivity Matrix..........................................................................................336
  A.3  Piezoelectric Constant Matrix.........................................................................336
Appendix B  Transformation of Material Properties ...........................................338
Appendix C  Integral of Analytical Flexural Modes .............................................342
  C.1  Integral of Exact Formula................................................................................342
  C.2  Integral of Approximate Formula....................................................................346
Appendix D  PWAS Charge Amplifier...................................................................350
  D.1  Charge Amplifier Principle..............................................................................350
  D.2  Operational Amplifier Open-loop Gain...........................................................353
  D.3  PWAS Charge Amplifier.................................................................................354
Appendix E  Cauchy’s Residue Theorem for Complex Integrals Evaluation............357
  E.1  Cauchy’s Integral Theorem..............................................................................357
  E.2  Cauchy’s Residues Theorem............................................................................361
# LIST OF TABLES

Table 4.1  Electrical power expression .................................................................47
Table 4.2  Typical material properties for aluminum alloy.................................48
Table 4.3  Piezoelectric properties of the PWAS material (APC-850).................49
Table 4.4  Transmitter simulation parameters....................................................49
Table 4.5  Transmitter size effect simulation parameters....................................65
Table 5.1  Pitch-catch simulation parameters......................................................123
Table 8.1  7mm square PWAS (APC-850) resonances and anti-resonances.......166
Table 8.2  PWAS capacitance after poling............................................................172
Table 9.1  Comparison of PWAS providers.........................................................186
Table 9.2  Theoretical free capacitance for 7 mm round and square PWAS........189
Table 9.3  PWAS capacitance on plate #1 and #2...............................................190
Table 9.4  Max and Min values for the magnitude of the impedance and admittance of the 7 mm square PWAS on plate #1 .................................................................190
Table 10.1 List of factors and their ranges for DOE factorial designs....................199
Table 10.2 Measurement of free PWAS used in DOE experiment......................201
Table 10.3 List of factors and responses in factorial design experiment...............202
Table 11.1 Adhesives operating temperature ......................................................215
Table 11.2  Circular plate specimens for outdoors test ..................................................228
Table 11.3  Summary of PWAS specimens (experiment started in Nov 2003).................................230
Table 11.4  Fatigue specimens overview (R = 0.1)..........................................................235
Table 12.1  Comparison of different materials’ parameters..............................................253
Table 12.2  Comparison of the strain gauge, PVDF PWAS, PZT PWAS responses .....................257
Table 12.3  Comparison of PWAS responses ......................................................................257
Table 12.4  Comparison of the cantilever beam natural frequencies......................................258
Table 12.5  Comparison of PZT-PWAS and PVDF-PWAS in pitch-catch experiment. ....................263
Table 13.1  MOSD experiment: BST 80/20 on Ti: 16 total films...........................................287
LIST OF FIGURES

Figure 2.1 Schematic of a piezoelectric wafer active sensor (Giurgiutiu and Zagrai, 2000).................................................................................................8

Figure 2.2 PWAS impedance modeling: (a) PZT disk wafer active sensor constrained by the structural stiffness; (b) Experimental and calculated impedance for vibration (Giurgiutiu and Zagrai, 2002)...........................................................................................................9

Figure 2.3 PWAS interaction with Lamb waves in a plate: (a) S0 mode; (b) A0 mode. (Giurgiutiu and Bao, 2003). .........................................................10

Figure 3.1 Schematic of a free piezoelectric wafer active sensor.................................15

Figure 3.2 Schematic of a single PWAS attached to a bar with pin force.................................16

Figure 3.3 Schematic of PWAS pin force that contributes to axial force and bending moment. ..............................................................................................17

Figure 3.4 Schematic of a PWAS connected with measurement equipment (input impedance $Z_e$). .................................................................20

Figure 3.5 Schematic of a PWAS pitch-catch setup on a bar.............................................24

Figure 3.6 Non-harmonic excitation signal (3-count tone burst signal) and its frequency spectrum. .................................................................27

Figure 4.1 Schematic of a single PWAS attached to an infinite bar.................................28

Figure 4.2 Contour for evaluating the axial displacement solution under ideally bonded PWAS excitation. The residue at positive wavenumber is included, which the residue for the negative wavenumber is excluded.................................................................31
Figure 4.3 Contour for evaluating the flexural displacement solution under ideally bonded PWAS excitation. The residue at positive wavenumber, \( +\xi_F \), is included, while the residue for the negative wavenumber, \( -\xi_F \), is excluded. Also included is the residue at the imaginary wavenumber, \( +i\xi_F \).

Figure 4.4 PWAS transmitter power and energy flow chart.

Figure 4.5 Transmitter PWAS active power, reactive power, power rating and peak power.

Figure 4.6 Transmitter PWAS power and reactive and active power ratio.

Figure 4.7 Schematic of a single PWAS attached to an infinite bar.

Figure 4.8 Transmitter power simulation: (a) electrical active power; (b) mechanical power at \( A \) and \( A' \) (note \( P_{\text{active}} = \langle P_A \rangle + \langle P_{A'} \rangle \)).

Figure 4.9 Axial and flexural wave power.

Figure 4.10 Parametric study of transmitter size effect of admittance.

Figure 4.11 Parametric study of transmitter size effect for real part of admittance.

Figure 4.12 Parametric study of transmitter size effect for power rating.

Figure 4.13 Parametric study of transmitter size effect for wave power.

Figure 4.14 Axial wave power under constant 10-V voltage input.

Figure 4.15 Flexural wave power under constant 10-V voltage input.

Figure 4.16 Input voltage under constant 100-mA current input.

Figure 4.17 Power rating under constant 100-mA current input.

Figure 4.18 Wave power under constant 100-mA current input.
Figure 4.19 Input voltage under constant 10-W power rating input. ................................................................. 73

Figure 4.20 Input current under constant 10-W power rating input. ................................................................. 73

Figure 4.21 Wave power under constant 10-W power rating input. ................................................................. 74

Figure 4.22 Input voltage under constant 10-mW wave power output. ............................................................... 75

Figure 4.23 Input current under constant 10-mW wave power output. ............................................................... 75

Figure 4.24 Wave power under constant 10-mW wave power output. ............................................................... 76

Figure 4.25 Receiver output voltage under constant 100-microstrain axial wave input (receiver size varies for sensing). ........................................................................................................ 81

Figure 4.26 7-mm PWAS Receiver output voltage under constant 100-microstrain axial wave (measurement equipment impedance varies for power harvesting). ................................................................. 82

Figure 4.27 Receiver output voltage under constant 1-W axial wave input (receiver size varies for sensing). .................................................................................................................. 83

Figure 4.28 Receiver output voltage under constant 1-W axial wave input (measurement equipment impedance varies for power harvesting). ........................................................................ 84

Figure 4.29 Receiver output voltage under constant 100-microstrain flexural wave input (receiver size varies for sensing). ........................................................................................................ 85

Figure 4.30 7-mm receiver output voltage under constant 100-microstrain flexural wave input (measurement equipment impedance varies for power harvesting). ................................................................. 85

Figure 4.31 Receiver output electrical power under constant 100-mW power axial wave input (receiver size varies for sensing). ........................................................................................................ 86
Figure 4.32 Receiver output electrical power under constant 100-mW power axial wave input (measurement equipment impedance varies for power harvesting) ........................................................................................................86

Figure 5.1 PWAS pitch-catch setup on an infinite bar .................................................................91
Figure 5.2 Pitch-catch power and energy flow chart ..................................................................106
Figure 5.3 Transmitter electrical active power and mechanical power ....................................124
Figure 5.4 Comparison of active electrical and mechanical power at PWAS edges ..................124
Figure 5.5 Comparison of active electrical power and mechanical power at PWAS edges ........125
Figure 5.6 Receiver mechanical and electrical output power .....................................................125
Figure 5.7 Frequency response function of a pitch-catch simulation ........................................126
Figure 5.8 Comparison of NME and wave propagation simulation of pitch-catch receiver output voltage .........................................................................................................................127
Figure 5.9 Comparison of flexural and A0 wave speed for 1mm thick aluminum .........................127
Figure 6.1 Schematic of a single PWAS attached to a beam ......................................................128
Figure 6.2 The total horizontal displacement, $u_p$, results from the superposition of axial displacement $u$ and the rotation $w'$ ...................................................................................................................133
Figure 6.3 A 100-mm aluminum beam (a) Real part of Impedance; (b) FRF .....................................138
Figure 7.1 PWAS pitch-catch setup on a beam ...........................................................................139
Figure 7.2 Simulation setup of a pitch-catch experiment ..............................................................147
Figure 7.3 Simulation results of FRF of a pitch-catch setup on a 1220-mm long beam ................148
Figure 7.4 Transmitter non-harmonic excitation signal (3-count 100-kHz Hanning window tone-burst) .................................................................................................................................148
Figure 7.5 Simulation of receiver output voltage in a pitch-catch setup.......................................................................................................149

Figure 8.1 1mm thick Aluminum plate (a) phase velocity (b) group velocity........................................................................................154

Figure 8.2 Comparison of impedance from theoretical calculation, coupled-field finite element simulation and experimental result, (a) 7 mm diameter, 0.2 mm thickness round APC 850 PWAS; (b) 7mm square, 0.2mm thickness square PWAS; (c) 25 mm x 5 mm x 0.15 mm rectangular PWAS........................................................................................................156

Figure 8.3 Experimental setup (a) illustration of PWAS bonded on a circular plate; (b) test specimen.....................................................158

Figure 8.4 Comparison of experimental and analytical impedance of PWAS bonded on a circular plate (Giurgiuțiu et al., 2002). ..............................................................................................................................158

Figure 8.5 Comparison of experimental and FEM impedance of the same structure. ...........................................................................159

Figure 8.6 (a) PWAS sample bonded on a circular plate with crack at 25mm from center (b) detail mesh in crack area.........................................................................................................................160

Figure 8.7 Comparison of the experimental results with FEM model impedance with and without incipient crack. ..................................................................................................................160

Figure 8.8 PWAS array (a) 9-element PWAS array with 7-mm square PWAS.........................................................................................161

Figure 8.9 Illustration of 2-D PWAS array (Total size 56 x 28 mm), (a) top view; (b) side view. ..........................................................161

Figure 8.10 Model and impedance spectra of the free PWAS array without bonding on plate (a) model of free PWAS, PWAS electrode at a corner and PWAS electrode in the center of array (b) impedance spectrum. ..................................................................................................................162

Figure 8.11 PWAS resonator electro-mechanical resonance mode shapes and admittance. ......................................................................164
Figure 8.12 PWAS electro-mechanical anti-resonance mode shapes and impedance. .................................................................165

Figure 8.13 Model and impedance spectra of the free PWAS array without bonding on plate (a) model of free PWAS, PWAS Electrode at a corner and PWAS Electrode in the center of array (b) impedance spectrum. .............................................................................167

Figure 8.14 (a) Typical IDT pattern (top view), (b) Multi-physics FEM simulation of the cross-section electric field lines and remnant polarization after poling. .................................................................167

Figure 8.15 Flowchart of SAW-type sensor poling process ..........................................................168

Figure 8.16 Experimental setup for PWAS poling .................................................................170

Figure 8.17 PWAS capacitance vs temperature. .................................................................171

Figure 8.18 Measured PWAS impedance after different poling conditions. (a) 100 kHz to 900 kHz; (b) 1 MHz to 15 MHz .............................................................................172

Figure 8.19 Mesh plot of PWAS attached to a bar ....................................................................173

Figure 8.20 FRF in 2D FEM model .....................................................................................173

Figure 8.21 Receiver voltage response under a 3-count 100 kHz tone-burst excitation. .............................................................................174

Figure 8.22 Pitch-catch experimental setup. In comparison with FEM results. .............................................................................174

Figure 8.23 FRF in 3-D FEM model .....................................................................................175

Figure 8.24 Comparison of FEM and experimental result of receiver output signal. .............................................................................175

Figure 9.1 Installation procedure for piezoelectric active sensors. .............................................181

Figure 9.2 The installation kit for strain gages (Measurements Group, Inc) was used in the bonding of piezoelectric active sensors. .............................................................................182

Figure 9.3 Impedance characteristic of a free 7-mm square PWAS: (a) real part of impedance; (b) imaginary part of impedance. .............................................................................187
Figure 9.4  EUSR experimental setup. .................................................................188

Figure 9.5  Specimen aluminum plat and the PWAS array: a) 4-ft square plate with 8-element PWAS array at its center; b) details of the 8-element PWAS array with 7-mm square PWAS; c) details of the 8-element PWAS array with 7-mm diameter PWAS ...........................................189

Figure 9.6  Real and imaginary impedance of 7-mm Square PWAS in EUSR array on Plate 1. .................................................................191

Figure 9.7  Real and imaginary Admittance of 7 mm Square PWAS in EUSR array on Plate 1. .................................................................192

Figure 10.1  PWAS attached to a substrate structure through the adhesive bond layer. .................................................................193

Figure 10.2  Installation factors DOE experimental setup. .................................200

Figure 10.3  Statistics graphical summary (a) diameter, (b) thickness, (c) capacitance. .................................................................203

Figure 10.4  Adhesive thickness based on $2^4$ full factorial designs (a) Pareto analysis (b) Normal probability plot. .................................205

Figure 10.5  Adhesive thickness based on $2^3$ full factorial designs (a) Pareto analysis (b) Normal probability plot. .................................206

Figure 10.6  Pitch (Sending from outside) and catch (receiving from center reference PWAS) signal based on $2^4$ full factorial designs. (a) Pareto analysis, (b) Normal probability plot. .................................................................208

Figure 10.7  Pitch (Sending from outside) and catch (receiving from center reference PWAS) signal based on $2^4$ full factorial designs. (a) Adhesive main effect, (b) Interaction plot.................................................................209

Figure 10.8  Pitch (Sending from outside) and catch (receiving from center reference PWAS) signal based on $2^3$ full factorial designs. (a) Pareto analysis, (b) Normal probability plot, (c) Adhesive main effect, (d) Interaction plot. .................................................................210

Figure 10.9  Catch signal (receiving the pitch from center reference) of $2^4$ full factorial designs; (a) Pareto analysis (b) Normal probability plot. .................................................................211
Figure 11.1 Free PWAS and PWAS bonded to a metallic plate. .........................214

Figure 11.2 Indication of PWAS operability while submerged in liquid nitrogen. .................................................................................................................216

Figure 11.3 The real part of impedance indication of operability through retention of resonant properties while submerged in liquid nitrogen. .................................................................217

Figure 11.4 The real part of impedance indication of survivability through resumption of resonant properties at room temperature after submersion in liquid nitrogen. .........................218

Figure 11.5 Real part impedance spectrum of a free PWAS: (a) Impedance at room temperature after heating. (b) Amplitude of impedance peak at room temperature after heating. .................................................................220

Figure 11.6 Real part impedance spectrum of a bonded PWAS: (a) Impedance at the elevated temperature, (b) Amplitude of impedance peak at the elevated temperature. ..................................................................................222

Figure 11.7 Temperature cycling .............................................................................223

Figure 11.8 C-Scan adhesive and PWAS interface images of (a) bad bond specimen; (b) good bond specimen. .........................................................225

Figure 11.9 E/M impedance spectrum of PWAS after exposure to temperature cycling: (a) free PWAS; (b) PWAS bonded to a metallic plate .................................................................................................226

Figure 11.10 (a) Environmental testing of free and bonded PWAS: (a) outdoors test fixture: (b) temperature profile .................................................227

Figure 11.11 (a) E/M impedance spectrum of specimen Bond-8; (b) asymmetric displacement field. ...........................................................................229

Figure 11.12 PWAS submersion test: (a) test containers; (b) 5-mm diameter free PWAS specimen .................................................................................231

Figure 11.13 E/M impedance spectrum of PWAS after submersion exposure: (a) PWAS in saline water; (b) PWAS in distilled water .................................................................................................232

Figure 11.14 Large strains and fatigue cyclic loading tests: (a) large-strain test specimen; (b) fatigue cycling loading test specimen; (c) experimental setup .................................................................................233
Figure 11.15 Large strain tests: (a) Impedance signatures up to $6000 \mu \varepsilon$ (b) micrograph of the cracked PWAS at $7200 \mu \varepsilon$ ................................................................................................234

Figure 11.16 (a) Stress concentration factor for flat plate with hole; (b) S-N curve determined during reported work .................235

Figure 12.1 Optical photograph of directionally solidified BaTiO$_3$-CoFe$_2$O$_4$ eutectic grown in air at 10 mm/h showing lamellar structure of alternating BaTiO$_3$ and CoFe$_2$O$_4$ elongated grains (after Echigoya et al.)..........................................................................................................244

Figure 12.2 Nanoscale magnetoelectric composites consisting of multiferroic BaTiO$_3$-CoFe$_2$O$_4$ structures: (a) (A) Superlattice of a spinel structure (top) and a perovskite structure (middle) on a perovskite substrate (bottom). (B) resulting multilayer configuration; (C) Epitaxial alignment of a spinel (top left) and a perovskite (top right) on a perovskite substrate (bottom). (D) resulting pillars substrate; (b) TEM picture of an experimental sample showing self-assembled nanostructured CoFe$_2$O$_4$ pillars with ~20 nm diameter embedded in a BaTiO$_3$ matrix (after Zheng et al.).................................................................246

Figure 12.3 Composite PWAS experiments: materials used; (b) poling apparatus.....................................................................................248

Figure 12.4 Composite PWAS sample fabricated on a thin aluminum plate was used in measuring the impact waves .................................................................................................................249

Figure 12.5 The electric voltage signal measured by the impact hammer, (a) before poling (b) after poling..............................................250

Figure 12.6 Piezoelectric composite PWAS fabricated on a cantilever beam: (a) experimental setup; (b) close-up view of the top surface showing the piezoelectric composite PWAS; (c) close-up view of the bottom surface showing the conventional piezoceramic PWAS..............................................................................251

Figure 12.7 Measured electric signals during the plucked beam experiments: (a) conventional piezoceramic PWAS; (b) piezoelectric composite PWAS. .................................................................253
Figure 12.8  PZT-PWAS, PVDF-PWAS, and strain gauge on a cantilever beam: (a) experimental setup; (b) close-up view of the bottom surface showing the 200 \( \mu m \) PZT-PWAS and strain gauge; (c) close-up view of the top surface showing the 28 \( \mu m \) and 110 \( \mu m \) PVDF-PWAS.

Figure 12.9  Vibration signal recorded by strain gauge, PVDF-PWAS and PZT-PWAS.

Figure 12.10  Vibration signal and spectrum of magnitude (Fourier transform) recorded by (CH 1) strain gauge; (CH 2) 28 \( \mu m \) thick PVDF-PWAS; (CH 3) 110 \( \mu m \) thick PVDF-PWAS, (CH 4) 200 \( \mu m \) thick PZT-PWAS.

Figure 12.11  Rod impact experimental setup.

Figure 12.12  Stress wave propagation in a rod.

Figure 12.13  Impact responses for free-free boundary condition recorded by (CH 1) strain gauge; (CH 2) 28 \( \mu m \) thick PVDF-PWAS; (CH 3) 52 \( \mu m \) thick PVDF-PWAS, (CH 4) 110 \( \mu m \) thick PVDF-PWAS.

Figure 12.14  Impact responses recorded by (CH 1) strain gauge; (CH 2) 28 \( \mu m \) thick PVDF-PWAS; (CH 3) 52 \( \mu m \) thick PVDF-PWAS, (CH 4) 110 \( \mu m \) thick PVDF-PWAS.

Figure 12.15  Two PWAS mounted on an aluminum plate; the left PWAS as a transmitter, the right PWAS as a receiver.

Figure 13.1  Diagram of sputtering.

Figure 13.2  One possible configuration of a PLD deposition chamber.

Figure 13.3  (a) pitch-catch method; (b) pulse-echo method; (c) boded interface between PWAS and structure.

Figure 13.4  The schematic and real PZT nano film sample.
Figure 13.5 (a) PZT layer observed through scanning electron microscope (SEM). Note the defects in the surface. (b) AFM surface roughness analysis.................................283

Figure 13.6 Three dimensional image (a) area 5 x 5 microns, (b) area 1 x 1 microns.................................................................285

Figure 13.7 USC 15 sample (a) FESEM image (b) AFM image.................................................................289

Figure 13.8 The atomic force microscopy (AFM) results detailing the roughness and grain size as a function of the annealing temperature for samples that underwent a standard annealing treatment, had a thermal oxide layer deposited before undergoing a standard annealing treatment, and underwent a Rapid Thermal Annealing (RTA) treatment.........................................290

Figure 13.9 Rutherford backscattering spectrometry results for the titanium stubs with (a) BST, trace Pt layer, no thermal oxide, and standard anneal; (b) BST, trace Pt layer, no thermal oxide layer, and RTA; (c) BST, trace Pt layer, thermal oxide, and standard anneal; and, (d) unannealed Ti stub and unannealed Ti stub with trace Pt layer. .................................................................292

Figure 13.10 Glancing Angle X-Ray Diffraction patterns for Ti stubs with BST for (a) standard annealing conditions and no thermal oxide, (b) standard annealing conditions with a thermal oxide, and (c) RTA with no thermal oxide. The diffraction peaks for BST are the lower set of numbers, while the diffraction peaks for rutile are shown in italics as the higher set of numbers. (d) Shows a representative planar FE-SEM image of the grain structure of the BST thin films on the Ti stubs. .................................................................295

Figure 13.11 Pulsed laser deposition system experimental setup.................................................................299

Figure 13.12 TEM image of the BTO films on Ni with NiO interlayer (a) Plane-view (b) Cross-section. Inset is SAED pattern.................................................................299

Figure 13.13 TEM image of the BTO films on Ni with no NiO interlayer (a) Cross-section and (b) plan-view. Inset is SAED pattern.................................................................302
Figure 13.14 (a) X-ray diffraction $\theta$-2$\theta$ scans of randomly oriented BTO/NiO/Ni thin film; (b) X-ray diffraction $\theta$-2$\theta$ scans of oriented BTO/Ni thin film; (c) PRM images of the films with randomly oriented BTO/NiO/Ni; (d) orientation preferred BTO/Ni nanostructures. .......................................................... 303

Figure 13.15 Dielectric property showing the hysteresis loop achieved on the ferroelectric BaTiO$_3$ thin films on (a) Ni and (b) Ti. .......................................................... 304

Figure 13.16 BTO on Ti (a) Cross-section and (b) plan-view TEM image of the BTO films on Ti. Inset is SAED pattern of (b). .......................................................... 306

Figure 13.17 $\theta$ - 2$\theta$ XRD pattern of the BTO films on Ti substrate. .......................................................... 307

Figure 13.18 Sequential development of the multi-layer battery-less nano-PWAS phased array. .......................................................... 308

Figure 13.19 (a) PZT plate with SAW type electrode; (b) Impedance resonance of SAW type sensor. .......................................................... 309

Figure 13.20 Proposed thin-film layered nano-PWAS would require orders of magnitude lower voltage (0.015 V vs. 100 V) to achieve the same inplane strain ($S_\parallel = 87.5 \mu \varepsilon$). .......................................................... 311

Figure 13.21 nano-PWAS phased array showing quasi-annular interdigitated electrodes to ensure axisymmetric wave propagation from each transducer. .......................................................... 313

Figure D.1 The charge amplifier consists of an op-amp. In this simplified schematic, $V_o = \text{output voltage}; C_{PWAS} = \text{PWAS capacitance}; C_f = \text{feedback capacitor}; R_f = \text{time constant resistor}; Q = \text{charge generated by PWAS}. .......................................................... 351

Figure D.2 The open loop gain vs frequency (a) measurement circuit (b) results. .......................................................... 353

Figure D.3 Two PWAS on an aluminum plate; left PWAS: transmitter, right PWAS, receiver. .......................................................... 355

Figure D.4 Charge amplifier experimental results. .......................................................... 356
Figure D.5  Comparison of the theory and experiment voltage gain of the specific charge amplifier. ....................................................356

Figure E.1  A crosscut from any closed curve $C$ to unit circle $C_1$. .........................................................................................................360

Figure E.2  A finite number of interior isolated singularities in contour $C$ ...........................................................................................................363
LIST OF SYMBOLS AND ABBREVIATIONS

1-D One-dimensional

2-D Two-dimensional

APC American Piezo Ceramic (abbreviation for a piezoceramic type)

BSTO Barium Strontium Titanate

BTO Barium Titanate

$C_0$ Free PWAS capacitance

c Wave speed in medium

CF-FEM Coupled-field finite element method

$d_{31}$ Piezoelectric constant

$D_i$ Electrical displacement component

DOE Design of experiments

DOF Degree of freedom

$E$ Elastic modulus of the structure

$E_i$ Electric field component

E/M Electro-Mechanical
I, V  Electric current and voltage

$k_i$  Internal stiffness of PWAS actuator

$k_{31}$  Electromechanical coupling coefficient

NDE  Nondestructive evaluation

PLD  Pulsed laser deposition

PVDF  Polyvinylidene fluoride

PWAS  Piezoelectric wafer active sensors

PZT  Lead Zirconate Titanate, (denotes piezoelectric wafer active sensor)

$Q$  PWAS charge

SHM  Structural health monitoring

$S_i$  Strain component

$s_{ij}^E$  Elastic compliance at zero electric field

$T_i$  Stress component

$Y(\omega)$  Electro-mechanical admittance

$Y_{ij}^E$  Complex elastic modulus of the sensor at zero electric field

$Z(\omega)$  Electro-mechanical impedance

$\text{Re}Z(\omega)$  Real part of electro-mechanical impedance

$\text{Im}Z(\omega)$  Imaginary part of electro-mechanical impedance
\[ \varepsilon_{33} \] Permittivity component at zero stress

\[ \varepsilon_0 \] Permittivity of free space: \( \varepsilon_0 = 8.84194 \text{ pF/m} \)

\[ \dot{U} = \frac{\partial}{\partial t} U \]

\[ U' = \frac{\partial}{\partial x} U \]

\[ U = \hat{U} e^{i\omega t} \]

\[ \overline{U} = \text{conj}(U) \]
1 INTRODUCTION

The goal of this research is to develop the scientific and engineering basis for piezoelectric wafer active sensors (PWAS) in structural health monitoring (SHM). This dissertation describes the power and energy transduction of PWAS in modeling and application aspects.

1.1 MOTIVATION

SHM is a method of determining the health of a structure from the readings of a set of permanently-attached sensors that are embedded into the structure and monitored over time. SHM can be performed in basically two ways: passive and active. Passive SHM consists of monitoring a number of parameters (loading stress, environment action, performance indicators, acoustic emission from cracks, etc.) and infers the state of structural health from a structural model. In contrast, active SHM performs proactive interrogation of the structure, detects damage, and determines the state of structural health from the evaluation of damage extension and intensity. Both approaches aim at performing a diagnosis of the structural safety and health, to be followed by a prognosis of the remaining life. Passive SHM uses passive sensors which only “listen” but do not interact with the structure. Therefore, they do not provide direct measurement of the damage presence and intensity. Active SHM uses active sensors that interact with the structure and thus determine the presence or absence of damage. The methods used for active SHM resemble those of nondestructive evaluation (NDE), e.g., ultrasonics, eddy
currents, etc., only that they are used with embedded sensors. Hence, active SHM can be seen as a method of embedded NDE. One widely used active SHM method employs piezoelectric wafer active sensors (PWAS US 7,024,315 B2, USCRF 00330), which send and receive Lamb waves and determine the presence of cracks, delaminations, disbonds, and corrosion. Due to its similarities with NDE ultrasonics, this approach is also known as embedded ultrasonics.

Structure health monitoring plays a significant role in maintaining the safety of a structural system. The mounting costs of maintaining our aging infrastructure and the associated safety issues are a growing national concern. Over 27% of our nation’s bridges are structurally deficient of functionally obsolete. Deadly accidents are still marring our everyday life. In response to these growing concerns, SHM sets forth to determine the health of a structure by monitoring over time a set of structural sensors and assessing the remaining useful life and the need for structural actions. Built-in SHM system capable of detecting and quantifying damage would increase the operational safety and reliability, would conceivably reduce the number of unscheduled repairs, and would bring down maintenance cost.

The type and efficiency of the SHM sensors play a crucial role in the SHM system success. Ideally, SHM sensors should be able to actively interrogate the structure and find its state of health, its remaining life, and the effective margin of safety. Essential in this determination is to find out the presence and extension of structural damage. Currently, structural damage is determined during scheduled inspections with sophisticated NDE equipment and extensive labor costs. The challenge of SHM is to develop inexpensive
active sensors that can be permanently placed on the monitored structure and assess, continuous or on-demand, the state of structural health.

1.2 RESEARCH GOAL, SCOPE AND OBJECTIVES

This dissertation presents an interdisciplinary research that crosses the engineering and science boundaries and creates the foundation for the PWAS power and energy modeling, fabrication and survivability for SHM. The long-term goal of the research described in this dissertation is to develop the scientific basis and knowledge for a wireless miniaturized active-sensor system-on-a-chip.

The scope of this dissertation is to address the issues of PWAS power and energy simulation and development, the durability and survivability of the PWAS and the PWAS-structure interface, and novel in-situ miniaturized fabrication of PWAS for structural health monitoring.

The research objectives are defined as follows:

1. To present the detailed modeling of the PWAS with analytical and coupled-field finite element method (CF-FEM).

2. To trace the power and energy flow of PWAS and structure interface.

3. To investigate the durability and survivability of PWAS under various exposures (temperature cycling, freeze-thaw, outdoor environment, operational fluids, large strains, fatigue load cycling, etc.)

4. To develop the fabrication and optimum design of novel active sensors for structural health monitoring applications.
5. To improve piezoelectric properties through an ordered crystalline structure with quasi single-domain orientation using nano-technology.

6. To demonstrate solutions for the miniaturization and integration of the active sensors with the electronic modules in a chip-size unit (e.g., layered thin-film technology).

1.3 Research Areas and Dissertation Organization

The research focuses on three areas:

1. The first area of this research concentrates on modeling of power and energy of PWAS in correlation with the monitored structure using both analytical and coupled-field finite element methods.

2. The second area of this research focuses on the installation methods and durability study of PWAS under various exposures that examined thoroughly with statistical method.

3. The third area of the research addresses issues of novel miniaturized PWAS approaches for in-situ miniaturized fabrication of PWAS on structural materials.

In corresponding with the three focused areas, the dissertation is organized in three parts:

1. PWAS modeling

2. PWAS installation and quality study

PART I

PWAS Modeling
Piezoelectric wafer active sensors (PWAS) are inexpensive transducers and receivers that operate on the piezoelectric principle. PWAS can be used for vibration control and damage detection. PWAS operated on the piezoelectric principle that couples the electrical and mechanical variables in the material. In linear piezoelectricity, the equations of elasticity are coupled to the charge equation of electrostatics by means of piezoelectric constant (IEEE Standard on Piezoelectricity). PWAS utilize the $d_{31}$ coupling between in-plane strain and transverse electric field for the in-plane mode and the $d_{33}$ coupling for the thickness mode. The use of PWAS for structural health monitoring has followed three main paths: (a) modal analysis and transfer function; (b) electromechanical E/M impedance; (c) wave propagation. The modeling of PWAS is useful for (1) understanding the electrical response of the sensor; (2) sensor screening and quality control prior to installation on the monitored structure.

2.1 State of the Art of PWAS Transducer Modeling

Initially, PWAS were used for vibration control as pioneered by Crawley and deLuis (1987) and Fuller et al. (1990). Modeling of piezoelectric sensor/actuator design for dynamic measurement control was done by Tzou and Tseng (1990) and Lester and Lefebvre (1993). For damage detection, Banks et al. (1996) used PZT wafers to excite a structure and then sense the free decay response. The use of PWAS for damage detection
with Lamb-wave propagation was pioneered by Chang and his coworkers (Chang, 1995, 1998, 2001; Wang and Chang 2000; Ihn and Chang, 2002). They have studied the use of PWAS for generation and reception of elastic waves in composite materials. Passive reception of elastic wave was used for impact detection. Pitch-catch transmission-reception of low-frequency Lamb waves was used for damage detection. PWAS wave propagation was also studied by Culshaw et al. (1998), Lin and Yuan (2001), Osmont et al. (2000), and Diamanti et al. (2002). The use of PWAS for high-frequency local modal sensing with the electromechanical impedance method was pursued by Liang et al. (1994), Sun et al. (1994), Chaudhry et al. (1995), Park and Inman (2001), Giurgiutiu et al. (1998-2010), and others.

PWAS couple the electrical and mechanical effects through the piezoelectric constitutive equation. In linear piezoelectricity (IEEE Standard on Piezoelectricity), the constitutive equation is

\[
\{S\} = \left[s^E\right]\{T\} + \left[d^E\right]\{E\}
\]

\[
\{D\} = [d^T]\{T\} + \left[e^T\right]\{E\}
\]

(2.1)

where:

\{S\} = mechanical strain vector (six components x, y, z, yz, zx, xy)

\{T\} = mechanical stress vector (six components x, y, z, yz, zx, xy)

\{E\} = electrical field vector (three components x, y, z)

\{D\} = electrical displacement vector (three components x, y, z)
\[
[s^E] = \text{mechanical compliance matrix evaluated at zero electric field, i.e. short circuit}
\]

\([d]\) = piezoelectric matrix relating strain/electric field

\([d]^T\) = transposed \([d]\)

\[
[\varepsilon^T] = \text{dielectric matrix evaluated at zero mechanical stress, i.e. mechanically free}
\]

PWAS can be used in several ways. As resonators, PWAS can perform resonant mechanical vibration under direct electrical excitation. Thus, very precise frequency standards can be created with a simple setup consisting of the PWAS and the signal generator. The resonant frequencies depend only on the wave speed (which is material constant) and the geometric dimensions. Precise frequency values can be obtained through precise machining of PWAS geometry.

Free PWAS resonances could be two types: mechanical resonances and electromechanical resonances. The electromechanical resonances of free PWAS can be determined from its admittance. The detailed derivation of complex formulas for admittance and impedance of piezoelectric bar was given in Giurgiutiu and Zagrai (2000).
As embedded modal sensors, PWAS can directly measure the high-frequency modal spectrum of a support structure. This is achieved with the E/M impedance method, which reflects the mechanical impedance of the support structure into the real part of the E/M impedance measured at PWAS terminals. On standing waves, Giurgiutiu and Zagrai (2001) determined an analytical expression for PWAS admittance on a 1-D structure undergoing axial and flexural vibrations. For standing 1-D waves, the effective structural stiffness was calculated using the conventional axial and flexural vibration modes of a 1-D beam.

Giurgiutiu and Zagrai (2002) also considered PWAS radial mode vibration of a thin disk. The radial vibrations of piezoelectric disk are governed by one-dimensional mode where displacement is varying with radius. In this case, solution of the differential equation is developed in terms of Bessel functions: The 2-D analysis of PWAS admittance for circular-crested Lamb waves was performed in cylindrical coordinates using the Bessel functions formulation.

Figure 2.2 PWAS impedance modeling: (a) PZT disk wafer active sensor constrained by the structural stiffness; (b) Experimental and calculated impedance for vibration (Giurgiutiu and Zagrai, 2002).
Giurgiutiu and Bao (2003) developed the theoretical background of the interaction between the PWAS and Lamb waves and its practical implementation for damage detection. For embedded NDE applications, PWAS couple their in-plane motion, excited by the applied oscillatory voltage through the piezoelectric effect, with the Lamb-waves particle motion on the material surface. Lamb waves can be either quasi-axial (S₀, S₁, S₂, …) or quasi-flexural (A₀, A₁, A₂, …). Figure 2.3 shows the interaction between surface mounted PWAS and S₀ and A₀ guided Lamb waves.

Figure 2.3 PWAS interaction with Lamb waves in a plate: (a) S₀ mode; (b) A₀ mode. (Giurgiutiu and Bao, 2003).

Giurgiutiu (2005) explored the capability of embedded PWAS to excite and detect tuned Lamb waves for structure health monitoring. A model of Lamb wave tuning mechanism with PWAS transducers was developed using space domain Fourier transform performed in the wavenumber space. A general solution was obtained for a generic expression of the interface shear stress distribution.

Xu and Giurgiutiu (2007) developed a theoretical model for the analysis of PWAS-related Lamb waves time reversal to predict the existence of single wave mode. Santoni, etc. (2007) used the shear lag transfer of traction and strains to derive Lamb wave-mode tuning curves. Giurgiutiu and Santoni-Bottai (2009) extended Crawley's shear-lag
solution to overcome the limitations of the current shear-lag model and derive a generic solution for the ultrasonic excitation transmitted between a PWAS and a thin-wall structure through an adhesive layer in the presence of multiple guided Lamb-wave modes. Sohn and Lee (2009) proposed a calibration technique to examine the difference of numerical simulations and experimental results.

2.2 State of the Art of PWAS Energy Transfer Modeling

Up to date work on PWAS SHM technology has not yet systematically addressed the modeling of power and energy transduction. However, this topic has been addressed to a certain extent in classical NDE. Viktorov (1967) mentioned the transmissibility function between an NDE transducer and the guided waves in the structure, but did not give an analytical expression but rather relies on experimentally determination. Auld (1973) treated comprehensibly the power and energy of ultrasonic acoustic fields and developed the complex reciprocity approach to their calculation. He developed some predictive models for surface acoustic wave (SAW) devices that relied on the simplifying assumption of single mode non-dispersive Rayleigh wave propagation. Rose (1999) developed a model for coupling between an angle-beam ultrasonic transducer and a system of guided Lamb waves in the structure using the normal modes expansion approach. However, this model was not specific about how the shear transfer takes place through the gel coupling between the transducer and the structure. Classical NDE analysis has not studied in detail the power flow between transducer and structure because of the following reasons:

(a) The coupling-gel interface did not have clearly predictable behavior
(b) Power was not generally an issue, since NDE devices are not meant to operate autonomously on harvested power

Although not much addressed by classical NDE analysis, the understanding and mastering of the power and energy flow is of paramount importance for the design of autonomous SHM systems employing structurally integrated active sensors.

### 2.3 STATE OF THE ART OF PWAS FINITE ELEMENT MODELING

The analytical results are good for simple structures. However, the analytical results were not perfect, as for example in the approximate analysis of a square PWAS resonator using 1-D analysis. An alternative to analytical methods has developed in the form of the finite element method (FEM), which can perform the numerical analysis of complicated structures using small-element discretization. The FEM approach has gained wide-ranging acceptance especially for its ease of use in modeling complicated structures.

The state of the art in numerical verification encompasses the wide-spread use of numerical methods based on space and time discretization that transforms the solving of continuum problem into the solving of a large-size matrix system. Commercial codes based on finite elements method (FEM) and finite differences approach that can handle multi-physics problems are widely available (e.g., ANSYS, ABAQUS, COMSOL, FEMLAB, FLUENT, etc.). The accuracy of these programs is directly dependent on the number of degrees of freedom (dof), i.e., on how fine the discretization is, or how elaborated the interpolation functions are, or both. Thus, their zoom-in capabilities are achieved through increased computational expense. For a given well-defined problem,
these numerical programs can offer a controlled-accuracy solution that, within their fundamental assumptions, is almost as good as a detailed experiment.


2.4 Research Challenges in PWAS Modeling

The previous researchers have developed analytical models that can predict the electromechanical behavior of PWAS and bonded PWAS on the structure. However, the analysis methods to understand and master of the power and energy flow in PWAS active and passive SHM sensing was not developed. It is important for the design of autonomous SHM systems employing PWAS with optimum power and energy flow. The following issues were addressed:

(a) Predictive modeling of the frequency transfer function and the power and energy transduction at the interface between PWAS and the structure in the presence of axial and flexural waves

(b) Identification of maximum energy flow

(c) Knowing that energy flows under impedance match conditions
3 SIMPLIFIED PWAS MODEL

Piezoelectric wafer active sensors (PWAS) are both transducers and receivers that operate on the piezoelectric principle. PWAS can be used for vibration control and damage detection. This chapter introduces the 1-D piezoelectric constitutive equation and PWAS pin-force model under the ideal bonding assumption for PWAS attached to the structure. The relation between applied pin force and PWAS displacement is modeled for PWAS acting as both transmitter and receiver.

3.1 1-D CONSTITUTIVE EQUATION

Consider a PWAS of length $l$, width $b$ and thickness $t$, undergoing longitudinal expansion, $u_1$, induced by the thickness polarization electric field, $E_3$ (Figure 3.1). The electric field is produced by the application of a harmonic voltage $V(t) = \hat{V}e^{j\omega t}$ between the top and bottom surface electrodes. Recall the constitutive Equation (2.1), its 1-D form is

$$
S_1 = s_{11}^{E} T_1 + d_{31} E_3 \\
D_3 = d_{31} T_1 + \varepsilon_{33}^{T} E_3
$$

(3.1)

where $S_1$ is the strain, $T_1$ is the stress, $D_3$ is the electrical displacement, $s_{11}^{E}$ is the mechanical compliance at zero field, $\varepsilon_{33}^{T}$ is the dielectric constant at zero stress, $d_{31}$ is the induced strain coefficient.
For compactness, we are going to the following notation:

\[
\begin{align*}
\dot{U} &= \frac{\partial}{\partial t} U \\
U' &= \frac{\partial}{\partial x} U \\
U &= \hat{U} e^{i\omega t} \\
\overline{U} &= \text{conj}(U)
\end{align*}
\] (3.2)

We are also going to use the notation \( \tilde{f} \) for the space domain Fourier transform and \( h(t) * x(t) \) for the convolution of two variables, i.e.,

\[
\tilde{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-i\xi x} dx \\
\mathcal{F}\{h(t) * x(t)\} = H(\omega) X(\omega)
\] (3.3)

### 3.2 PWAS Pin-Force Model

The transmission of actuation and sensing between the PWAS and the structure is achieved through the adhesive layer. The adhesive layer acts as a shear layer, in which the mechanical effects are transmitted through shear effects. Under static and low-frequency dynamic conditions, the usual hypothesis associates with simple axial and flexural motions, i.e., constant displacement for axial motion and linear displacement strain for flexural motion. The shear-lag analysis indicates that, as the bond thickness decrease, the shear stress transfer becomes concentrated over some infinitesimal distances at the ends of the PWAS actuator. The concept of ideal bonding (also known as
the pin-force model) assumed that all the load transfer take place over an infinitesimal region at the PWAS ends, and the induced-strain action is assumed to consist of a pair of concentrated forces applied at the ends (Figure 3.2).

![Figure 3.2 Schematic of a single PWAS attached to a bar with pin force.](image)

The PWAS with length $l$ locates its left edge at $x_0$. The PWAS right edge is at $x_0 + l$.

The applied voltage is $V = \hat{v} e^{i\omega t}$. The induced-strain action is assumed to consist of a pair of concentrated forces applied at the ends, i.e.,

$$F(x) = F_{PWAS} \left[ -H(x - x_0) + H(x - x_0 - l) \right]$$

where $H$ is the Heaviside step function, $x_0$ is the location of one PWAS edge, and $l$ is the length of the PWAS.

The axial force and bending moment associated with the ideal-bonding hypothesis are

$$N = F \quad \text{(axial force)} \quad (3.5)$$

$$M = \frac{F}{2} h \quad \text{(bending moment)} \quad (3.6)$$

The axial force and bending moment here represent the excitation induced by the PWAS into the structure under the ideal-bonding hypothesis (Figure 3.3)
Figure 3.3 Schematic of PWAS pin force that contributes to axial force and bending moment.

Under ideal-bonding hypothesis, PWAS has a pair of pin-end forces. Under harmonic assumptions, we can write the axial force and bending moment as

\[ N = \hat{F} e^{-i\alpha} \]  
\[ M = \frac{\hat{F}}{2} h e^{-i\alpha} \]

The overall axial force and bending moment applied to the structure can be written as

\[ N_e(x,t) = \left\{ \hat{F} \left[ -H(x-x_0) + H(x-x_0-l) \right] \right\} e^{-i\alpha} \]  
\[ M_e(x,t) = -\left\{ \frac{\hbar}{2} \hat{F} \left[ -H(x-x_0) + H(x-x_0-l) \right] \right\} e^{-i\alpha} \]

3.3 PWAS Transmitter and Receiver

As high-bandwidth strain sensors and strain exciters, PWAS convert directly mechanical energy to electric energy and vice versa. High frequency waves and vibrations are easily excited by directly connecting PWAS to a function generator and are easily detected with a high-impedance measuring instrument, such as an oscilloscope. The dual sensing and excitation characteristic of PWAS justifies their name of ‘active sensors’.
3.3.1 PWAS Transmitter Analysis

Consider a PWAS of length $l$, width $b$ and thickness $t$, undergoing longitudinal expansion, $u_1$, induced by the thickness polarization electric field, $E_3$ (Figure 3.1). The electric field is produced by the application of a harmonic voltage $V = \hat{V} e^{i\omega t}$, between the top and bottom surfaces (electrodes). Assume that the length, width and thickness have widely separated values $(t \ll b \ll l)$ such that the length, width and thickness motions are practically uncoupled.

Under the 1-D assumptions, the general constitutive equations reduce to the simpler expressions

$$S_1 = s_{11}T_1 + d_{31}E_3 \quad \text{(Strain)} \quad (3.11)$$
$$D_3 = d_{31}T_1 + \varepsilon_{33}E_3 \quad \text{(Electric displacement)} \quad (3.12)$$

At zero mechanical stress $T_1$ for free boundary condition, the in-plane actuated strain is

$$S_1 = d_{31} \frac{V(t)}{l} \quad (3.13)$$

The elongation of PWAS end (induced displacement) is the relative displacement of both ends, i.e.,

$$u_{IS4} = \Delta u_1 = S_1 l = d_{31} l \frac{V(t)}{l} \quad (3.14)$$

The PWAS actuator works together with the actuated structure, the interaction between PWAS and structure needs to be considered. Under quasi-static assumption, i.e., no PWAS resonances within range, we can recall the constitute Equation(3.11) in the form

$$S_1 = s_{11}^E T_1 + d_{31}E_3 \quad (3.15)$$
Under harmonic assumptions, the strain, stress and electrical field is

\[ S_i = \frac{\Delta \hat{u}}{l} \]  \hspace{1cm} (3.16)

\[ T_i = \frac{\hat{F}}{tb} \]  \hspace{1cm} (3.17)

\[ E_s = \frac{\dot{V}}{t} \]  \hspace{1cm} (3.18)

Substitution of Equation (3.16) - (3.18) to (3.15) yields

\[ \frac{\Delta \hat{u}}{l} = s_{11}^{E} \frac{\hat{F}}{tb} + d_{31} \frac{\dot{V}}{t} \]  \hspace{1cm} (3.19)

Define \( k_i \) as the internal stiffness of PWAS actuator, i.e.,

\[ k_i = \frac{tb}{s_{11}^{E} l} \]  \hspace{1cm} (3.20)

The induced displacement under harmonic excitation is

\[ u_{ISA} = S_{ISA} l = d_{31} E_s l = d_{31} \frac{\dot{V}}{t} l \]  \hspace{1cm} (3.21)

Substitution of Equation (3.20) and (3.14) into (3.19) yields

\[ \Delta \hat{u} = k_i^{-1} \hat{F} + u_{ISA} \]  \hspace{1cm} (3.22)

We get the relation of the pin-end force and the elongation of PWAS when PWAS was constrained on the structure, i.e.,

\[ \hat{F} = k_i (\Delta \hat{u} - u_{ISA}) \]  \hspace{1cm} (3.23)
3.3.2 PWAS Receiver Analysis

PWAS can be used for strain and stress measurements because they directly convert the mechanical energy into electrical energy. Since the resulting voltage is proportional to the strain rate, this type of measurement would especially beneficial at high frequency.

PWAS in dynamic regime is also analyzed using 1-D assumptions. If one PWAS that is undergoing longitudinal expansion \( u_1 \), the result voltage is harmonic with frequency \( f \) when vibration is harmonic with natural frequency \( \omega \), The derivative of voltage \( \dot{V} = i\omega V \).

![Figure 3.4 Schematic of a PWAS connected with measurement equipment (input impedance \( Z_e \)).](image)

Under the 1-D assumptions, considering the harmonic dynamic strain \( S_1 \) and taking the time derivative, the constitutive Equations (3.11), (3.12) can be written as

\[
\dot{S}_1 = s_{11}\dot{T}_1 + d_{31}\dot{E}_3 \tag{3.24}
\]

\[
\dot{D}_3 = d_{31}\dot{T}_1 + \varepsilon_{33}\dot{E}_3 \tag{3.25}
\]

When PWAS is under harmonic strain and connected with external measurement equipment, it will generate an AC current in measurement equipment. The current flowing through the circuit is given by the product between the time-derivative of the electric displacement \( \dot{D}_3 \) and the electrode area \( A \), i.e.,
\( I = \dot{D}_3 A \)  \hspace{1cm} (3.26)

Take the time-derivative of Equation (3.18), we get

\[ \dot{E}_3 = -\frac{\dot{V}}{t} \]  \hspace{1cm} (3.27)

The relation between voltage \( V \) and the internal admittance of the measuring equipment is

\[ I = Y_e V \]  \hspace{1cm} (3.28)

We want to find the voltage \( V \) as a function of applied strain. Eliminate \( \dot{T}_1 \) between (3.11) and (3.12) to get

\[ d_{31} \dot{S}_1 - s_1 \dot{D}_3 = (d_{31}^2 - s_{11} e_{33}) E_3 \]  \hspace{1cm} (3.29)

Combine Equations (3.26) and (3.28) to write

\[ \dot{D}_3 = \frac{I}{A} = \frac{Y_e}{A} V \]  \hspace{1cm} (3.30)

Substitute Equation (3.27) and (3.30) into (3.29) to obtain

\[ d_{31} \dot{S}_1 - s_1 \frac{Y_e}{A} V = (d_{31}^2 - s_{11} e_{33}) (-\frac{\dot{V}}{t}) = (1-k_{31}^2) s_{11} e_{33} \frac{\dot{V}}{t} \]  \hspace{1cm} (3.31)

where \( k_{31}^2 = \frac{d_{31}^2}{s_{11} e_{33}} \) is the electromechanical coupling coefficient.

Upon rearrangement, Equation (3.31) becomes

\[ d_{31} \frac{A}{s_1} \dot{S}_1 = Y_e V + (1-k_{31}^2) e_{33} \frac{A}{h} \dot{V} = Y_e V + (1-k_{31}^2) C_0 \dot{V} \]  \hspace{1cm} (3.32)
where $C_0$ is the free capacitance of the PWAS.

Under harmonic assumptions, the voltage can be written as

$$V(t) = \hat{V}_e^{i\omega}$$  \hspace{1cm} (3.33)

$$\dot{V}(t) = i\omega \hat{V}_e^{i\omega} = i\omega V(t)$$  \hspace{1cm} (3.34)

Substitution of Equation (3.34) into (3.32) gives

$$d_{31} \frac{A}{s_1^0} \dot{S}_1 = Y_e V + (1 - k_{31}^2) i\omega C_0 V = \left[ Y_e + (1 - k_{31}^2) Y_0 \right] V$$  \hspace{1cm} (3.35)

where $Y_0$ is the admittance of the unconstrained PWAS transducer given by

$$Y_0 = i\omega C_0$$  \hspace{1cm} (3.36)

Upon solution, Equation (3.35) gives the expression of voltage as a function of applied strain rate with the value of the instrument admittance as a parameter, i.e.,

$$V = \frac{1}{Y_e + (1 - k_{31}^2) Y_0} \frac{d_{31} A}{s_1} \dot{S}_1$$  \hspace{1cm} (3.37)

Recall $A = lb$ to get

$$\dot{V} = \frac{1}{Y_e + (1 - k_{31}^2) Y_0} \frac{lb}{s_1} \frac{d_{31}}{s_1} \dot{S}$$  \hspace{1cm} (3.38)

Under harmonic assumptions, the strain is

$$\dot{S} = i\omega S = i\omega \frac{\Delta u}{l}$$  \hspace{1cm} (3.39)

The relation of output voltage and PWAS elongation is

22
\[ \dot{V} = \frac{1}{Y_e + (1 - k_{31}^2)Y_0} - i\omega b d_{31}^{s_{11}} \Delta u \]  

(3.40)

Recall constitutive Equation (3.11), i.e.,

\[ S_1 = s_{11}^{E} T_1 + d_{31} E_3 \]  

(3.41)

Notice that, \( S_1 = \frac{\Delta \dot{u}}{l} \), \( T_1 = \frac{\dot{F}}{tb} \), \( E_3 = \frac{\dot{V}}{t} \); hence, Equation (3.41) becomes

\[ \frac{\Delta \dot{u}}{l} = \frac{s_{11}^{E}}{tb} \dot{F} + \frac{d_{31}}{t} \frac{\dot{V}}{t} \]  

(3.42)

Substitution of Equation (3.40) into (3.42) gives

\[ \frac{\Delta \dot{u}}{l} = \frac{s_{11}^{E}}{tb} \dot{F} + \frac{d_{31}}{t} \frac{1}{Y_e + (1 - k_{31}^2)Y_0} \frac{i\omega b d_{31}^{s_{11}}} {s_{11}} \Delta u \]  

(3.43)

Recall \( k_{31}^2 = \frac{d_{31}^2}{s_{11}^{E} e_{33}} \), \( Y_0 = i\omega C_0 = i\omega e_{33} \frac{b l}{t} \) to get

\[ k_{31}^2 Y_0 = i\omega \frac{b l d_{31}^2}{s_{11} t} \]  

(3.44)

Substitution of Equation (3.44) into (3.43) yields

\[ \Delta \ddot{u} = k_{31}^{-1} \dot{F} + \frac{k_{31}^2 Y_0}{Y_e + (1 - k_{31}^2)Y_0} \Delta u \]  

(3.45)

Upon rearrangement, we get the relation of the pin force at PWAS sensor and the elongation of PWAS, i.e.,

\[ \dot{F} = (1 - \frac{k_{31}^2 Y_0}{Y_e + (1 - k_{31}^2)Y_0}) k_t \Delta \ddot{u} \]  

(3.46)
Define \( r_\omega (\omega) \) and \( R(\omega) \) as

\[
    r_\omega (\omega) = \frac{Y_\omega (\omega)}{Y_0}
\]

\[
    R(\omega) = 1 - \frac{k_{31}^2 Y_0}{Y_0 + (1 - k_{31}^2) Y_0} = \frac{Y_\omega + (1 - 2k_{31}^2) Y_0}{Y_0 + (1 - k_{31}^2) Y_0} = \frac{r_\omega + 1 - 2k_{31}^2}{r_\omega + 1 - k_{31}^2}
\]

The pin force of a PWAS receiver can be written as

\[
    \hat{F} = R(\omega)k \Delta \hat{u}
\]

### 3.4 Pitch-catch Model

PWAS pitch-catch setup in 1-D model is shown in Figure 3.5. PWAS transmitter A with length \( l_A \) locates at location \( x_A \), PWAS receiver B with length \( l_B \) locates at location \( x_B \). For pitch-catch setup, we have two forces at PWAS A and B.

![Figure 3.5 Schematic of a PWAS pitch-catch setup on a bar.](image)

#### 3.4.1 Pitch-catch Pin Forces

Under ideal-bonding hypothesis, both PWAS A and B have pin-end forces. Under harmonic assumptions, we can write the axial force and bending moment as

\[
    N_A = \hat{F}_A e^{-i \omega t}
\]

\[
    N_B = \hat{F}_B e^{-i \omega t}
\]

\[
    (3.50)
\]
\[
M_A = \frac{h}{2} \hat{F}_A e^{-i\omega t} \\
M_B = \frac{h}{2} \hat{F}_B e^{-i\omega t}
\]

(3.51)

The overall axial force and bending moment applied to the structure can be written as

\[
N_{x}(x,t) = \left\{ \frac{h}{2} \hat{F}_A \left[ -H(x-x_A) + H(x-x_A-l_A) \right] \right\} e^{-i\omega t}
\]

(3.52)

\[
M_{x}(x,t) = \left\{ \frac{h}{2} \hat{F}_A \left[ -H(x-x_A) + H(x-x_A-l_A) \right] \right\} e^{-i\omega t}
\]

(3.53)

### 3.4.2 Pitch-catch Transmitter

For pitch-catch setup, we use PWAS A as transmitter. PWAS transmitter works together with the actuated structure, the interaction between PWAS and structure is considered. Under quasi-static assumption, the internal stiffness of the PWAS A is defined as

\[
k_{iA} = \frac{t_A b_A}{s_{11} l_A}
\]

(3.54)

The relation of the pin-end force and the elongation of PWAS in pitch-catch setup is

\[
\hat{F}_A = k_{iA} (\Delta \hat{u}_A - u_{iA})
\]

(3.55)

### 3.4.3 Pitch-catch Receiver

For receiver PWAS B in pitch-catch setup, we have sensing/reception voltage with the strain rate at B.
\[
\hat{V}_B = \frac{1}{Y_e + (1 - k_{31}^2)Y_0} l_b b_B \frac{d_{31}}{s_{11}} \hat{S}_B
\]  

(3.56)

Under harmonic assumptions, recall \( Y_{0B} = i\omega C_{0B} = i\omega e_{35} b_B l_B / t_B \) and \( k_{ib} = \frac{t_b b_B}{s_{11} l_B} \), we get the relation of the pin-end force at PWAS B and the elongation of PWAS B, i.e.,

\[
\hat{F}_B = (1 - \frac{k_{31}^2 Y_{0B}}{Y_e + (1 - k_{31}^2)Y_0}) k_{ib} \Delta u_B = R(\omega) k_{ib} \Delta u_B
\]  

(3.57)

Here \( R(\omega) \) in pitch-catch setup is,

\[
R(\omega) = 1 - \frac{k_{31}^2 Y_{0B}}{Y_e + (1 - k_{31}^2)Y_0} = \frac{Y_e + (1 - 2k_{31}^2)Y_{0B}}{Y_e + (1 - k_{31}^2)Y_0} = \frac{r_y + 1 - 2k_{31}^2}{r_y + 1 - k_{31}^2}
\]  

(3.58)

### 3.5 Frequency Response Function

A frequency response function (FRF) is a mathematical representation in terms of frequency of the relation between the input and output of a (linear time-invariant) system. Under harmonic excitation, the FRF is the measure of any system's output spectrum in response to an input signal, i.e.,

\[
FRF(\omega) = \frac{OUT(\omega)}{IN(\omega)}
\]  

(3.59)

### 3.6 Non-harmonic Excitation

Under non-harmonic excitation, the instantaneous signal can be derived from the FRF. The instantaneous response can be calculated using the inverse Fourier transform of the product of excitation signal spectrum and FRF. Recall the convolution property of the Fourier transform, i.e.,
The function \( f_{rf}(t) \) is called the impulse response function. If the input voltage signal is an impulse function \( in(t) = \delta(t) \), then the output voltage signal \( out(t) \) is

\[
out(t) = f_{rf}(t) \ast in(t) = \mathcal{F}^{-1} \left[ FRF(\omega) \cdot IN(\omega) \right]
\]  
(3.60)

Hence, the response of the current signal to an impulsive input voltage signal is the impulse voltage response function \( f_{rf}(t) \).

The output response under non-harmonic excitation is

\[
out(t) = f_{rf}(t) \ast in(t) = \mathcal{F}^{-1} \left[ FRF(\omega) \cdot IN(\omega) \right]
\]  
(3.62)

Non-harmonic excitation, e.g., a tone-burst excitation, is popular to use to generate the wave propagation in SHM. In the dissertation, we use a 3-count Hanning windowed tone burst. A 100 kHz 3-count 10-V tone-burst excitation voltage signal and its Fourier spectrum is shown in Figure 7.4.

![Excitation signal and frequency spectrum](image)

**Figure 3.6** Non-harmonic excitation signal (3-count tone burst signal) and its frequency spectrum.
4 SINGLE PWAS ANALYSIS USING WAVE PROPAGATION MODEL

In this chapter, the PWAS models are based on 1-D axial and flexural wave propagation assumption. The simple cases are studied here: (a) axial waves in a bar and (b) flexural waves in a bar. The space-domain Fourier transform is used to resolve the problem using complex contour integrals. A close-form solution is obtained for the ideal bonding between the PWAS and structure using the pin-force model.

4.1 WAVE PROPAGATION METHOD

Consider the pin force on an infinite bar of width \( b \) and thickness \( h \), we get the axial and flexural waves. One PWAS is mounted on a uniform bar at \( x_0 \) under ideal bonding hypothesis. The waves generated by the PWAS travel without reflections in the infinite bar (Figure 4.1).

![Figure 4.1 Schematic of a single PWAS attached to an infinite bar.](image)

4.1.1 Axial Wave

The axial wave equation is
where \( A = bh \) is the beam cross-section area, \( \rho \) is material density and \( E \) is the Young's modules of the beam.

Rearrangement of Equation (4.1) to get
\[
\ddot{u}(x,t) - c^2 \frac{\rho}{\rho A} u''(x,t) = \frac{N'_e}{\rho A}
\]  
(4.2)

where \( c = \sqrt{\frac{E}{\rho}} \) is the axial wave speed.

Recall axial force Equation (3.9), i.e.,
\[
\{0\} \hat{\omega} \dot{\theta} = -\dot{\theta} - \ddot{\theta} + \theta + \phi(x) \xi \xi \xi \xi
\]  
(4.3)

Taking derivative and Equation (4.3) becomes
\[
\{0\} \hat{\omega} \delta' \delta + \ddot{\delta} = -\dot{\theta} - \ddot{\theta} + \theta + \phi(x) \xi \xi \xi \xi
\]  
(4.4)

where \( \delta \) is Dirac's function and \( \delta' = H' \).

Assume harmonic variation in the time domain of the form \( e^{-i\omega t} \). Hence, the equation becomes
\[
-\omega^2 u(x,t) - c^2 u''(x,t) = \frac{N'_e}{\rho A}
\]  
(4.5)

Define \( \xi_0^2 = \omega^2 / c^2 \), that is the wavenumber of axial waves in the 1-D medium. Recall that \( c^2 \rho A = EA \), the wave equation is written as
\[-u'' - \xi_0^2 u = \frac{N_e'(x,t)}{EA}\]  \hspace{1cm} (4.6)

Define

\[h(x,t) = \frac{N'_e(x,t)}{EA}\]  \hspace{1cm} (4.7)

Recall the space-domain Fourier transform, i.e.,

\[\tilde{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-ix\xi}dx\]  \hspace{1cm} (4.8)

\[f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\xi)e^{ix\xi}d\xi\]  \hspace{1cm} (4.9)

The space-domain Fourier transform has property \(\tilde{f}' = i\xi \tilde{f}\). Taking the space-domain Fourier transform of Equation (4.6), it is written in Fourier domain as

\[\xi^2 \tilde{u} - \xi_0^2 \tilde{u} = \tilde{h}\]  \hspace{1cm} (4.10)

Rearrangement of Equation (4.10) yields

\[\tilde{u} = \frac{1}{\xi^2 - \xi_0^2} \tilde{h}\]  \hspace{1cm} (4.11)

Equation (4.11) represents the solution in the Fourier domain. Taking the inverse space-domain Fourier transform yields the solution in the space domain. The solution is

\[u(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\xi^2 - \xi_0^2} \tilde{h}(\xi)e^{i\xi x}d\xi\]  \hspace{1cm} (4.12)

The space-domain Fourier transform of \(h(x)\) is
\[ \tilde{h}(\xi) = \int_{-\infty}^{\infty} h(x) e^{-i\xi x} dx = \int_{-\infty}^{\infty} N'(x) e^{-i\xi x} dx = \frac{1}{EA} \int_{-\infty}^{\infty} \left\{ \hat{F} \left[ -\delta(x - x_0) + \delta(x - x_0 - l) \right] e^{-i\xi x} dx \right\} e^{-i\xi x} dx \]  

(4.13)

Recall the delta function property, i.e.,

\[ \int_{-\infty}^{\infty} f(x) \delta(x - a) dx = f(a) \]  

(4.14)

Use of the delta function property and the Equation (4.13) becomes

\[ \tilde{h}(\xi) = \frac{1}{EA} \hat{F}(-e^{-i\xi x_0} + e^{-i\xi(x_0+l)}) \]  

(4.15)

Substitution (4.15) into (4.12) yields

\[ u(x,t) = \frac{1}{2\pi EA} e^{-i\xi t} \int_{\xi_0 - \xi}^{\xi_0 + \xi} \frac{1}{\xi - \xi_0} \left[ \hat{F}(-e^{-i\xi x_0} + e^{-i\xi(x_0+l)}) \right] e^{i\xi x} d\xi \]  

(4.16)

Figure 4.2 Contour for evaluating the axial displacement solution under ideally bonded PWAS excitation. The residue at positive wavenumber is included, which the residue for the negative wavenumber is excluded.
The integral in Equation (4.16) can be resolved analytically using the residues theorem and a semicircular contour $C$ in the complex $\xi$ domain. We note that the integrand in Equation (4.16) has two poles, corresponding to the wavenumbers $-\xi_0$ and $+\xi_0$. We will resolve Equation (4.16) for the forward wave, which exists for $x > 0$ and generates a solution containing $i(\xi x - \omega t)$ in the exponential function. Hence, we will retain the positive pole, $+\xi_0$, inside the integration contour but exclude the negative pole, $-\xi_0$, from the integration contour. The resulting integration contour $C$ is shown in Figure 4.2.

We note that the integration along the semicircular portion of the contour vanishes as the radius of integration becomes very large, i.e., $R \to \infty$. Therefore, the integration on the contour resolves into the integration along the real $\xi$ axis from $-\infty$ to $\infty$.

According to the residue theorem, the integration around the contour equals the sum of the residues inside the contour times a multiplicative factor $2\pi i$. We only have retained one pole inside the contour, the pole $+\xi_0$, the contour integral takes the expression

$$\oint_C = 2\pi i \text{Res} \bigg|_{\xi = \xi_0}$$

(4.17)

The residue is

$$\text{Res} \left( \frac{1}{\xi^2 - \xi_0^2} \left[ \hat{F}(-e^{-i\xi_0} + e^{-i\xi_0(t+1)}) e^{i\xi x} \right] \right)$$

$$= \left( \frac{1}{\xi + \xi_0} \left[ \hat{F}(-e^{-i\xi_0} + e^{-i\xi_0(t+1)}) e^{i\xi x} \right] \right)_{\xi = \xi_0}$$

(4.18)

$$= \frac{1}{2\xi_0} \left[ \hat{F}(-e^{-i\xi_0} + e^{-i\xi_0(t+1)}) \right] e^{i\xi_0 x}$$

Substitute Equation (4.17) and (4.18) into (4.16) gives
\[ u(x) = \frac{i}{2EA\xi_0} \left\{ \hat{F}(-e^{-i\xi_0 x_0} + e^{-i\xi_0 (x_0 + l)}) \right\} e^{i(\xi_0 x - \omega t)} \] (4.19)

Special case, only one PWAS at location \( x_0 = -\frac{l}{2} = -a \), the axial displacement is

\[ u(x) = \frac{i}{2EA\xi_0} \left\{ \hat{F}(-e^{i\xi_0 a} + e^{-i\xi_0 a}) \right\} e^{i(\xi_0 x - \omega t)} \] (4.20)

The strain in this situation is

\[ \varepsilon = u' = \frac{i(i\xi_0)}{2EA\xi_0} \left\{ \hat{F}(-e^{i\xi_0 a} + e^{-i\xi_0 a}) \right\} e^{i(\xi_0 x - \omega t)} = i \frac{\hat{F}}{EA} \sin(\xi_0 a) e^{i(\xi_0 x - \omega t)} \] (4.21)

Define \( \varepsilon = \frac{\hat{F}}{EA} \) and substitute it into Equation (4.21), the strain becomes

\[ \varepsilon = i\varepsilon a \sin(\xi_0 a) e^{i(\xi_0 x - \omega t)} \] (4.22)

which is consistent with previous work on axial waves excited by PWAS (Giurgiutiu, 2008, page 315).

### 4.1.2 Flexural Wave

The flexural wave equation is

\[ \rho A\ddot{w}(x,t) + EI\dddot{w}(x,t) = -M_e''(x,t) \] (4.23)

where \( \dot{w} = \frac{\partial}{\partial t} w \) and \( w' = \frac{\partial}{\partial x} w \), \( I = \frac{bh^3}{12} \).

Assume harmonic variation in the time domain of the form \( e^{-i\omega t} \). Hence, Equation (4.23) becomes

\[ EI\dddot{w'} - \omega^2 \rho A w = -M_e'' \] (4.24)
Recall

\[ M_c(x, t) = -\left\{ \frac{h}{2} \hat{F} \left[ -H(x - x_0) + H(x - x_0 - l) \right] \right\} e^{iat} \]  (4.25)

Divide by \( EI \), Equation (4.24) becomes

\[ w'' - \frac{\rho A}{EI} w = \frac{h}{EI} \hat{F} \left[ -\delta'(x - x_0) + \delta'(x - x_0 - l) \right] \]  (4.26)

Introducing the notation

\[ \xi^4_F = \omega^2 \frac{\rho A}{EI} = \omega^2 / \bar{a}^2 \]  where \( \bar{a}^2 = \frac{EI}{\rho A} = \frac{bh^3}{12\rho(bh)} = \frac{h^2 E}{12\rho} \)  (4.27)

Define

\[ g = -\frac{M''}{EI} = \frac{h}{EI} \hat{F} \left[ -\delta'(x - x_0) + \delta'(x - x_0 - l) \right] \]  (4.28)

Substitution of Equation (4.27) and (4.28) into Equation (4.26) yields

\[ w'' - \xi^4_F w = g \]  (4.29)

Taking the space-domain Fourier, the transform of (4.29) becomes

\[ \xi^4 \tilde{w} - \xi^4_F \tilde{w} = \tilde{g} \]  (4.30)

The solution is

\[ \tilde{w} = \frac{1}{\xi^4 - \xi^4_F} \tilde{g} \]  (4.31)

Equation (4.31) represents the solution in the Fourier domain. Taking the inverse space-domain Fourier transform yields the solution in the space domain.
The integral in Equation (4.32) can be resolved using the residues theorem and a semicircular contour \( C \) in the complex \( \xi \) domain. The integrand in (4.32) has four poles, corresponding to the wavenumbers \( +\xi_F \), \( -\xi_F \), \( +i\xi_F \), \( -i\xi_F \). We will resolve Equation (4.32) for the forward traveling wave, which exists for \( x > 0 \). Hence, we will retain the positive poles, \( +\xi_F \) and \( +i\xi_F \) inside the integration contour, but exclude the negative poles. The resulting contour is shown in Figure 4.3.

\[
w(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\xi^4 - \xi^4 F} \xi^4 F (\xi) e^{i\xi x} d\xi
\]  

(4.32)

Figure 4.3 Contour for evaluating the flexural displacement solution under ideally bonded PWAS excitation. The residue at positive wavenumber, \( +\xi_F \), is included, while the residue for the negative wavenumber, \( -\xi_F \), is excluded. Also included is the residue at the imaginary wavenumber, \( +i\xi_F \).

The integration contour includes the real pole \( +\xi_F \) and imaginary pole \( +i\xi_F \). We note that the integration along the semicircular portion of the contour vanishes as the radius of integration becomes very large. According to the residue theorem, the integration around the contour equals the sum of the residues inside the contour times a multiplicative factor.
We only have retained two poles inside the contour, the pole $+\xi_F$ and imaginary pole $+i\xi_F$, the contour integral takes the expression

$$\hat{f}_c = 2\pi i (\text{Res}_{\xi=\xi_F} + \text{Res}_{\xi=i\xi_F})$$  \hspace{1cm}(4.33)$$

The residue at $+\xi_F$ is calculated as

$$\text{Res}_{\xi=\xi_F} \left( \frac{1}{\xi^4 - \xi_F^4} g(\xi) e^{i\xi x} \right) = \frac{1}{4\xi_F^3} \tilde{g}(\xi_F) e^{i\xi_F x}$$  \hspace{1cm}(4.34)$$

The residue at $+i\xi_F$ is calculated as

$$\text{Res}_{\xi=i\xi_F} \left( \frac{1}{\xi^4 - \xi_F^4} g(\xi) e^{i\xi x} \right) = \frac{1}{-4i\xi_F^3} \tilde{g}(i\xi_F) e^{-\xi_F x}$$  \hspace{1cm}(4.35)$$

Substitution of Equations (4.33)-(4.35) into (4.32) yields

$$w(x) = \frac{1}{2\pi} 2\pi i \left( \frac{1}{4\xi_F^3} \tilde{g}(\xi_F) e^{i\xi_F x} - \frac{1}{-4i\xi_F^3} \tilde{g}(i\xi_F) e^{-\xi_F x} \right) e^{i\omega x}$$  \hspace{1cm}(4.36)$$

Note that the first term in Equation (4.36) represents a propagating wave, while the second term does not. In fact, the second term represents a vibration that is decaying fast with $x$. This term represents a local vibration that does not propagate. It is called an evanescent wave. Thus, we will retain only the propagating wave part of Equation (4.36)

$$w(x) = i \frac{1}{4\xi_F^3} \tilde{g} e^{i(\xi_F x - \omega t)}$$  \hspace{1cm}(4.37)$$

The space domain Fourier transform of flexural excitation is

$$\tilde{g}(\xi_F) = \int_{-\infty}^{\infty} g(x) e^{-i\xi_F x} dx = \int_{-\infty}^{\infty} \frac{h}{2EI} \left[ -\delta'(x-x_0) + \delta'(x-x_0-l) \right] e^{-i\xi_F x} dx$$  \hspace{1cm}(4.38)$$
The delta function has a property as

\[ \int_{-\infty}^{\infty} f(x) \delta'(x-a) dx = -f'(a) \]  

(4.39)

The space-domain Fourier transform of Equation (4.38) yields

\[ \tilde{\tilde{g}}(\xi_F) = -\frac{h}{2EI} \left[ \hat{F}(-e^{-i\xi_F x_0} + e^{-i\xi_F (x_0 + l)}) \right] = i\xi_F \frac{h}{2EI} \left[ \hat{F}(-e^{-i\xi_F x_0} + e^{-i\xi_F (x_0 + l)}) \right] \]  

(4.40)

Substitution of Equation (4.40) into (4.37) yields the flexural-displacement solution

\[ w(x) = -\frac{1}{4\xi_F^2} \frac{h}{2EI} \left[ \hat{F}(-e^{-i\xi_F x_0} + e^{-i\xi_F (x_0 + l)}) \right] e^{i(\xi_F x - ax)} \]  

(4.41)

Recall that

\[ I = \frac{bh^3}{12} \quad \text{and} \quad A = bh \]  

(4.42)

Then

\[ \frac{h}{2EI} = \frac{h}{2E \frac{bh^3}{12}} = \frac{6}{Ebh^2} = \frac{6}{EAh} \]  

(4.43)

Substitute Equation (4.43) into (4.41) yields

\[ w(x) = -\frac{1}{\xi_F^2} \frac{3}{EAh} \left[ \hat{F}(-e^{-i\xi_F x_0} + e^{-i\xi_F (x_0 + l)}) \right] e^{i(\xi_F x - ax)} \]  

(4.44)

Special case, PWAS at location \( x_0 = -\frac{l}{2} = -a \), the flexural-displacement solution is

\[ w(x) = -\frac{1}{\xi_F^2} \frac{3}{EAh} \hat{F}(-e^{-i\xi_F x_0} + e^{-i\xi_F (x_0 + l)}) e^{i(\xi_F x - ax)} \]  

\[ = -\frac{1}{\xi_F^2} \frac{3}{EAh} \hat{F}(e^{i\xi_F a} + e^{-i\xi_F a}) e^{i(\xi_F x - ax)} = \frac{1}{\xi_F^2} \frac{6}{EAh} \hat{F} i \sin(\xi_F a) e^{i(\xi_F x - ax)} \]  

(4.45)
The displacement due to the flexural wave is

\[ u_x = -\frac{h}{2} w'(x) = \frac{1}{\xi_p^2 EAh} \hat{F} \sin(\xi_p a) e^{i(\xi_p x - \omega t)} \] (4.46)

Recall Equation \( \varepsilon_x = \frac{\hat{F}}{EA} \), the strain due to the flexural wave is

\[ \varepsilon_x = u'_x = \left( \frac{1}{\xi_p} \frac{3}{EA} i \hat{F} \sin(\xi_p a) e^{i(\xi_p x - \omega t)} \right)' = i3\varepsilon_a \sin(\xi_p a) e^{i(\xi_p x - \omega t)} \] (4.47)

which is consistent with previous work on flexural waves excited by PWAS (Giurgiutiu, 2008, page 320).

4.1.3 In-plane Surface Displacement under Wave Propagation

Kinematic analysis gives the horizontal displacement of a generic point P on the infinite bar surface in terms of the axial and flexural displacement as

\[ u_p(t) = u(x) - \frac{h}{2} w'(x) \] (4.48)

Taking derivative of Equation (4.44) yields

\[ w' = -\frac{i}{\xi_p EAh} \left[ \hat{F}(-e^{-i\xi_p x_0} + e^{-i\xi_p (x_0 + t)}) \right] e^{i(\xi_p x - \omega t)} \] (4.49)

The in-plane displacement due to the flexural wave at the material surface is

\[ u_p \bigg|_{y=h/2} = -\frac{h}{2} w'(x) = \frac{3i}{2 EAh \xi_p} \left[ \hat{F}(-e^{-i\xi_p x_0} + e^{-i\xi_p (x_0 + t)}) \right] e^{i(\xi_p x - \omega t)} \] (4.50)

The in-plane surface displacement of a generic point P on the infinite bar surface in terms of the axial and flexural displacement is
\[ u_p(t) = u(x) - \frac{h}{2} w'(x) \]

\[ = -\frac{i}{2EA\xi_0} \hat{F}(e^{-i\xi_0 x_0} + e^{-i\xi_0 (x_0 + l)}) e^{i\xi x - \alpha x} + \frac{3i}{2EA\xi_F} \hat{F}(e^{-i\xi_F x_0} + e^{-i\xi_F (x_0 + l)}) e^{i\xi x - \alpha x} \]

\[ = \left( \frac{-e^{-i\xi_0 x_0} + e^{-i\xi_0 (x_0 + l)}}{\xi_0} \right) e^{i\xi x - \alpha x} + 3 \left( \frac{-e^{-i\xi_F x_0} + e^{-i\xi_F (x_0 + l)}}{\xi_F} \right) e^{i\xi x - \alpha x} \]

The elongation of PWAS is

\[ \Delta u = u(x_0 + l) - u(x_0) \]

\[ = -\frac{i}{2EA} \hat{F} \left( \frac{-e^{-i\xi_0 x_0} + e^{-i\xi_0 (x_0 + l)}}{\xi_0} \left( e^{i\xi_0 (x_0 + l)} - e^{i\xi_0 x_0} \right) \right) + 3 \left( \frac{-e^{-i\xi_F x_0} + e^{-i\xi_F (x_0 + l)}}{\xi_F} \left( e^{i\xi_F (x_0 + l)} - e^{i\xi_F x_0} \right) \right) \]

Define the coefficient

\[ C_{\Delta u}(\omega) = \frac{i}{2EA} \left( \frac{e^{-i\xi_0 l} - 1}{\xi_0} \right) + 3 \left( \frac{e^{-i\xi_F l} - 1}{\xi_F} \right) \]

The relation of \( \Delta u(\hat{F}; \omega) \) is

\[ \Delta u = \hat{F}C_{\Delta u}(\omega)e^{-i\omega x} \]

Because of the infinite boundary for the infinite bar, the wave generated by the PWAS would not be reflected from the boundary. Recall transmitter pin-force Equation (3.23), i.e.

\[ F = k_i(\Delta u - u_{SSA}) \]
\[
\Delta \hat{u} = k_i (\Delta \hat{u} - u_{ISA}) C_{AA}(\omega)
\]  
(4.56)

Rearrangement of Equation (4.56) yields

\[
(k_i C_{AA}(\omega) - 1)\Delta \hat{u}(\omega) = k_i C_{AA}(\omega)u_{ISA}
\]  
(4.57)

The expression of \(\Delta \hat{u}(\omega)\) is

\[
\Delta \hat{u}(\omega) = \frac{k_i C_{AA}(\omega)}{k_i C_{AA}(\omega) - 1} u_{ISA} = \left(1 - \frac{1}{k_i C_{AA}(\omega)}\right)^{-1} u_{ISA}
\]  
(4.58)

The strain is

\[
\varepsilon(\omega) = \frac{\Delta \hat{u}(\omega)}{l} = \left(1 - \frac{1}{k_i C_{AA}(\omega)}\right)^{-1} \frac{u_{ISA}}{l} = \left(1 - \frac{1}{k_i C_{AA}(\omega)}\right)^{-1} \varepsilon_{ISA}
\]  
(4.59)

where \(\varepsilon_{ISA}\) is the induced strain, i.e.,

\[
\varepsilon_{ISA} = \frac{u_{ISA}}{l} = \frac{d_{31}}{t_A} \hat{V}_A
\]  
(4.60)

Special case, PWAS at location \(x_0 = -\frac{l}{2} = -a\), the \(C_{AA}(\omega)\) coefficient is

\[
C_{AA}(\omega) = \frac{i}{2EA} \left( \frac{e^{-i\xi_0 a} - 1}{\xi_0} \frac{e^{i\xi_0 a} - 1}{\xi_F} + 3 \frac{e^{-i\xi_F 2a} - 1}{\xi_F} \frac{e^{i\xi_F 2a} - 1}{\xi_F} \right) = \frac{i}{EA\xi_0} 2 \sin^2(\xi_0 a) + \frac{i}{EA\xi_F} 6 \sin^2(\xi_F a)
\]  
(4.61)

Recall PWAS stiffness equation (3.20), we get
\[-\frac{1}{k_i C_{ia} (\omega)} = -\frac{1}{s_{i1}} \left( \frac{i E A \xi_0^2}{6\sin^2(\xi_0 a)} \right) + \frac{i E A \xi_0^2}{2\sin^2(\xi_0 a)} \right) \right]^{-1} \]
\[= \frac{i s_{i1}^E 2a E \xi_0}{2\sin^2(\xi_0 a)} + \frac{i s_{i1}^E 2a E \xi_F^2}{6\sin^2(\xi_F a)} = i s_{i1}^E E \left( \frac{\xi_0 a}{\sin^2(\xi_0 a)} + \frac{\xi_F a}{3\sin^2(\xi_F a)} \right) \]

Substitution of Equation (4.62) into (4.59) yields
\[\hat{e}(\omega) = \left( 1 + i s_{i1}^E E \left( \frac{\xi_0 a}{\sin^2(\xi_0 a)} + \frac{\xi_F a}{3\sin^2(\xi_F a)} \right) \right) \]
4.2.1 Transmitter Input Admittance

Consider the constrained PWAS transmitter under harmonic electric excitations. Recall constitutive Equation (3.1) representing the electrical displacement, i.e.,

\[
S_1 = s_{11}^E T_1 + d_{31} E_3 \\
D_3 = d_{31} T_1 + \varepsilon_{33}^T E_3
\]  

(4.64)

The stress can be rewritten as

\[
T_1 = \frac{1}{s_{11}^E} (S_1 - d_{31} E_3)
\]  

(4.65)

Hence, the electrical displacement can be expressed as

\[
D_3 = \frac{d_{31}}{s_{11}^E} (S_1 - d_{31} E_3) + \varepsilon_{33}^T E_3
\]  

(4.66)

Recall the strain-displacement relation, i.e.,

\[
S_1 = u_1'
\]  

(4.67)

Substitution of the strain-displacement relation yields

\[
D_3 = \frac{d_{31}}{s_{11}^E} u_1' - \frac{d_{31}^2}{s_{11}^E} E_3 + \varepsilon_{33}^T E_3
\]  

(4.68)
Rearrange to get

\[ D_3 = \varepsilon_{33}^r E_3 \left[ 1 + k_{31}^2 \left( \frac{u_t'}{d_{31} E_3} - 1 \right) \right] \]  \hfill (4.69)

where \( k_{31}^2 = \frac{d_{31}^2}{s_{13}^r \varepsilon_{33}} \) is the electromechanical coupling coefficient. Integration of Equation (4.69) over the electrodes area yields the total charge.

\[ Q = \int_{x_0}^{x_0 + l} \int_0^b D_3 \, dx \, dy = \varepsilon_{33}^r \frac{bl}{t} V \left[ 1 + k_{31}^2 \left( \frac{1}{l d_{31} E_3} \frac{u_t'}{d_{31} E_3} \right) \right] \]  \hfill (4.70)

Assuming harmonic time dependence, we write

\[ \hat{Q} = C_0 \hat{V} \left[ 1 + k_{31}^2 \left( \frac{1}{l d_{31} E_3} \left[ \hat{u}(x_0 + l) - \hat{u}(x_0) \right] \right) - 1 \right] \]  \hfill (4.71)

where \( C_0 = \varepsilon_{33} \frac{bl}{t} \) is the conventional stress free capacitance of the PWAS. Using the definitions \( u_{lsd} = d_{31} E_3 l \) and \( E_3 = \frac{\hat{V}}{t} \), we obtain

\[ \hat{Q} = C_0 \hat{V} \left[ 1 - k_{31}^2 + k_{31}^2 \left( \frac{\hat{u}(x_0 + l) - \hat{u}(x_0)}{u_{lsd}} \right) \right] \]  \hfill (4.72)

The electrical current is obtained as the time derivative of the electrical charge, i.e.,

\[ I = \hat{Q} = i \omega Q \]  \hfill (4.73)
\[ \hat{I} = i\omega C_0 \hat{V} \left[ 1 - k_{31}^2 + k_{31}^2 \left( \frac{\hat{u}(x_0 + t) - \hat{u}(x_0)}{u_{ISA}} \right) \right] \]

\[ = i\omega C_0 \hat{V} \left[ 1 - k_{31}^2 + k_{31}^2 \frac{\Delta \hat{u}(\omega)}{u_{ISA}} \right] \quad (4.74) \]

Recall transmitter Equation (4.58), i.e.,

\[ \Delta \hat{u}(\omega) = \frac{k_{C_A4}(\omega)}{k_{C_A4}(\omega) - 1} u_{ISA} \quad (4.75) \]

Substitution of Equation (4.75) into (4.74), we get

\[ \hat{I}(\omega) = i\omega C_0 \hat{V} \left[ 1 - k_{31}^2 \left( 1 - \frac{k_i C_{A4}(\omega)}{k_{C_A4}(\omega) - 1} \right) \right] \quad (4.76) \]

The admittance is defined as the ratio between the current and voltage, i.e.,

\[ Y(\omega) = \frac{\hat{I}}{\hat{V}} = i\omega C_0 \left[ 1 - k_{31}^2 \left( 1 - \frac{k_i C_{A4}(\omega)}{k_{C_A4}(\omega) - 1} \right) \right] \quad (4.77) \]

### 4.2.2 Transmitter Electrical Response

Under dynamic operation, the flow of electric energy, that is the electric power, must be analyzed. The electrical power to be discussed in connection with actuators can have several descriptors: power rating, active power, reactive power, and maximum instantaneous power.

We write with the expression for \textbf{instantaneous electrical power}, i.e.,

\[ p(t) = v(t) i(t) \quad (4.78) \]

Consider the expression of instantaneous electrical power defined by the product of voltage of current. During dynamic operation, alternating electric voltage is applied and
the electrical charge flows as electric current in and out of the induced-strain actuator.

The voltage and current are not necessarily in phase, hence,

\[ v(t) = \hat{V} \cos \omega t \]
\[ i(t) = \hat{I} \cos (\omega t + \phi) \]  

(4.79)

We write the instantaneous electrical power as

\[ p(t) = v(t) i(t) = \hat{V} \cos \omega t \hat{I} \cos (\omega t + \phi) \]  

(4.80)

Using trigonometric identities, we express Equation (4.80) in the form

\[ p(t) = \frac{1}{2} \hat{V} \hat{I} \cos \phi + \frac{1}{2} \hat{V} \hat{I} \cos (2\omega t + \phi) \]  

(4.81)

The electrical system is voltage controlled; the voltage is the reference signal. The electrical current is expressed in terms of voltage and admittance, that is

\[ I(t) = Y(\omega)V(t) \quad \text{and} \quad \cos \phi = Y_r / |Y| \]  

(4.82)

In this case, the power expression of Equation (4.81) takes the admittance-voltage power form, that is

\[ p(t) = \frac{1}{2} Y_r \hat{V}^2 - \frac{1}{2} |Y| \hat{V}^2 \cos (2\omega t - \phi) \]  

(4.83)

Using complex notation with the implied convention \( \cos \omega t = \text{Re}(e^{j\omega t}) \), we write

\[ v(t) = \hat{V} e^{j\omega t} \]
\[ i(t) = \hat{I} e^{j(\omega t + \phi)} \]  

(4.84)

Recall Euler's identity, we understand that Equation (4.84) implies the convention of using only the real part of the complex exponential since
\[ v(t) = \hat{V} \cos \omega t = \text{Re}\left(\hat{V}e^{i\omega t}\right) \]
\[ i(t) = \hat{I} \cos(\omega t + \phi) = \text{Re}\left(\hat{I}e^{i(\omega t + \phi)}\right) \]  
(4.85)

The **electrical active power** is

\[ P_{\text{active}} = P_{\text{average}} = \frac{1}{T} \int_0^T P(t) \, dt = \frac{1}{2} |Y| \hat{V}^2 \cos \phi = \frac{1}{2} Y_R \hat{V}^2 \]  
(4.86)

The active power is related to the real part of admittance.

The **electrical reactive power** is

\[ P_{\text{reactive}} = \frac{1}{2} Y_I \hat{V}^2 \]  
(4.87)

The active power is the power that converts to the mechanical power at the interface. The power rating is the power requirement of the power supply without distortion. In induced-strain transmitter applications, the reactive power is the dominant factor, since the transmitter impedance is dominated by its capacitive behavior. Managing high reactive power requirements is one of the challenges of using induced-strain transmitters.

The **power rating** is

\[ P_{\text{rating}} = \frac{1}{2} Y_R \hat{V}^2 = \frac{1}{2} |Y| \hat{V}^2 \]  
(4.88)

The relation among active power, reactive power and power rating is

\[ P_{\text{rating}}^2 = P_{\text{active}}^2 + P_{\text{reactive}}^2 \]  
(4.89)

Of great importance in sizing an system is to know the maximum instantaneous power required by the system. Examination of Equation (4.81) reveals that it has a constant part and oscillatory part. The constant part is associated with the power uniformly dissipated...
by the actuator, that is, the active power. The oscillatory part represents power that flows in and out of the induced-strain actuator during its cyclic operation.

The **peak power** is

\[
P_{\text{max}} = P_{\text{rating}} + P_{\text{active}}
\]  

(4.90)

Under harmonic excitation, the time average product of the two harmonic variables defined by Equation (4.80) is one half the product of one variable times the conjugate of the other. Since the harmonic exponential is common in both \(V\) and \(I\). The time-averaged electrical response can be written as

\[
\langle P(\omega) \rangle = \frac{1}{2} \text{Re}\left( \bar{V}(\omega)I(\omega) \right)
\]  

(4.91)

<table>
<thead>
<tr>
<th>Table 4.1 Electrical power expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textbf{Name}</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Electrical active power</td>
</tr>
<tr>
<td>Electrical reactive power</td>
</tr>
<tr>
<td>Electrical power rating</td>
</tr>
<tr>
<td>Electrical peak power</td>
</tr>
<tr>
<td>Time-average electrical power</td>
</tr>
</tbody>
</table>

Under ideal excitation source assumption, we considered three kinds of excitation source: (a) constant voltage input; (b) constant current input; (c) constant power rating input.

Under **constant voltage input**, the power rating and active power is
\( \langle P_{\text{rating}} \rangle = \frac{1}{2} |Y| |V|^2 \) \hspace{1cm} (4.92)

\( \langle P_{\text{active}} \rangle = \frac{1}{2} Y_k |V|^2 = 2 \langle P_{\text{wave}} \rangle \) \hspace{1cm} (4.93)

Under constant current input, the voltage and power rating is

\[ V = \frac{1}{|Y|} I \] \hspace{1cm} (4.94)

\( \langle P_{\text{rating}} \rangle = \frac{1}{2} |Y| |V|^2 = \frac{1}{2 |Y|} |I|^2 \) \hspace{1cm} (4.95)

Under constant power rating input, the voltage is

\[ |V|^2 = \frac{2 \langle P_{\text{rating}} \rangle}{|Y|} \] \hspace{1cm} (4.96)

In simulation, the material properties are shown in Table 4.2. The piezoelectric material properties of the PWAS are given in Table 4.3

<table>
<thead>
<tr>
<th>Table 4.2</th>
<th>Typical material properties for aluminum alloy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aluminum</td>
</tr>
<tr>
<td>Modulus ((E))</td>
<td>70GPa</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>0.33</td>
</tr>
<tr>
<td>Density ((\rho))</td>
<td>2700</td>
</tr>
<tr>
<td>Yield Stress ((Y))</td>
<td>500MPa</td>
</tr>
</tbody>
</table>
Table 4.3  Piezoelectric properties of the PWAS material (APC-850)

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compliance, in plane</td>
<td>$s_{11}^E$</td>
<td>$15.30 \cdot 10^{-12} \text{Pa}^{-1}$</td>
</tr>
<tr>
<td>Compliance, thickness wise</td>
<td>$s_{33}^E$</td>
<td>$17.30 \cdot 10^{-12} \text{Pa}^{-1}$</td>
</tr>
<tr>
<td>Dielectric constant</td>
<td>$\varepsilon_{33}^T$</td>
<td>$1750 \varepsilon_0$</td>
</tr>
<tr>
<td>Thickness wise induced-strain coefficient</td>
<td>$d_{33}$</td>
<td>$400 \cdot 10^{-12} \text{m/V}$</td>
</tr>
<tr>
<td>In-plane induced-strain coefficient</td>
<td>$d_{31}$</td>
<td>$-175 \cdot 10^{-12} \text{m/V}$</td>
</tr>
<tr>
<td>Coupling factor, parallel to electric field</td>
<td>$k_{33}$</td>
<td>0.72</td>
</tr>
<tr>
<td>Coupling factor, transverse to electric field</td>
<td>$k_{31}$</td>
<td>0.36</td>
</tr>
<tr>
<td>Coupling factor, transverse to electric field, polar motion</td>
<td>$k_p$</td>
<td>0.63</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>$\nu$</td>
<td>0.35</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>$7700 \text{kg/m}^3$</td>
</tr>
<tr>
<td>Sound speed</td>
<td>$c$</td>
<td>$2900 \text{m/s}$</td>
</tr>
</tbody>
</table>

In transmitter simulation, an Aluminum alloy 2024 infinite bar is 40 mm width and 1 mm thickness. The PWAS is 7mm length, 40mm width and 0.2 mm thickness and ideal bond in the center. The location of PWAS is 0 mm. A harmonic voltage with 10V amplitude is applied on the PWAS. The admittance and power at harmonic situation are considered. The parameter is listed in Table 4.4.

Table 4.4  Transmitter simulation parameters

<table>
<thead>
<tr>
<th>Beam</th>
<th>Transmitter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Height</td>
<td>1 mm</td>
</tr>
<tr>
<td>Width</td>
<td>40 mm</td>
</tr>
<tr>
<td>Transmitter Receiver Distance</td>
<td>200 mm</td>
</tr>
<tr>
<td>Frequency</td>
<td>Frequency sweep 0-600 kHz</td>
</tr>
<tr>
<td>Measurement Instrument Resistance</td>
<td>1000 Ω</td>
</tr>
</tbody>
</table>
Under constant voltage input, the power is proportional with the impedance of PWAS. The active power, reactive power, power rating and peak power is shown in Figure 4.5. The active power is the power that converts to the mechanical power at the interface. The power rating is the power requirement of the power supply without distortion. In induced-strain transmitter applications, the reactive power is the dominant factor, since the transmitter impedance is dominated by its capacitive behavior. Managing high reactive power requirements is one of the challenges of using induced-strain transmitters. For example, the active power at 100 kHz is 20mW and the reactive power is 600mW. The
power rating and peak power is overlapped to reactive power (Figure 4.6) because of the small value of active power. The ratio of reactive power and active power at different frequency is shown in Figure 4.6

Figure 4.6 Transmitter PWAS power and reactive and active power ratio.

4.2.3 Transmitter Mechanical Response

By definition, mechanical power is

\[ p_{\text{mech}}(t) = F(t) v(t) \]  \hspace{1cm} (4.97)

Under harmonic excitation, the force is

\[ F(t) = \hat{F}(\omega) e^{-i\omega t} \]  \hspace{1cm} (4.98)

Recall the relation of the pin force and the elongation of PWAS Equation (3.23), i.e.,

\[ \hat{F}(\omega) = k_i \left( \Delta u(\omega) - u_{IS4} \right) \]  \hspace{1cm} (4.99)

Recall Equation (4.58), i.e.,

\[ \Delta \hat{u}(\omega) = \frac{k C_{AA}(\omega)}{k_i C_{AA}(\omega) - 1} u_{IS4} \]  \hspace{1cm} (4.100)
Substitution of Equation (4.100) into (4.99) yields

\[ \hat{F}(\omega) = k_i \left( \frac{k_i C_{AA}(\omega)}{k_i C_{AA}(\omega) - 1} - 1 \right) u_{ts} = \frac{1}{k_i C_{AA}(\omega) - 1} k_i u_{ts} \]  \hspace{1cm} (4.101)

The frequency response function of pin force at the transmitter PWAS is

\[ F_{RF_{vv}}(\omega) = \frac{\hat{F}}{\hat{V}} = \frac{1}{k_i C_{AA}(\omega) - 1} k_i d_{31} \frac{l}{t} \]  \hspace{1cm} (4.102)

Recall the in-plane surface displacement Equation (4.51), i.e.,

\[ u_p(t) = \frac{i\hat{F}}{2EA} \left\{ \frac{(-e^{-i\xi_0 \xi_0} + e^{-i\xi_0 (x_0 + l)})}{\xi_0} e^{i(\xi_0 x - \omega t)} + 3 \frac{(-e^{-i\xi_F \xi_0} + e^{-i\xi_F (x_0 + l)})}{\xi_F} e^{i(\xi_F x - \omega t)} \right\} \]  \hspace{1cm} (4.103)

![Figure 4.7 Schematic of a single PWAS attached to an infinite bar.](image)

The displacement at \( A = x_0 \) is

\[ u_A(x_0, t) = \frac{i\hat{F}}{2EA} \left\{ \frac{(-e^{-i\xi_0 \xi_0} + e^{-i\xi_0 (x_0 + l)})}{\xi_0} e^{i(\xi_0 x_0 - \omega t)} \right\} \]

\[ + 3 \frac{(-e^{-i\xi_F \xi_0} + e^{-i\xi_F (x_0 + l)})}{\xi_F} e^{i(\xi_F x_0 - \omega t)} \]  \hspace{1cm} (4.104)

\[ = e^{-i\alpha} \left\{ \frac{-1 + e^{-i\xi_0 l}}{\xi_0} + 3 \frac{-1 + e^{-i\xi_F l}}{\xi_F} \right\} \]
The velocity at \( A = x_0 \) is

\[
v_A = \dot{u}_A(x_0,t) = -i\omega u_A(x_0,t) = -\frac{\omega}{2EA} \left[ 1 - e^{-i\xi_0t} + 3 \frac{1 - e^{-i\xi_Ft}}{\xi_0} \right] \hat{F}(\omega)e^{-iat} \tag{4.105}
\]

The FRF of the displacement \( FRF_{uVA}(\omega) \) and of velocity \( FRF_{vVA}(\omega) \) at \( A = x_0 \) are

\[
FRF_{uVA}(\omega) = \frac{\hat{u}_A}{V} \tag{4.106}
\]

\[
FRF_{vVA}(\omega) = \frac{\hat{v}_A}{V} \tag{4.107}
\]

Hence,

\[
FRF_{uVA}(\omega) = \frac{\hat{u}_A}{V} = -i \frac{\omega}{2EA} \left[ 1 - e^{-i\xi_0t} + 3 \frac{1 - e^{-i\xi_Ft}}{\xi_0} \right] \frac{\hat{F}}{V} \tag{4.108}
\]

\[
FRF_{vVA}(\omega) = \frac{\hat{v}_A}{V} = -\frac{\omega}{2EA} \left[ 1 - e^{-i\xi_0t} + 3 \frac{1 - e^{-i\xi_Ft}}{\xi_0} \right] \frac{\hat{F}}{V} \tag{4.109}
\]

At \( A' = x_0 + l \), the displacement is

\[
u_A(x_0+l,t) = \frac{i\hat{F}}{2EA} \left\{ \frac{(-e^{-i\xi_0x_0} + e^{-i\xi_0(x_0+l)})}{\xi_0} e^{i(\xi_0(x_0+l)-at)} + 3 \frac{(-e^{-i\xi_Fx_0} + e^{-i\xi_F(x_0+l)})}{\xi_F} e^{i(\xi_F(x_0+l)-at)} \right\} \tag{4.110}
\]

\[
= e^{-iat} \frac{i\hat{F}}{2EA} \left[ 1 - e^{-i\xi_0t} + 3 \frac{1 - e^{-i\xi_Ft}}{\xi_F} \right]
\]

53
The velocity at \( A' = x_0 + l \) is

\[
v_{x'} = \dot{u}_{x'}(x_0 + l, t) = -i \omega u_{x'}(x_0 + l, t) = \frac{\omega}{2EA} \left[ 1 - e^{i\xi_0 l} + 3 \frac{1 - e^{i\xi_F l}}{\xi_0} \right] \hat{F}(\omega) e^{-i\omega x} \quad (4.111)
\]

The FRF of the displacement \( FRF_{uA'}(\omega) \) and of velocity \( FRF_{vA'}(\omega) \) at \( A' = x_0 + l \) are

\[
FRF_{uA'}(\omega) = \frac{\hat{u}_{x'}}{V} \quad (4.112)
\]

\[
FRF_{vA'}(\omega) = \frac{\hat{v}_{x'}}{V} \quad (4.113)
\]

Hence,

\[
FRF_{uA'}(\omega) = \frac{\hat{u}_{x'}}{V} = \frac{i \hat{F}}{2EA \hat{V}} \left[ 1 - e^{i\xi_0 l} + 3 \frac{1 - e^{i\xi_F l}}{\xi_0} \right] FRF_{vA'}(\omega) \quad (4.114)
\]

\[
FRF_{vA'}(\omega) = \frac{\hat{v}_{x'}}{V} = \frac{\omega \hat{F}}{2EA \hat{V}} \left[ 1 - e^{i\xi_0 l} + 3 \frac{1 - e^{i\xi_F l}}{\xi_0} \right] FRF_{vA'}(\omega) \quad (4.115)
\]

The time-averaged product of two harmonic variables is one half the product of one variable times the conjugate of the other. The time averaged mechanical power under harmonic excitation for PWAS transmitter is

\[
\langle P_A \rangle = -\frac{1}{2} \bar{F}(\omega) \dot{v}_{A'}(\omega) \quad (4.116)
\]

\[
\langle P_{A'} \rangle = \frac{1}{2} \bar{F}(\omega) \dot{v}_{A'}(\omega) \quad (4.117)
\]
The numerical simulation uses the same simulation setup described in transmitter electrical response. The numerical simulation results (Figure 4.8) indicate that the time-averaged mechanical power at the PWAS-structure interface both ends are equal, i.e.,

\[ \langle P_A \rangle = \langle P_A' \rangle \]  

(4.118)

The time-averaged active power equals the sum of the time-averaged interface mechanical power, i.e.,

\[ P_{active} = \langle P_A \rangle + \langle P_A' \rangle = 2\langle P_A \rangle \]  

(4.119)

![Figure 4.8 Transmitter power simulation: (a) electrical active power; (b) mechanical power at A and A’ (note \( P_{active} = \langle P_A \rangle + \langle P_A' \rangle \)).](image)

4.2.4 Wave Power and Energy

The waves that propagate in a bar consist of axial and flexural waves. Each wave form travels with its energy. The power and energy of axial and flexural waves are derived next.

4.2.4.1 Axial Wave Energy

By definition, the kinetic and elastic densities per unit volume are given by
\[ k = \frac{1}{2} \rho \dot{u}^2 \quad \text{(kinetic energy per unit volume)} \]  \hspace{1cm} (4.120)

\[ v_e = \frac{1}{2} \sigma \epsilon \quad \text{(elastic energy per unit volume)} \]  \hspace{1cm} (4.121)

To obtain the energy distribution per unit length of bar, we integrate Equation (4.120) and (4.121) across the cross-section. The particle motion \(u(x,t)\) associated with the axial waves propagating in a bar is uniform over the bar cross-section. Hence,

\[ k(x,t) = \int_A \frac{1}{2} \rho \dot{u}^2 (x,t) \, dA = \frac{1}{2} m \dot{u}^2 (x,t) \]  \hspace{1cm} (4.122)

\( \text{(kinetic energy per unit length)} \)

\[ v_e (x,t) = \int_A \frac{1}{2} \sigma (x,t) \epsilon (x,t) \, dA = \frac{1}{2} (EA) u'^2 (x,t) \]  \hspace{1cm} (4.123)

\( \text{(elastic energy per unit length)} \)

The total energy is calculated by adding the kinetic and elastic energies, i.e.,

\[ e(x,t) = k(x,t) + v_e (x,t) \quad \text{(total energy per unit length)} \]  \hspace{1cm} (4.124)

Consider a forward propagating wave, i.e.,

\[ u(x,t) = f(x - ct) \quad \text{(forward wave)} \]  \hspace{1cm} (4.125)

Substitution of Equation (4.125) into Equation (4.122) and (4.123) yields

\[ k(x,t) = \frac{1}{2} m (-c)^2 f'^2 (x - ct) = \frac{1}{2} mc^2 f'^2 (x - ct) \]  \hspace{1cm} (4.126)

\[ v_e (x,t) = \frac{1}{2} (EA) f'^2 (x - ct) = \frac{1}{2} mc^2 f'^2 (x - ct) \]  \hspace{1cm} (4.127)
where the relation \(EA = mc^2\) is from the axial wave speed in a bar, and \(c = \sqrt{\frac{EA}{m}}\). It is apparent that the kinetic energy is equal to the elastic energy.

\[
k(x,t) = v_e(x,t) = \frac{1}{2} mc^2 f^2(x - ct)
\]

Equation (4.128) indicates that a propagating wave has its energy equally partitioned into kinetic and elastic components. Substitution into Equation (4.124) yields the total energy density per unit length of a propagating axial wave in a bar as

\[
e(x,t) = k(x,t) + v_e(x,t) = mc^2 f^2(x - ct)
\]

The case of the harmonic waves is a particular case of the general propagating waves, and hence the previous results hold. Assume forward harmonic wave in the form,

\[
u(x,t) = \text{Re} \left( \hat{u} e^{i(\gamma x - \omega t)} \right) = \hat{u} \cos(\gamma x - \omega t)
\]

Substitution of Equation (4.130) into (4.129) gives the total energy, i.e.,

\[
e(x,t) = k(x,t) + v_e(x,t) = m\omega^2 \hat{u}^2 \sin^2(\gamma x - \omega t)
\]

The amplitude of wave energy is given by

\[
\hat{e} = m\omega^2 \hat{u}^2
\]

It indicates that, for a harmonic wave, the energy amplitude is proportional to the frequency squared.

The time-averaged energy can be calculated using the product of two harmonic variables is one half the real part of the product of one variable times the conjugate of the other. The time-averaged energy is
\[ \langle e \rangle = \frac{1}{2} m \omega^2 (u \bar{u}) \]  

(4.133)

For a single PWAS attached to a bar, the displacement is

\[ u(x) = \frac{i}{2EA\xi_0} \left\{ \hat{F}(-e^{-i\xi_0x} + e^{-i\xi_0(x_0+t)}) \right\} e^{i(\xi x - \omega t)} \]  

(4.134)

For the special case, one PWAS at location \( x_0 = -l/2 = -a \), the displacement is

\[ u(x) = \frac{i}{2EA\xi_0} \left\{ \hat{F}(-e^{-i\xi_0a} + e^{-i\xi_0a}) \right\} e^{i(\xi_0 x - \omega t)} = \frac{-i\hat{F}}{EA\xi_0} \sin(\xi_0a) e^{i(\xi_0 x - \omega t)} \]  

(4.135)

The strain in this situation is

\[ \varepsilon = \varepsilon_0 \sin(\xi_0a) e^{i(\xi_0 x - \omega t)} \]  

(4.136)

The time-average power is

\[ \langle e \rangle = \frac{1}{2} mc^2 \varepsilon^2 a^2 \sin^2(\xi_0a) \]  

(4.137)

4.2.4.2 Axial Wave Power

The wave power is seen as the rate with which energy is transferred past a certain point during wave propagation. Assume that we cut the bar at a given point \( x = x_0 \) and study the energy flux across the interface. Consider the right hand part of the cut bar, and replace the removed part of the bar with the interface tractions \( \bar{r} = -\sigma \bar{e}_x \). The work done by the tractions onto the interface represents the energy input into the bar. The time rate of this interface work is the energy flux into the bar or the power input into the bar. We write that power is the product of tractions and particle velocity, \( \dot{u} = \bar{u} \bar{e}_x \), i.e.,
\[ P = \int_A \tilde{i} \tilde{u} dA = -\sigma \tilde{u} A = -EAu'\tilde{u} = -mc^2u'\tilde{u} \quad (4.138) \]

For the forward wave (4.125), we have \( u'(x,t) = f'(x-ct) \) and \( \tilde{u}(x,t) = -cf'(x-ct) \); hence, Equation (4.138) becomes

\[ P(x,t) = -mc^2 f'(x-ct)[-cf'(x-ct)] = mc^3 f^2(x-ct) \quad (4.139) \]

Comparison of Equation (4.139) and (4.129) reveals that the power is the product of energy and wave speed, i.e.,

\[ P(x,t) = ce(x,t) \quad (4.140) \]

The same conclusion can be reached via the definition of power as the rate of energy transferred from the left-hand part of the bar.

The power is

\[ \langle P_{\text{axial}} \rangle = c \langle e \rangle = \frac{1}{2} c m \omega^2 \tilde{u}^2 = \frac{1}{2} c \rho \omega^2 \tilde{u}^2 \quad (4.141) \]

Recall the displacement of axial wave's equation (4.20), i.e.,

\[ u(x) = \frac{i}{2EA\bar{\xi}_0} \left\{ \tilde{F}(-e^{-i\omega x_0} + e^{-i\omega(x_0+t)}) \right\} e^{i(\omega x - ct)} \quad (4.142) \]

For one PWAS on a bar setup, the time-averaged axial wave power generated by PWAS is

\[ \langle P_{\text{axial}} \rangle = c \langle e \rangle = \frac{1}{2} c m \omega^2 \langle u \tilde{u} \rangle \quad (4.143) \]
For the special case, only one PWAS at location \( x_0 = -\frac{l}{2} = -a \), the time-averaged power is

\[
\langle P_{\text{avg}} \rangle = c \langle e \rangle = \frac{1}{2} mc^3 a^2 \sin^2 (\xi_0 a)
\]  

\[ (4.144) \]

### 4.2.4.3 Flexural Wave Energy

The kinetic and elastic energy densities per unit volume is given by

\[
k = \frac{1}{2} \rho \left( \dot{u}^2 + \dot{w}^2 \right) \text{ (kinetic energy per unit volume)} 
\]

\[ (4.145) \]

\[
v_c = \frac{1}{2} \left( \sigma_{xx} \varepsilon_{xx} + \sigma_{zz} \varepsilon_{zz} \right) \text{ (elastic energy per unit volume)} 
\]

\[ (4.146) \]

To obtain the energy density per unit beam length, we integrate Equation\((4.145)\) and \((4.146)\) across the cross-section, i.e.,

\[
k(x,t) = \int A \frac{1}{2} \rho \left( \dot{u}^2 (x,z,t) + \dot{w}^2 (x,t) \right) dA 
\]

(kinetic energy per unit length)

\[ (4.147) \]

\[
v_c = \int A \frac{1}{2} \left( \sigma_{xx} (x,z,t) \varepsilon_{xx} (x,z,t) + \sigma_{zz} (x,z,t) \varepsilon_{zz} (x,z,t) \right) dA 
\]

(elastic energy per unit length)

\[ (4.148) \]

Assume harmonic wave motion is of the following form

\[
w = \text{Re} \left( \hat{w} e^{i(\gamma x - \omega t)} \right) 
\]

\[ (4.149) \]

\[
u = -zw' = \text{Re} \left( -iz\gamma \hat{w} e^{i(\gamma x - \omega t)} \right) = \text{Re} \left( -iz\gamma w \right) 
\]
Under the Euler-Bernoulli bending assumptions employed here, the shear deformation and rotary inertia are ignored, and the kinetic and elastic energy expressions only retain the terms in the $\dot{w}$ and $\sigma_{xx} \varepsilon_{xx}$, respectively. The time-averaged expressions of the kinetic energy and elastic energy are

$$\langle k \rangle = \frac{1}{4} m \omega^2 \langle w \, \bar{w} \rangle \quad (4.150)$$

$$\langle v_e \rangle = \frac{1}{4} EI \gamma^4 \langle w \, \bar{w} \rangle \quad (4.151)$$

For the flexural wave in a bar, $EI \gamma^4 = m \omega^2$. Hence,

$$\langle k \rangle = \langle v_e \rangle = \frac{1}{4} EI \gamma^4 \langle w \, \bar{w} \rangle \quad (4.152)$$

The time-averaged total energy is

$$\langle e \rangle = \langle k \rangle + \langle v_e \rangle = \frac{1}{2} m \omega^2 \langle w \, \bar{w} \rangle \quad (4.153)$$

Recall the special case, only one PWAS at location $x_0 = -\frac{l}{2} = -a$, the displacement is

$$w(x) = \frac{3}{\xi_p} \frac{E A h}{\hat{F}^2} 2i \sin(\xi_p a) e^{i(\xi_p x - \alpha x)} \quad (4.154)$$

The time-average power is

$$\langle e \rangle = \frac{1}{2} m \omega^2 \langle w \, \bar{w} \rangle = \frac{1}{2} m \omega^2 \left( \frac{36}{\xi_p^4} \frac{E A h^2}{\hat{F}^2} \varepsilon_a^2 \sin^2(\xi_p a) \right) \quad (4.155)$$

Recall the flexural wave number
\[
\xi^4_F = \omega^2 \frac{\rho A}{EI} = \omega^2 / \bar{a}^2 \text{ where } \bar{a}^2 = \frac{EI}{\rho A} = \frac{h^2 E}{24 \rho}
\] (4.156)

Substitution of Equation (4.156) into (4.155) yields

\[
\langle e \rangle = \frac{1}{2} m \omega^2 \left( \frac{36}{\xi^4_F h^2} \varepsilon_a^2 \sin^2 (\xi_F a) \right) = \frac{1}{2} m \bar{a}^2 \left( \frac{36}{h^2} \varepsilon_a^2 \sin^2 (\xi_F a) \right) = \frac{1}{2} \frac{m E h^2}{24 \rho} \left( \frac{36}{h^2} \varepsilon_a^2 \sin^2 (\xi_F a) \right) = \frac{3}{4} m c^2 e_a^2 \sin^2 (\xi_F a)
\] (4.157)

4.2.4.4 Flexural Wave Power

The wave power is seen as the rate with which energy is transferred past a certain point during wave propagation. Assume that we cut the bar at a given point \( x = x_0 \) and study the energy flux across the interface. Consider the right hand part of the cut bar, and replace the removed part of the bar with the interface tractions \( \tilde{t} = -\sigma \tilde{e}_x \). The work done by the tractions onto the interface represents the energy input into the bar. The time rate of this interface work is the energy flux into the bar or the power input into the bar. We write that power is the product of tractions and particle velocity, \( \dot{u} = \dot{u} \tilde{e}_x \), i.e.,

\[
P = \int_A \tilde{t} \dot{u} dA = -\sigma \dot{u} A = -EA \dot{u} \dot{u} = -mc^2 u \dot{u}
\] (4.158)

For the forward wave (4.125), we have \( u'(x,t) = f'(x-ct) \) and \( \dot{u}(x,t) = -cf'(x-ct) \); hence, Equation(4.138) becomes

\[
P(x,t) = -mc^2 f'(x-ct) \left[ -cf'(x-ct) \right] = mc^3 f'^2 (x-ct)
\] (4.159)

Comparison of Equation (4.139) and (4.129) reveals that the power is the product of energy and wave speed, i.e.,
\[
P(x,t) = ce(x,t)
\]

(4.160)

The same conclusion can be reached via the definition of power as the rate of energy transferred from the left-hand part of the bar.

The power flow associated with flexural waves is calculated by taking the integral over the cross-section of the product between the tractions vector \( \vec{t} = -\sigma_{xx} \vec{e}_x - \sigma_{xz} \vec{e}_z \) and velocity vector \( \vec{u} = \dot{u}_x \vec{e}_x + \dot{u}_z \vec{e}_z \), i.e.,

\[
P = \int_A \vec{t} \cdot \dot{\vec{u}} dA = -\int_A (\sigma_{xx} \dot{u}_x + \sigma_{xz} \dot{u}_z) dA
\]

(4.161)

Where \( u_x = u \) and \( u_z = w \). The negative sign is implied by the tractions sign convention.

Recalling the stress-strain relations, we write

\[
\sigma_{xx} = E\varepsilon_{xx} = E\gamma^2 zw \quad \text{and} \quad \sigma_{xz} = 2G\varepsilon_{zx} = 0
\]

(4.162)

For harmonic flexural wave, the time-averaged power is

\[
\langle P \rangle = -\frac{1}{2} \int_A \vec{\sigma}_{xx} \dot{u}_x dA = \frac{1}{2} EI \gamma^2 \omega \langle w \dot{w} \rangle = \frac{1}{2} c_F m \omega \dot{\omega} \langle w \dot{w} \rangle
\]

(4.163)

Comparison of Equation (4.163) and (4.153) yields

\[
\langle P \rangle = c_F \langle e \rangle
\]

(4.164)

This indicates that the power of flexural waves is the product between energy and flexural wave speed. Recall the flexural wave displacement under one PWAS harmonic excitation

For the special case, only one PWAS at location \( x_0 = -\frac{l}{2} = -a \), the power is

63
\[
\langle P \rangle = c_F \langle \varepsilon \rangle = \frac{3}{4} mc_F c^2 \varepsilon_0^2 \sin^2(\xi r a)
\]  
(4.165)

Under ideal excitation source assumption, we considered four kinds of excitation source: 
(a) constant voltage input; (b) constant current input; (c) constant power rating input; (d) 
constant wave power output.

Using the same simulation setup described in transmitter electrical response, the axial and 
flexural wave power is shown in Figure 4.9.

![Figure 4.9 Axial and flexural wave power.](image)

**4.2.5 Parametric Study of Transmitter Behavior**

The following simulation is under ideal excitation source at the transmitter PWAS and 
fully-resistive external load at the receiver PWAS. The ideal excitation source has four 
types: (a) constant voltage input; (b) constant current input; (c) constant power rating 
input; (d) source can provide constant output wave power. The transmitter size plays an
import factors to the power and energy. The following transmitter size parametric study is to find the optimum transmitter size under ideal excitation source.

Using the same example, an Aluminum alloy 2024 infinite bar is 40 mm wide and 1 mm thick. The transmitter is 40 mm wide and 0.2 mm thick and ideal bond in the center. The transmitter length varies from 5-25 mm to trace the PWAS transmitter size effect. The location of PWAS is 0 mm. A harmonic voltage with 10V amplitude is applied on the PWAS. The admittance and power at harmonic situation are considered. All the parameter is listed in Table 4.5.

<table>
<thead>
<tr>
<th>Table 4.5 Transmitter size effect simulation parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Height</td>
</tr>
<tr>
<td>Width</td>
</tr>
<tr>
<td>Frequency</td>
</tr>
</tbody>
</table>

### 4.2.5.1 Admittance

The transmitter size is related to the admittance of embedded PWAS on structure. The absolute value of admittance (Figure 4.10) contributes to the power rating. The absolute value of admittance increases when PWAS transmitter size increases. It increases with frequency increasing as well.
Figure 4.10 Parametric study of transmitter size effect of admittance.

The real part of admittance (Figure 4.11) contributes to the active power. It indicates that the PWAS transmitter size and frequency has a tuning effect. The admittance peak and valley is associated with PWAS size and frequency.

Figure 4.11 Parametric study of transmitter size effect for real part of admittance.
4.2.5.2  Constant Voltage Input

Under constant 10-V voltage input, the power rating, active power and wave power is

\[
P_{\text{rating}} = \frac{1}{2} |Y||V|^2
\]

\[
P_{\text{active}} = \frac{1}{2} Y_R |V|^2 = 2 \langle P_{\text{wave}} \rangle
\]

\[
P_{\text{wave}} = \frac{1}{4} Y_R |V|^2
\]

Based on Equation (4.166), we know that the power rating is proportional to the absolute value of input admittance. The active power is proportional to the real part of input admittance. The wave power is half of the active power. The power rating and wave power in this simulation is shown in Figure 4.12 and Figure 4.13. For a certain transmitter size, the power rating increases when the frequency increases. At a certain frequency, the power rating increases when the transmitter size increases. To drive a 25mm length PWAS at 600 kHz with 10 V constant voltage input, the power amplifier need to provide 12.5 W powers. Figure 4.13 shows the total axial and flexural wave power that PWAS generates into the structure. The wave power output is determined by the tuning effect of transmitter size and excitation frequency. The maximum wave output in this experimental setup is about 40 mW. The active power is twice the wave power since the wave propagates in both directions. It shows that the reactive power is the dominant factor, since the transmitter impedance is dominated by its capacitive behavior. It also indicates the increasing the PWAS size solo does not mean to generate more wave power. It gives the guideline for the transmitter size and excitation frequency for maximum wave output.
Figure 4.14 and Figure 4.15 show the axial and flexural wave power separately under constant voltage input. In PWAS SHM applications, a single mode is often used to reduce the complexity. These figures give the guideline for the optimum size and frequency for mode excitation.

![Graph showing the parametric study of transmitter size effect for power rating.](image)

**Figure 4.12** Parametric study of transmitter size effect for power rating.
Figure 4.13 Parametric study of transmitter size effect for wave power.

Figure 4.14 Axial wave power under constant 10-V voltage input.
4.2.5.3 **Constant Current Input**

Under constant 100-mA current input, the voltage, power rating and wave power is

\[
V = \frac{I}{|\mathbf{Y}|}
\]  

(4.169)

\[
\langle P_{\text{rating}} \rangle = \frac{1}{2} |\mathbf{Y}| |\mathbf{V}|^2 = \frac{1}{2} \frac{|\mathbf{I}|^2}{|\mathbf{Y}|}
\]  

(4.170)

\[
\langle P_{\text{wave}} \rangle = \frac{1}{4} Y_{\text{r}} |\mathbf{V}|^2 = \frac{1}{4} \frac{Y_{\text{r}}}{|\mathbf{Y}|} |\mathbf{I}|^2
\]  

(4.171)

The power rating and wave power under constant 100-mA current input is shown in Figure 4.16 - Figure 4.18.
Figure 4.16 Input voltage under constant 100-mA current input.
4.2.5.4 Constant Power Rating Input

Under constant 10-W power rating input, the voltage, current and wave power are

\[ |V|^2 = \frac{\langle P_{\text{rating}} \rangle}{|Y|} \]  
\[ |I|^2 = 2 |Y| \langle P_{\text{rating}} \rangle \]  
\[ \langle P_{\text{wave}} \rangle = \frac{1}{4} Y_r 2 \frac{\langle P_{\text{rating}} \rangle}{|Y|} = \frac{1}{2} Y_r \frac{\langle P_{\text{rating}} \rangle}{|Y|} \]

The input voltage, current and wave power under constant 10-W power rating input are shown in Figure 4.19 - Figure 4.21
Figure 4.19 Input voltage under constant 10-W power rating input.

Figure 4.20 Input current under constant 10-W power rating input.
Figure 4.21 Wave power under constant 10-W power rating input.

4.2.5.5 Constant Wave Power Output

Under constant 10-mW wave power output, the voltage and power rating is

\[ |V|^2 = 4 \frac{P_{\text{wave}}}{Y_R} \]  \hspace{1cm} (4.175)

\[ \langle P_{\text{rating}} \rangle = \frac{1}{2} |Y| |V|^2 = 2 \frac{|Y|}{Y_R} \langle P_{\text{wave}} \rangle \]  \hspace{1cm} (4.176)

The input voltage current and wave power under constant 10-mW wave power output are shown in Figure 4.22 - Figure 4.24. It shows a peak for PWAS size between 15-20 mm.
Figure 4.22 Input voltage under constant 10-mW wave power output.

Figure 4.23 Input current under constant 10-mW wave power output.
4.3 **Receiver Power and Energy**

In this part, we consider the mechanical power and energy and the electrical power and energy output of the receiver. The PWAS receiver structural interface acoustic and electrical energy transduction is studied. Using PWAS as a strain sensor, the electrical power, the mechanical power at the PWAS and structure interface is developed. The simulation of the power and energy in each stage was calculated here.

### 4.3.1 Receiver Electrical Response

Strain wave $\varepsilon = \varepsilon_e e^{(i k y-x_0)} + \varepsilon_i e^{(i k y-x_0)}$ travels to one receiver PWAS at location $x_0$, and calculate the output voltage $V$. Recall receiver output Equation (3.40), i.e.,

$$\hat{V} = \frac{1}{Y_e + (1-k_{51}^2)Y_0} i \omega b \frac{d_{31}}{s_{11}} \Delta u$$

(4.177)
For sensing application, we assume the PWAS does not affect the wave propagation. The PWAS receiver elongation is generated by input strain wave, it is written as

\[
\Delta u = \varepsilon_d \left( e^{i\xi_0(x_0 + l)} - e^{i\xi_0 x_0} \right) + \varepsilon_F l \left( e^{i\xi_F(x_0 + l)} - e^{i\xi_F x_0} \right) \tag{4.178}
\]

The receiver output voltage due to the input strain wave is

\[
\hat{V} = \frac{1}{Y_e + (1-k_{31}^2)Y_0} i\omega b \frac{d_{31}}{s_{11}} \left( e^{i\xi_0(x_0 + l)} - e^{i\xi_0 x_0} \right) + \varepsilon_F l \left( e^{i\xi_F(x_0 + l)} - e^{i\xi_F x_0} \right) \tag{4.179}
\]

Under constant axial excitation, the axial strain is \( \varepsilon = \varepsilon_0 e^{i(\xi_0 + \alpha t)} \), the displacement is \( |u| = \varepsilon_d l \). The output voltage is

\[
\hat{V} = \frac{1}{Y_e + (1-k_{31}^2)Y_0} i\omega b \frac{d_{31}}{s_{11}} \varepsilon_d l e^{i\xi_0 x_0} \left( e^{i\xi_0 l} - 1 \right) \tag{4.180}
\]

Under constant flexural wave input, the output voltage is

\[
\hat{V} = \frac{1}{Y_e + (1-k_{31}^2)Y_0} i\omega b \frac{d_{31}}{s_{11}} \varepsilon_F l e^{i\xi_F x_0} \left( e^{i\xi_F l} - 1 \right) \tag{4.181}
\]

The current generated in the measurement equipment is

\[
I = Y_e V \tag{4.182}
\]

The time-average electrical response for receiver is

\[
\langle P(\omega) \rangle = \frac{1}{2} \text{Re} \left( \tilde{P}(\omega) I(\omega) \right) \tag{4.183}
\]

### 4.3.2 Receiver Mechanical Response

Receiver can undergoes constant strain wave or constant power wave; the mechanical responses under these two situations are shown next.
4.3.2.1 Constant Strain Wave Input

Recall the receiver equation

\[
\hat{F}_2(\omega) = (1 - \frac{k_{31}^2 Y_0}{Y_e + (1 - k_{31}^2)Y_0})k_i \Delta u(\omega) = k_i R(\omega) \Delta u(\omega)
\] (4.184)

The elongation of PWAS under harmonic strain excitation

\[
\Delta u = \epsilon_a l \left( e^{i\xi_0(x_0 + l)} - e^{i\xi_0 x_0} \right) + \epsilon_F l \left( e^{i\xi_F(x_0 + l)} - e^{i\xi_F x_0} \right)
\] (4.185)

The force for a PWAS receiver under harmonic strain excitation is

\[
\hat{F}_2(\omega) = \left[ \epsilon_a l \left( e^{i\xi_0(x_0 + l)} - e^{i\xi_0 x_0} \right) + \epsilon_F l \left( e^{i\xi_F(x_0 + l)} - e^{i\xi_F x_0} \right) \right] k_i R(\omega)
\] (4.186)

The displacement of PWAS at \( A = x_0 \) of strain wave is

\[
u(x_A, t) = l\epsilon_a e^{i\xi_0 x_0} + l\epsilon_F e^{i\xi_F x_0}
\] (4.187)

The velocity at \( A = x_0 \) is

\[
\nu(x_A, t) = \left( -i \omega \right) \left[ l\epsilon_a e^{i\xi_0 x_0} + l\epsilon_F e^{i\xi_F x_0} \right]
\] (4.188)

\[
\dot{\nu}(x_A, \omega) = \left( -i \omega l \right) \left[ \epsilon_a e^{i\xi_0 x_0} + \epsilon_F e^{i\xi_F x_0} \right]
\] (4.189)

The time-average mechanical response for receiver \( A = x_0 \) is

\[
\langle P_A(\omega) \rangle = \frac{1}{2} \Re \{ \hat{F}_2(\omega) \nu_A(\omega) \}
\] (4.190)

The displacement of PWAS at \( A' = x_0 + l \) of strain wave is

\[
u(x_{A'}, t) = l\epsilon_a e^{i\xi_0(x_0 + l)} + l\epsilon_F e^{i\xi_F(x_0 + l)}
\] (4.191)
The velocity at $A' = x_0 + l$ is

\[ v(x', t) = (-i\omega l) \left[ \varepsilon_x e^{i(\xi_0(x_0+l)-\omega t)} + \varepsilon_x e^{i(\xi_l(x_0+l)-\omega t)} \right] \]  

(4.192)

\[ \hat{v}(x', \omega) = (-i\omega l) \left[ \varepsilon_x e^{i\xi_0(x_0+l)} + \varepsilon_x e^{i\xi_l(x_0+l)} \right] \]  

(4.193)

The time-average mechanical response for receiver $A' = x_0 + l$ is

\[ \langle P'(\omega) \rangle = \frac{1}{2} \text{Re}\left( \tilde{F}_z(\omega)v'(\omega) \right) \]  

(4.194)

4.3.2.2 Constant Power Axial Wave Input

Recall the time-average axial wave energy Equation (4.143), i.e.,

\[ \langle e \rangle = \frac{1}{2} m\omega \hat{\delta} (u \bar{u}) \]  

(4.195)

The time-average axial wave power is

\[ \langle P_{\text{axial}} \rangle = c \langle e \rangle = \frac{1}{2} cm\omega \hat{\delta} (u \bar{u}) = \frac{1}{2} cm\omega \left( e_0 l \right)^2 \]  

(4.196)

The force applied on the receiver edges is

\[ \hat{F} = (1 - k_3^2 Y_0) \frac{k_l \Delta \hat{u}}{Y_e + (1 - k_3^2) Y_0} = R(\omega) k_l \Delta \hat{u} \]  

(4.197)

The velocity at the receiver edges is

\[ v_\ell = -i\omega \varepsilon_\ell e^{i(\xi_\ell - \omega t)} \]  

(4.198)

Hence, the time-average mechanical power is
\[
\langle P_d(\omega) \rangle = \frac{1}{2} \Re \left( \overline{F(\omega)} v_d(\omega) \right)
\]  
(4.199)

If the axial wave power is constant, the wave strain is

\[
\varepsilon_a = \sqrt{\frac{2 \langle P_{\text{axial}} \rangle}{cm\omega^2 l^2}}
\]  
(4.200)

4.3.2.3 Constant Power Flexural Wave Input

Recall the time-average wave power, i.e.,

\[
\langle P \rangle = \frac{1}{2} c_p m \omega^2 (w \bar{w})
\]  
(4.201)

The displacement for flexural is \(|w| = \varepsilon_f l\), the time-average flexural power is

\[
\langle P_{\text{flexural}} \rangle = \frac{1}{2} cm \omega^2 (\varepsilon_f l)^2
\]  
(4.202)

The force applied on the receiver edges is

\[
\hat{F} = (1 - \frac{k_{31}^2 Y_0}{Y} + (1 - k_{31}^2) Y_0) k_i \Delta \hat{u} = R(\omega) k_i \Delta \hat{u}
\]  
(4.203)

The velocity at the receiver edges is

\[
v_d = -i \omega \varepsilon_f e^{(\xi r - i \omega t)}
\]  
(4.204)

Hence, the time-average mechanical power is

\[
\langle P_d(\omega) \rangle = \frac{1}{2} \Re \left( \overline{F(\omega)} v_d(\omega) \right)
\]  
(4.205)

If the flexural wave power is constant, the flexural strain is
\[ \varepsilon_F = \sqrt{\frac{2P_{\text{flexural}}}{cm^2l^2}} \] (4.206)

4.3.3 Parametric Study of Receiver Behavior

The parametric study is based on the simplified model. Only one wave form, either axial or flexural wave, exists in the input. For constant strain, the strain amplitude is a constant. For constant power, the wave power is known. The receiver is connected to a fully resistive load.

4.3.3.1 Constant Strain Axial Wave Input

Under constant strain axial wave input, the receiver size and resistive load effects are considered separately. First, we assume the receiver size varies and the external load is fixed. Second, we assume the external load varies when the receiver size is 7 mm.

Figure 4.25 Receiver output voltage under constant 100-microstrain axial wave input (receiver size varies for sensing).
Figure 4.26 7-mm PWAS Receiver output voltage under constant 100-microstrain axial wave (measurement equipment impedance varies for power harvesting).

The receive size varies from 5-25mm and the output voltage is monitored to show the receive size effect. The electrical resistive load is set at high impedance to get the maximum voltage output. The output voltage vs. PWAS size and frequency under constant axial strain input is shown in Figure 4.25.

For power harvesting application, the PWAS receiver size is 7 mm in simulation. The measurement equipment impedance is fully resistive and varies from 1 to 1 MΩ. The electrical output power is shown in Figure 4.26. It shows that the peak output power is at 300 kHz when the resistive load is around 100 Ω.

4.3.3.2 **Constant Power Axial Wave Input**

Under constant power axial wave input, we use the same simulation setup to evaluate the receiver size and resistive load effects.
The receive size varies from 5-25mm and the output voltage is monitored to show the receive size effect under constant power axial wave. The electrical resistive load is set at high impedance to get the maximum voltage output. The output voltage vs. PWAS size and frequency under constant axial strain input is shown in Figure 4.25.

For power harvesting application, the PWAS receiver size is 7 mm in simulation. The measurement equipment impedance is fully resistive and varies from 1 to 1 MΩ. The electrical output power is shown in Figure 4.26. It shows that the peak output power is at 300 kHz when the resistive load is around 100 Ω.

Figure 4.27 Receiver output voltage under constant 1-W axial wave input (receiver size varies for sensing).
Figure 4.28 Receiver output voltage under constant 1-W axial wave input (measurement equipment impedance varies for power harvesting).

4.3.3.3 Constant Strain Flexural Wave Input

The output voltage vs. PWAS size and frequency under constant flexural strain input is shown in Figure 4.29. For power harvesting application, the electrical output power is shown in Figure 4.26.

4.3.3.4 Constant Power Flexural Wave Input

Under constant wave power input, the receiver power harvesting capability is considered. The output voltage vs. PWAS size and frequency under constant flexural strain input is shown in Figure 4.31. The receiver size is fixed at 7mm, the load varies from 1 $\Omega$ to 1 M$\Omega$ were considered. The output electrical power was monitored under constant power flexural wave (Figure 4.32).
Figure 4.29 Receiver output voltage under constant 100-microstrain flexural wave input. (receiver size varies for sensing).

Figure 4.30 7-mm receiver output voltage under constant 100-microstrain flexural wave input (measurement equipment impedance varies for power harvesting).
Figure 4.31 Receiver output electrical power under constant 100-mW power axial wave input (receiver size varies for sensing).

Figure 4.32 Receiver output electrical power under constant 100-mW power axial wave input (measurement equipment impedance varies for power harvesting).
4.4 Power and Energy Under Non-harmonic Voltage Excitation

Under non-harmonic excitation, the instantaneous power and energy can be derived from the FRF. Use the method in non-harmonic excitation for FRF, we can get the instantaneous response.

The admittance is defined as

\[ Y(\omega) = \frac{I(\omega)}{V(\omega)} \]  \hspace{1cm} (4.207)

Rearrange to get

\[ I(\omega) = Y(\omega)V(\omega) \]  \hspace{1cm} (4.208)

Apply inverse Fourier transform on both side, we get

\[ i(t) = y(t) * v(t) = \mathcal{F}^{-1} \left[ Y(\omega)V(\omega) \right] \]  \hspace{1cm} (4.209)

If the input voltage signal is an impulse function \( v(t) = \delta(t) \), then the output current signal \( i(t) \) is

\[ i(t) = \int_{-\infty}^{\infty} y(t-\lambda)v(\lambda)d\lambda = \int_{-\infty}^{\infty} y(t-\lambda)\delta(\lambda)d\lambda = y(t) \]  \hspace{1cm} (4.210)

Hence, the response of the current signal to an impulsive input voltage signal is the impulse voltage response function of \( y(t) \). The function \( y(t) \) is called the impulse voltage response function. The admittance is

\[ Y(\omega) = \mathcal{F}[y(t)] \]  \hspace{1cm} (4.211)
4.4.1.1 Transmitter Instantaneous Electrical Response

For transmitter, the electrical instantaneous power is

\[ p(t) = v(t)i(t) \]  \hspace{1cm} (4.212)

Substitution of Equation (4.209) into (4.212) yields

\[ p(t) = v(t)[y(t) \ast v(t)] = v(t)\mathcal{F}^{-1}\left[Y(\omega)V(\omega)\right] \]  \hspace{1cm} (4.213)

Taking Fourier transform of equation (4.213) gives the complex power.

\[ P(\omega) = \mathcal{F}\left\{v(t)[y(t) \ast v(t)]\right\} \]
\[ = \mathcal{F}[v(t)] \ast \mathcal{F}[y(t) \ast v(t)] = V(\omega) \ast Y(\omega)V(\omega) \]  \hspace{1cm} (4.214)

Since the voltage applied is assumed a real number, the active instantaneous power is

\[ p_{active}(t) = v(t)\text{Re}[i(t)] \]  \hspace{1cm} (4.215)

The reactive instantaneous power is

\[ p_{reactive}(t) = v(t)\text{Im}[i(t)] \]  \hspace{1cm} (4.216)

The instantaneous power rating is

\[ p_{rating}(t) = v(t)|i(t)| \]  \hspace{1cm} (4.217)

The average power is

\[ P_{avg} = \frac{1}{T} \int_{0}^{T} p(t) \, dt \]  \hspace{1cm} (4.218)

The transmitter electrical response under non-harmonic excitation is

\[ p^{elec}(t) = v(t)i(t) = \mathcal{F}^{-1}\left[\hat{V}(\omega)\right] \mathcal{F}^{-1}\left[I(\omega)\right] \]  \hspace{1cm} (4.219)
4.4.1.2 Transmitter Instantaneous Mechanical Response

Recall the FRF of pin-force at the PWAS A and applied voltage at the PWAS A

\[ FRF_{FV} (\omega) = \frac{\hat{F} (\omega)}{V (\omega)} \]  
(4.220)

Rearrangement

\[ \hat{F} (\omega) = FRF_{FV} (\omega) \hat{V} (\omega) \]  
(4.221)

Inverse Fourier transform on both side yields

\[ f (t) = \mathcal{F}^{-1} [ \hat{F} (\omega) ] = \mathcal{F}^{-1} [ FRF_{FV} (\omega) \hat{V} (\omega) ] \]  
(4.222)

Similar for instantaneous velocity

\[ v_A (t) = \mathcal{F}^{-1} [ \hat{v}_A (\omega) ] = \mathcal{F}^{-1} [ FRF_{vA} (\omega) \hat{V} (\omega) ] \]  
(4.223)

The instantaneous power at \( A = x_0 \) is

\[ p_A (t) = f (t) v_A (t) = \mathcal{F}^{-1} [ FRF_{FV} (\omega) \hat{V} (\omega) ] \mathcal{F}^{-1} [ FRF_{vA} (\omega) \hat{V} (\omega) ] \]  
(4.224)

The instantaneous power at \( A' = x_0 + l \) is

\[ p_A (t) = f (t) v_A (t) = \mathcal{F}^{-1} [ FRF_{FV} (\omega) \hat{V} (\omega) ] \mathcal{F}^{-1} [ FRF_{vA} (\omega) \hat{V} (\omega) ] \]  
(4.225)

4.4.1.3 Receiver Instantaneous Mechanical Response

Under strain excitation, the instantaneous power at \( A = x_0 \) is

\[ p_A (t) = f (t) v_A (t) = \mathcal{F}^{-1} [ F_z (\omega) ] v (x_A, t) \]  
(4.226)
The instantaneous power at $A' = x_0 + l$ is

$$p_{A'}(t) = f(t) v_{A'}(t) = \mathcal{F}^{-1}[F_2(\omega)] v(x_{A'}, t) \quad (4.227)$$

### 4.4.1.4 Receiver Instantaneous Electrical Response

Under strain excitation, the instantaneous electrical power is

$$\hat{V}(\omega) = \frac{1}{Y_e + (1 - k_3^2)Y_0} \left( e^{i\beta(x_0 + l)} - e^{i\beta x_0} \right) + e^{i\beta x_0} \left( e^{i\beta(x_0 + l)} - e^{i\beta x_0} \right) \quad (4.228)$$

The instantaneous output voltage is

$$v_{out}(t) = \mathcal{F}^{-1}[V(\omega)] \quad (4.229)$$

The electrical power is

$$p_{out}(t) = \mathcal{F}^{-1}[V(\omega)] \mathcal{F}^{-1} \left[ \frac{V(\omega)}{Z_{ext}} \right] \quad (4.230)$$

$Z_{ext}$ is the measurement equipment impedance.
5 PITCH-CATCH ANALYSIS USING WAVE PROPAGATION MODEL

This chapter extends previous chapter to model a complete pitch-catch system setup. A close-form solution is obtained for the ideal bonding between the PWAS and structure using the pin-force model. The power flow from electrical source into piezoelectric power at the transmitter, the mechanical interface power of the transmitter PWAS, the acoustic wave power were traced. After the wave power arrives at the receiver PWAS, the receiver is captured at the mechanical interface between the receiver PWAS at the structure, the mechanical power captured is converted back into electrical power in the receiver PWAS and captured at the receivers electric instrument. The reflection of PWAS is considered during the wave propagation.

5.1 PITCH-CATCH WAVE PROPAGATION ANALYSIS

The pitch-catch model is considered on an infinite bar with two PWAS under ideal bonding hypothesis, shown in Figure 5.1.

Figure 5.1 PWAS pitch-catch setup on an infinite bar.
5.1.1 Axial Wave

The axial wave equation is

$$\rho A \dddot{u}(x,t) - E A u''(x,t) = N'_e$$  \hspace{1cm} (5.1)$$

Upon rearrangement to get

$$\dddot{u}(x,t) - c^2 u''(x,t) = \frac{N'_e}{\rho A}$$  \hspace{1cm} (5.2)$$

Recall the pin force Equation (3.52), i.e.,

$$N_e = \left\{ \hat{F}_A \left[ -H(x-x_A) + H(x-x_A-l_A) \right] + \hat{F}_b \left[ -H(x-x_B) + H(x-x_B-l_B) \right] \right\} e^{-i\omega t} \hspace{1cm} (5.3)$$

$$N'_e = \left\{ \hat{F}_A \left[ -\delta(x-x_A) + \delta(x-x_A-l_A) \right] + \hat{F}_b \left[ -\delta(x-x_B) + \delta(x-x_B-l_B) \right] \right\} e^{-i\omega t} \hspace{1cm} (5.4)$$

Assume harmonic variation in the time domain of the form $e^{-i\omega t}$. Hence, the equation becomes

$$-\omega^2 u(x,t) - c^2 u''(x,t) = \frac{N'_e}{\rho A}$$  \hspace{1cm} (5.5)$$

define $\xi_0^2 = \frac{\omega^2}{c^2}$ is the wavenumber of axial waves in the 1-D medium Recall $c^2 \rho A = EA$, the wave equation can be written as

$$-u'' - \xi_0^2 u = \frac{N'_e(x,t)}{EA} \hspace{1cm} (5.6)$$

Define

$$h(x,t) = \frac{N'_e(x,t)}{EA} \hspace{1cm} (5.7)$$
Recall the space-domain Fourier transform

\[ \tilde{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-i\xi x} \, dx \]  
(5.8)

\[ f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\xi)e^{i\xi x} \, d\xi \]  
(5.9)

The space-domain Fourier transform has property \( \tilde{f}' = i\xi \tilde{f} \), Equation (4.6) can be written in Fourier domain as

\[ \xi^2 \tilde{u} - \xi_0^2 \tilde{u} = \tilde{h} \]  
(5.10)

Rearrange to get

\[ \tilde{u} = \frac{1}{\xi^2 - \xi_0^2} \tilde{h} \]  
(5.11)

Equation (4.11) represents the solution in the Fourier domain. Taking the inverse space-domain Fourier transform yields the solution in the space domain. The solution is

\[ u(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\xi^2 - \xi_0^2} \tilde{h}(\xi)e^{i\xi x} \, d\xi \]  
(5.12)

The space-domain Fourier transform of \( h \) is

\[ \tilde{h}(\xi) = \int_{-\infty}^{\infty} h(x)e^{-i\xi x} \, dx = \frac{N'(x)}{EA} e^{-i\xi x} \, dx \]  
\[ = \frac{1}{EA} \int_{-\infty}^{\infty} \left\{ \tilde{F}_a [-\delta(x-x_a) + \delta(x-x_a-l_a)] \right\} e^{-i\xi x} \, dx \]  
(5.13)

Recall the delta function property, i.e.,

\[ \int_{-\infty}^{\infty} f(x)\delta(x-a) \, dx = f(a) \]  
(5.14)
The space-domain Fourier transform of Equation (4.13) is in the form

\[
\tilde{h}(\xi) = \frac{1}{EA} \left[ \hat{F}_A(-e^{-i\xi x_A} + e^{-i\xi(x_A+l_A)}) + \hat{F}_B(-e^{-i\xi x_B} + e^{-i\xi(x_B+l_B)}) \right]
\]

Substitution (5.15) into (5.12) yields

\[
u(x,t) = \frac{1}{2\pi EA} e^{-i\omega t} \int_{-\xi_0}^{\xi_0} \frac{1}{\xi^2 - \xi_0^2} \left[ \hat{F}_A(-e^{-i\xi x_A} + e^{-i\xi(x_A+l_A)}) + \hat{F}_B(-e^{-i\xi x_B} + e^{-i\xi(x_B+l_B)}) \right] e^{i\xi x} d\xi
\]

The integral in Equation (5.16) can be resolved analytically using the residues theorem and a semicircular contour C in the complex \(\xi\) domain. We note that the integrant in Equation (5.16) has two poles, corresponding to the wavenumbers \(-\xi_0\) and \(+\xi_0\). We will resolve Equation (5.16) for the forward wave, which exists for \(x > 0\) and generates a solution containing \(i(\xi x - \omega t)\) in the exponential function. Hence, we will retain the positive pole, \(+\xi_0\), inside the integration contour but exclude the negative pole, \(-\xi_0\), from the integration contour. We note that the integration along the semicircular portion of the contour vanishes as the radius of integration becomes very large, i.e., \(R \to \infty\).

Therefore, the integration on the contour resolves into the integration along the real \(\xi\) axis from \(-\infty\) to \(\infty\).

According to the residue theorem, the integration around the contour equals the sum of the residues inside the contour times a multiplicative factor \(2\pi i\). We only have retained one pole inside the contour, the pole \(+\xi_0\), the contour integral takes the expression as

\[
\hat{\xi}|_{\xi=\xi_0} = 2\pi i \text{Res}
\]

The residue is
\[
\text{Res} \left( \frac{1}{\xi^2 - \xi_0^2} \left[ \hat{F}_A(-e^{-i\xi x_A} + e^{-i\xi (x_A + l_A)}) + \hat{F}_B(-e^{-i\xi x_B} + e^{-i\xi(x_B + l_B)}) \right] e^{i\xi x_A} \right) \\
= \left( \frac{1}{\xi + \xi_0} \left[ \hat{F}_A(-e^{-i\xi x_A} + e^{-i\xi (x_A + l_A)}) + \hat{F}_B(-e^{-i\xi x_B} + e^{-i\xi(x_B + l_B)}) \right] e^{i\xi x_A} \right)_{\xi = \xi_0} \\
= \frac{1}{2\xi_0} \left\{ \hat{F}_A(-e^{-i\xi_0 x_A} + e^{-i\xi_0 (x_A + l_A)}) + \hat{F}_B(-e^{-i\xi_0 x_B} + e^{-i\xi_0(x_B + l_B)}) \right\} e^{i\xi_0 x} \\
\text{(5.18)}
\]

Substitute Equation (5.17) and (5.18) into (5.16) gives

\[
u(x) = \frac{i}{2EA\xi_0} \left\{ \hat{F}_A(-e^{-i\xi_0 x_A} + e^{-i\xi_0 (x_A + l_A)}) + \hat{F}_B(-e^{-i\xi_0 x_B} + e^{-i\xi_0(x_B + l_B)}) \right\} e^{i(\xi_0 x-\omega t)} \\
\text{(5.19)}
\]

### 5.1.2 Flexural Wave

The flexural wave equation is

\[
\rho A \ddot{w}(x,t) + EI \dddot{w}(x,t) = -M''_e(x,t) \\
\text{(5.20)}
\]

where \( \ddot{w} = \frac{\partial w}{\partial t} \) and \( \dot{w} = \frac{\partial w}{\partial x} \)

Assume harmonic variation in the time domain of the form \( e^{-i\omega t} \). Hence, Equation (5.20) becomes

\[
EI \dddot{w} - \omega^2 \rho A \ddot{w} = -M''_e \\
\text{(5.21)}
\]

Recall the pin moment Equation (3.53), i.e.,

\[
M_e(x,t) = -\left\{ \frac{h}{2} \hat{F}_A \left[ -H(x-x_A) + H(x-x_A-l_A) \right] + \frac{h}{2} \hat{F}_B \left[ -H(x-x_B) + H(x-x_B-l_B) \right] \right\} e^{i\omega t} \\
\text{(5.22)}
\]

Divide by \( EI \) of Equation (5.21) and we get
\[
\ddot{w} - \omega^2 \frac{\rho A}{EI} w = \frac{1}{2EI} h \left[ \hat{F}_A \left( -\delta'(x-x_A) + \delta(x-x_A-l_A) \right) + \hat{F}_B \left( -\delta'(x-x_B) + \delta(x-x_B-l_B) \right) \right]
\]  

(5.23)

Introducing the notation \( \xi_F^4 = \omega^2 \frac{\rho A}{EI} = \omega^2 / \bar{a}^2 \), where \( \bar{a}^2 = \frac{EI}{\rho A} = \frac{h^2 E}{24 \rho} \)

Define

\[
g = -M^*_{\xi} = \frac{1}{2EI} h \left[ \hat{F}_A \left( -\delta'(x-x_A) + \delta(x-x_A-l_A) \right) + \hat{F}_B \left( -\delta'(x-x_B) + \delta(x-x_B-l_B) \right) \right]
\]

(5.24)

Hence, Equation (5.23) becomes

\[
\ddot{w} - \xi_F^4 w = g
\]

(5.25)

The space-domain Fourier transform has property \( \tilde{f}' = i\xi f \), the Equation (5.25) becomes

\[
\xi^4 \tilde{w} - \xi_F^4 \tilde{w} = \tilde{g}
\]

(5.26)

The solution is

\[
\tilde{w} = \frac{1}{\xi^4 - \xi_F^4} \tilde{g}
\]

(5.27)

Equation (5.27) represents the solution in the Fourier domain. Taking the inverse space-domain Fourier transform yields the solution in the space domain as

\[
w(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\xi^4 - \xi_F^4} \tilde{g}(\xi) e^{i\xi x} d\xi
\]

(5.28)

The integral in Equation (5.28) can be resolved using the residues theorem and a semicircular contour \( C \) in the complex \( \xi \) domain. We note that the integrant in (5.28) has four poles, corresponding to the wavenumbers \( +\xi_F, -\xi_F, +i\xi_F, -i\xi_F \). We will resolve
Equation (5.28) for the forward traveling wave, which exists for \( x > 0 \). Hence, we will retain the positive poles, \( +\xi_F \) and \( +i\xi_F \) inside the integration contour, but exclude the negative poles.

The integration contour includes the real pole \( +\xi_F \) and imaginary pole \( +i\xi_F \). We note that the integration along the semicircular portion of the contour vanishes as the radius of integration becomes very large. According to the residue theorem, the integration around the contour equals the sum of the residues inside the contour times a multiplicative factor \( 2\pi i \). We only have retained two poles inside the contour, the pole \( +\xi_F \) and imaginary pole \( +i\xi_F \) the contour integral takes the expression as

\[
\oint_C \mathbf{f} = 2\pi i (\text{Res}_{\xi = \xi_F} + \text{Res}_{\xi = i\xi_F})
\]

The residue at \( +\xi_F \) is calculated as

\[
\text{Res}_{\xi = \xi_F} \left( \frac{1}{\xi^4 - \xi_F^4} \hat{g}(\xi) e^{i\xi x} \right) = \frac{1}{(\xi + \xi_F)(\xi^2 + \xi_F^2)} \hat{g}(\xi_F) e^{\xi_F x} = \frac{1}{4\xi_F^3} \hat{g}(\xi_F) e^{i\xi_F x}
\]

The residue at \( +i\xi_F \) is calculated as

\[
\text{Res}_{\xi = i\xi_F} \left( \frac{1}{\xi^4 - \xi_F^4} \hat{g}(\xi) e^{i\xi x} \right) = \frac{1}{(\xi + i\xi_F)(\xi^2 + \xi_F^2)} \hat{g}(\xi_F) e^{i\xi_F x} = \frac{1}{-4i\xi_F^3} \hat{g}(i\xi_F) e^{-i\xi_F x}
\]

Substitution of Equations (5.29)-(5.31) into (5.28) yields
\[ w(x) = \frac{1}{2\pi} \int \frac{1}{4\xi_F} \tilde{g}(\xi) e^{i\xi x} - \frac{1}{4i\xi_F} \tilde{g}(i\xi) e^{-i\xi x})e^{-i\omega t} \] (5.32)

Note that the first term in Equation (5.32) represents a propagating wave, while the second term does not. In fact, the second term represents a vibration that is decaying fast with \( x \). This term represents a local vibration that does not propagate. It is called an evanescent wave. Thus, we will retain only the propagating wave part of Equation (5.32), i.e.,

\[ w(x) = i \frac{1}{4\xi_F} \tilde{g}e^{i(\xi_F x - \omega t)} \] (5.33)

The space domain Fourier transform of flexural excitation is

\[ \tilde{g} (\xi_F) = \int_{-\infty}^{\infty} g(x)e^{-i\xi_F x} dx \]

\[ = \int_{-\infty}^{\infty} \frac{1}{h} \left[ \hat{F}_a \left[ -\delta'(x-x_A) + \delta'(x-x_A-l_A) \right] + \hat{F}_b \left[ -\delta'(x-x_B) + \delta'(x-x_B-l_B) \right] \right] e^{-i\xi_F x} dx \] (5.34)

Recall the delta function property, i.e.,

\[ \int_{-\infty}^{\infty} f(x)\delta'(x-a)dx = -f'(a) \] (5.35)

The space-domain Fourier transform of Equation is the following from

\[ \tilde{g} (\xi_F) = i\xi_F \frac{h}{2EI} \left[ \hat{F}_a \left( e^{-i\xi_F x_A} + e^{-i\xi_F (x_A+l_A)} \right) + \hat{F}_b \left( e^{-i\xi_F x_B} + e^{-i\xi_F (x_B+l_B)} \right) \right] \] (5.36)

Substitution of Equation (5.36) into (5.33) yields the flexural-displacement solution as

\[ w(x) = -\frac{1}{4\xi_F} \frac{h}{2EI} \left[ \hat{F}_a \left( e^{-i\xi_F x_A} + e^{-i\xi_F (x_A+l_A)} \right) + \hat{F}_b \left( e^{-i\xi_F x_B} + e^{-i\xi_F (x_B+l_B)} \right) \right] e^{i(\xi_F x - \omega t)} \] (5.37)

Recall
\[ I = \frac{bh^3}{12} \quad \text{and} \quad A = bh \] (5.38)

Hence

\[ \frac{h}{2EI} = \frac{h}{2E \frac{bh^3}{12}} = \frac{6}{Eb} = \frac{6}{Eah} \] (5.39)

Substitute Equation (5.39) into (5.37) to get

\[ w(x) = \frac{-1}{\xi_F^2 EAh} \left[ \hat{F}_A(-e^{-i\xi_F x_a} + e^{-i\xi_F (x_s + l_a)}) + \hat{F}_B(-e^{-i\xi_F x_a} + e^{-i\xi_F (x_s + l_b)}) \right] e^{i(\xi_F x - \omega x)} \] (5.40)

### 5.1.3 In-plane Surface Displacement

Kinematic analysis gives the in-plane surface displacement of a generic point \( P \) on the infinite bar surface in terms of the axial and flexural displacement as

\[ u_p(t) = u(x) - \frac{h}{2} w'(x) \] (5.41)

Take derivative of Equation (5.40) yields

\[ w' = \frac{i}{\xi_F} \frac{3}{EAh} \left[ \hat{F}_A(-e^{-i\xi_F x_a} + e^{-i\xi_F (x_s + l_a)}) + \hat{F}_B(-e^{-i\xi_F x_a} + e^{-i\xi_F (x_s + l_b)}) \right] e^{i(\xi_F x - \omega x)} \] (5.42)

The x-displacement from the flexural wave at the material surface is

\[ u_p \bigg|_{y=\frac{h}{2}} = \frac{-h}{2} w'(x) = \frac{h}{2} \frac{i}{\xi_F} \frac{3}{EAh} \left[ \hat{F}_A(-e^{-i\xi_F x_a} + e^{-i\xi_F (x_s + l_a)}) + \hat{F}_B(-e^{-i\xi_F x_a} + e^{-i\xi_F (x_s + l_b)}) \right] e^{i(\xi_F x - \omega x)} \] (5.43)

The in-plane surface displacement of a generic point \( P \) on the infinite bar surface in terms of the axial and flexural displacement is
Comparison of in-plane surface displacement in pitch-catch setup Equation (5.44) and that in one PWAS setup Equation (4.51).

5.2 PITCH-CATCH FREQUENCY RESPONSE FUNCTION

The elongation of PWAS A is

\[
\Delta u_A = u(x_A + l_A) - u(x_A) = \frac{i}{2EA}\left\{ \hat{F}_A(e^{-i}\phi_A x_A + e^{-i}\phi_A(x_A + l_A))e^{i(\xi_0 x_A - \omega t)} + \hat{F}_B(e^{-i}\phi_B x_B + e^{-i}\phi_B(x_B + l_B))e^{i(\xi_0 x_B - \omega t)} \right\}
\]

\[
+ \frac{3i}{2EA} \left[ \hat{F}_A(e^{-i}\phi_A x_A + e^{-i}\phi_A(x_A + l_A))e^{i(\xi_0 x_A - \omega t)} + \hat{F}_B(e^{-i}\phi_B x_B + e^{-i}\phi_B(x_B + l_B))e^{i(\xi_0 x_B - \omega t)} \right] (5.44)
\]

\[
- \frac{i\hat{F}_A}{2EA} \left\{ \frac{(-e^{-i}\phi_A x_A + e^{-i}\phi_A(x_A + l_A))}{\xi_0} (e^{i\phi_A(x_A + l_A)} - e^{i\phi_A x_A}) + 3\left( -e^{-i}\phi_A x_A + e^{-i}\phi_A(x_A + l_A)) \right) \frac{e^{i\phi_A(x_A + l_A)} - e^{i\phi_A x_A}}{\xi_F} \right\}
\]

\[
+ \frac{i\hat{F}_B}{2EA} \left\{ \frac{(-e^{-i}\phi_B x_B + e^{-i}\phi_B(x_B + l_B))}{\xi_0} (e^{i\phi_B(x_B + l_B)} - e^{i\phi_B x_B}) + 3\left( -e^{-i}\phi_B x_B + e^{-i}\phi_B(x_B + l_B)) \right) \frac{e^{i\phi_B(x_B + l_B)} - e^{i\phi_B x_B}}{\xi_F} \right\} (5.45)
\]

The elongation of PWAS B is
\[ \Delta \tilde{u}_B = u(x_B + l_B) - u(x_B) \]
\[ = \frac{i \tilde{F}_A}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ -e^{-i\xi_0 x_B} + e^{i\xi_0 (x_B + l_B)} \right] \left( e^{i\xi_F x_B} - e^{i\xi_F x_B} \right) \right\} + 3 \left[ -e^{-i\xi_0 x_B} + e^{i\xi_0 (x_B + l_B)} \right] \left( e^{i\xi_F x_B} - e^{i\xi_F x_B} \right) \]
\[ + \frac{i \tilde{F}_B}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ -e^{-i\xi_0 x_B} + e^{i\xi_0 (x_B + l_B)} \right] \left( e^{i\xi_F x_B} - e^{i\xi_F x_B} \right) \right\} + 3 \left[ -e^{-i\xi_0 x_B} + e^{i\xi_0 (x_B + l_B)} \right] \left( e^{i\xi_F x_B} - e^{i\xi_F x_B} \right) \]
\[ = \frac{i \tilde{F}_A}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) \right\} + \frac{i \tilde{F}_B}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) \right\} \]
\[ + \frac{i \tilde{F}_A}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) \right\} + \frac{i \tilde{F}_B}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) \right\} \]

Define the following coefficients as

\[ C_{AA}(\omega) = \frac{i}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) + 3 \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) \right\} \]  
(5.47)

\[ C_{BA}(\omega) = \frac{i}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) \right\} \]  
(5.48)

\[ C_{AB}(\omega) = \frac{i}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) \right\} \]  
(5.49)

\[ C_{BB}(\omega) = \frac{i}{2EA} \left\{ \frac{\xi_0}{\xi_F} \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) + 3 \left[ e^{-i\xi_0 x_a} - 1 \right] \left( e^{i\xi_F x_a} - 1 \right) \right\} \]  
(5.50)
The expression of $\Delta u_A$ and $\Delta u_B$ is written as

$$\Delta u_A (\hat{F}_A, \hat{F}_B; \omega) = (\hat{F}_A C_{AA} (\omega) + \hat{F}_B C_{BA} (\omega)) e^{-i \alpha} \quad (5.51)$$

$$\Delta u_B (\hat{F}_A, \hat{F}_B; \omega) = (\hat{F}_A C_{AB} (\omega) + \hat{F}_B C_{BB} (\omega)) e^{-i \alpha} \quad (5.52)$$

Recall the pin force and PWAS elongation relation in pitch-catch of Equation (3.55) and (3.57), i.e.,

$$F_A = k_{ia} (\Delta u_A - u_{ISA}) \quad (5.53)$$

$$F_B = (1 - \frac{k_{iA}^2 Y_0}{Y_c + (1 - k_{iB}^2) Y_0}) k_{ib} \Delta \hat{u}_B = R(\omega) k_{ib} \Delta u_B \quad (5.54)$$

Substitute Equation (5.53) and (5.54) into (5.51) and (5.52) to get

$$\Delta \hat{u}_A (\omega) = k_{ia} (\Delta \hat{u}_A (\omega) - u_{ISA}) C_{AA} (\omega) + R(\omega) k_{ib} \Delta \hat{u}_B (\omega) C_{BA} (\omega) \quad (5.55)$$

$$\Delta \hat{u}_B (\omega) = k_{ia} (\Delta \hat{u}_A (\omega) - u_{ISA}) C_{AB} (\omega) + R(\omega) k_{ib} \Delta \hat{u}_B (\omega) C_{BB} (\omega) \quad (5.56)$$

Rearrangement yields

$$(k_{ia} C_{AA} (\omega) - 1) \Delta \hat{u}_A (\omega) + R(\omega) k_{ib} C_{BA} (\omega) \Delta \hat{u}_B (\omega) = k_{ia} C_{AA} (\omega) u_{ISA} \quad (5.57)$$

$$k_{ia} C_{AB} (\omega) \Delta \hat{u}_A (\omega) + R(\omega) k_{ib} C_{BB} (\omega) - 1) \Delta \hat{u}_B (\omega) = k_{ia} C_{AB} (\omega) u_{ISA} \quad (5.58)$$

We have two unknown variables $\Delta \hat{u}_A (\omega)$ and $\Delta \hat{u}_B (\omega)$, solve the linear Equation (5.57) and (5.58). To simplify the notation, define

$$a_i = k_{ia} C_{AA} (\omega) - 1 \quad (5.59)$$

$$b_i = R(\omega) k_{ib} C_{BA} (\omega) \quad (5.60)$$

$$c_i = k_{ia} C_{AA} (\omega) u_{ISA} \quad (5.61)$$
\[ a_2 = k_{ia} C_{AB}(\omega) \]  
(5.62)

\[ b_2 = R(\omega)k_{ib} C_{BB}(\omega) - 1 \]  
(5.63)

\[ c_2 = k_{ia} C_{AB}(\omega)u_{IS4} \]  
(5.64)

The Equation (5.57) and (5.58) becomes

\[ a_1 \Delta \hat{u}_A + b_1 \Delta \hat{u}_B = c_1 \]
\[ a_2 \Delta \hat{u}_A + b_2 \Delta \hat{u}_B = c_2 \]  
(5.65)

The expression of \( \Delta \hat{u}_A(\omega) \) is

\[
\Delta \hat{u}_A(\omega) = \frac{b_2 c_1 - b_1 c_2}{b_2 a_1 - b_1 a_2} 
\]
\[
= \left( \frac{R(\omega)k_{ib} C_{BB}(\omega) - 1)k_{ia} C_{AA}(\omega) - R(\omega)k_{ib} C_{BA}(\omega)k_{ia} C_{AB}(\omega)}{R(\omega)k_{ib} k_{ia} (C_{BB}(\omega)C_{AA}(\omega) - C_{BA}(\omega)C_{AB}(\omega)) - k_{ia} C_{AA}(\omega)} \right) u_{IS4} 
\]  
(5.66)

The expression of \( \Delta \hat{u}_B(\omega) \) is

\[
\Delta \hat{u}_B = \frac{a_2 c_1 - a_1 c_2}{a_2 b_1 - a_1 b_2} 
\]
\[
= \frac{k_{ia} C_{AB}(\omega)k_{ia} C_{AA}(\omega) - (k_{ia} C_{AA}(\omega) - 1)k_{ia} C_{AB}(\omega)}{R(\omega)k_{ib} k_{ia} (C_{BA}(\omega)C_{AB}(\omega)) - (k_{ia} C_{AA}(\omega) - 1)(R(\omega)k_{ib} C_{BB}(\omega) - 1)} u_{IS4} 
\]  
(5.67)

It can be proved that \( C_{AB}(\omega)C_{BA}(\omega) - C_{AA}(\omega)C_{BB}(\omega) = 0 \).
\[ C_{AA}(\omega)C_{BB}(\omega) = \frac{i}{2EA} \left( \frac{\left( e^{-i\xi_0^a} - 1 \right) \left( e^{i\xi_0^a} - 1 \right)}{\xi_0} + 3 \frac{\left( e^{-i\xi_F^a} - 1 \right) \left( e^{i\xi_F^a} - 1 \right)}{\xi_F} \right) \]

\[ = \left( \frac{i}{2EA} \right)^2 \left( \frac{\left( e^{-i\xi_0^a} - 1 \right) \left( e^{i\xi_0^a} - 1 \right) \left( e^{-i\xi_0^b} - 1 \right) \left( e^{i\xi_0^b} - 1 \right)}{\xi_0} + \frac{\left( e^{-i\xi_F^a} - 1 \right) \left( e^{i\xi_F^a} - 1 \right) \left( e^{-i\xi_F^b} - 1 \right) \left( e^{i\xi_F^b} - 1 \right)}{\xi_F} \right) \]

\[ = \left( \frac{i}{2EA} \right)^2 \left( \frac{\left( e^{-i\xi_0^a} - 1 \right) \left( e^{i\xi_0^a} - 1 \right) \left( e^{-i\xi_0^b} - 1 \right) \left( e^{i\xi_0^b} - 1 \right)}{\xi_0} + \frac{\left( e^{-i\xi_F^a} - 1 \right) \left( e^{i\xi_F^a} - 1 \right) \left( e^{-i\xi_F^b} - 1 \right) \left( e^{i\xi_F^b} - 1 \right)}{\xi_F} \right) \]

(5.68)

\[ C_{AB}(\omega)C_{BA}(\omega) = \frac{i}{2EA} \left( \frac{\left( e^{-i\xi_0^a (x_a - x_b)} \right) \left( e^{i\xi_0^b (x_a - x_b)} - 1 \right) \left( e^{i\xi_0^b (x_a - x_b)} - 1 \right)}{\xi_0} + \frac{\left( e^{-i\xi_F^a (x_a - x_b)} \right) \left( e^{i\xi_F^b (x_a - x_b)} - 1 \right) \left( e^{i\xi_F^b (x_a - x_b)} - 1 \right)}{\xi_F} \right) \]

\[ = \left( \frac{i}{2EA} \right)^2 \left( \frac{\left( e^{-i\xi_0^a (x_a - x_b)} \right) \left( e^{i\xi_0^b (x_a - x_b)} - 1 \right) \left( e^{i\xi_0^b (x_a - x_b)} - 1 \right)}{\xi_0} + \frac{\left( e^{-i\xi_F^a (x_a - x_b)} \right) \left( e^{i\xi_F^b (x_a - x_b)} - 1 \right) \left( e^{i\xi_F^b (x_a - x_b)} - 1 \right)}{\xi_F} \right) \]

(5.69)
Hence,

\[ C_{AB}(\omega)C_{BA}(\omega) = C_{AA}(\omega)C_{BB}(\omega) \]  \hspace{1cm} (5.70)

Substitution of \( C_{AB}(\omega)C_{BA}(\omega) - C_{AA}(\omega)C_{BB}(\omega) = 0 \) into (5.66)(5.67) respectively yields

\[ \Delta \hat{u}_A(\omega) = \frac{k_{ia}C_{AA}(\omega)}{k_{ia}C_{AA}(\omega) + R(\omega)k_{ib}C_{BB}(\omega) - 1} u_{ISA} \]  \hspace{1cm} (5.71)

\[ \Delta \hat{u}_B(\omega) = \frac{k_{ia}C_{AB}(\omega)}{k_{ia}C_{AA}(\omega) + R(\omega)k_{ib}C_{BB}(\omega) - 1} u_{ISA} \]  \hspace{1cm} (5.72)

Recall Equation (3.40), i.e.,

\[ \hat{V}_B(\omega) = i\omega \frac{1}{Y_e + (1 - k_{31}^2)Y_{0b}} \frac{b_d d_{31}}{s_{11}} \Delta \hat{u}_B(\omega) \]  \hspace{1cm} (5.73)

Substitute Equation (5.72) into (5.73) to get

\[ \hat{V}_B(\omega) = i\omega \frac{1}{Y_e + (1 - k_{31}^2)Y_{0b}} \frac{b_d d_{31}}{s_{11}} \frac{k_{ia}C_{AB}(\omega)}{k_{ia}C_{AA}(\omega) + R(\omega)k_{ib}C_{BB}(\omega) - 1} u_{ISA} \]  \hspace{1cm} (5.74)

Recall Equation (3.44) to simplify, Equation (5.74) becomes

\[ \hat{V}_B(\omega) = \frac{k_{31}^2 Y_{0b}}{Y_e + (1 - k_{31}^2)Y_{0b}} \frac{k_{ia}C_{AB}(\omega)}{k_{ia}C_{AA}(\omega) + R(\omega)k_{ib}C_{BB}(\omega) - 1} \hat{V}_A(\omega) \]  \hspace{1cm} (5.75)

Substitution of Equation \( r_y = \frac{Y_e}{Y_{0b}} \) and the FRF is

\[ FRF(\omega) = \frac{V_B(\omega)}{V_A(\omega)} = \frac{k_{31}^2}{r_y + (1 - k_{31}^2) \frac{k_{ia}C_{AB}(\omega)}{k_{ia}C_{AA}(\omega) + R(\omega)k_{ib}C_{BB}(\omega) - 1}} \]  \hspace{1cm} (5.76)
5.3 Pitch-catch Power and Energy Analysis

The power and energy transduction flow chart for a complete pitch-catch setup is shown in Figure 5.2. Under 1-D assumption, the electro-acoustic power and energy transduction of the PWAS transmitter and receiver are examined.

In pitch-catch mode, the power flow converts from electrical source into piezoelectric power at the transmitter, the piezoelectric transduction converts the electrical power into the mechanical interface power at the transmitter PWAS and then into acoustic wave power travelling in the structure. The wave power arrives at the receiver PWAS and is captured at the mechanical interface between the receiver PWAS at the structure, the mechanical power captured is converted back into electrical power in the receiver PWAS and captured at the receivers electric instrument. The time-averaged electrical power, mechanical power at the transmitter and wave power can be calculated from the frequency response function. The time-averaged mechanical power and electrical power
at the receiver PWAS can be calculated as well. The power and energy analysis of pitch-catch setup is considered under harmonic excitation and non-harmonic excitation.

5.3.1 Power and Energy under Harmonic Excitation

The power and energy conversation for PWAS pitch-catch model includes the electrical energy and mechanical energy conversation of the transmitter PWAS and the mechanical energy and electrical energy conversation of the receiver. The applied voltage on transmitter provides the electrical energy for excitation; the electrical energy generates the mechanical energy into the structure. The mechanical energy travels along with the axial and flexural waves and receiver pick up the mechanical energy and transfer back to electrical energy. The whole pitch-catch power flow can be considered as five stages. The first stage is the input electrical power. The second stage is the power at the PWAS A structure interface. The third stage is the wave propagation power. The forth stage is the power received at PWAS B. The fifth stage is the electrical power output. The power and energy in each stage were calculated here.

5.3.1.1 Input Electrical Energy and Power

The Electrical power is the product of voltage and current. The voltage and current can be derived from the admittance and input voltage.

5.3.1.1.1 Input Admittance

Recall the single constrained PWAS transmitter under harmonic electric excitations, the current is

\[
\hat{I} = i\omega C_0 \hat{V}_A \left[ 1 - k_{31}^2 + k_{31}^2 \frac{\Delta \hat{u}_A(\omega)}{u_{ISA}} \right]
\]  

(5.77)
where $C_0 = \varepsilon_{33} b_d l_d / t_A$ is the conventional stress free capacitance of the PWAS,

$$u_{\text{IS4}} = d_3 E_A l_d \text{ and } E_A = \frac{\hat{V}_A}{t_A}.$$  

Recall the transmitter elongation Equation (5.71), i.e.,

$$\Delta \hat{u}_A (\omega) = \frac{k_{iA} C_{AA} (\omega)}{k_{iA} C_{AA} (\omega) + R(\omega) k_{ib} C_{BB} (\omega) - 1} u_{\text{IS4}}$$  

(5.78)

Substitution of Equation (5.78) into (4.74), we get

$$\hat{I} (\omega) = i \omega C_0 \hat{V}_A \left[ 1 - k_{31}^2 \left( 1 - \frac{k_{iA} C_{AA} (\omega)}{k_{iA} C_{AA} (\omega) + R(\omega) k_{ib} C_{BB} (\omega) - 1} \right) \right]$$  

(5.79)

The admittance is defined as the ratio between the current and voltage, i.e.,

$$Y(\omega) = \frac{\hat{I}}{\hat{V}_A} = i \omega C_0 \left[ 1 - k_{31}^2 \left( 1 - \frac{k_{iA} C_{AA} (\omega)}{k_{iA} C_{AA} (\omega) + R(\omega) k_{ib} C_{BB} (\omega) - 1} \right) \right]$$  

(5.80)

In comparison with one PWAS on the beam, the existence of PWAS B changes the input admittance of PWAS.

5.3.1.1.2 Electrical Power under Harmonic Excitation

The time average product of the two variables defined by Equation (4.80) is one half the product of one variable times the conjugate of the other. Since the harmonic exponential is common in both $V$ and $I$. The time-averaged electrical response is

$$\langle P_{\text{elec}} (\omega) \rangle = \frac{1}{2} \text{Re} \left( \overline{\hat{V}}_A (\omega) I_A (\omega) \right)$$  

(5.81)

Hence, the complex power for transmitter under constant input voltage is
\[
P = \frac{1}{2} i \omega C_0 \hat{v}_A^2 \left[ 1 - k_{31}^2 \left( \frac{k_{iA} C_{AA}(\omega)}{k_{iA} C_{AA}(\omega) + R(\omega)k_{ib} C_{BB}(\omega) - 1} \right) \right] \quad (5.82)
\]

### 5.3.1.2 Power and Energy at Transmitter A Structure Interface

The mechanical power is the product of force and velocity. The force FRF, velocity FRF under harmonic excitation at transmitter is derived first. The time-average mechanical power is derived based on force and velocity FRF.

#### 5.3.1.2.1 Force FRF

At transmitter position \(x_A\), recall harmonic excitation, i.e.,

\[
F_A(t) = \hat{F}_A(\omega) e^{-i\omega t} \quad (5.83)
\]

\[
\hat{F}_A(\omega) = k_{iA} (\Delta u_A(\omega) - u_{ISA}) \quad (5.84)
\]

And recall the transmitter elongation equation, i.e.,

\[
\Delta u_A(\omega) = \frac{k_{iA} C_{AA}(\omega)}{k_{iA} C_{AA}(\omega) + R(\omega)k_{ib} C_{BB}(\omega) - 1} u_{ISA} \quad (5.85)
\]

Substitution of Equation (5.85) into (5.84) yields

\[
\hat{F}_A(\omega) = k_{iA} \left( \frac{k_{iA} C_{AA}(\omega)}{k_{iA} C_{AA}(\omega) + R(\omega)k_{ib} C_{BB}(\omega) - 1} - 1 \right) u_{ISA} \quad (5.86)
\]

Define FRF of pin-force at the PWAS A and applied voltage at the PWAS A as

\[
FRF_{FVA}(\omega) = \frac{\hat{F}_A}{V_A} = \frac{1 - R(\omega)k_{ib} C_{BB}(\omega)}{k_{iA} C_{AA}(\omega) + R(\omega)k_{ib} C_{BB}(\omega) - 1} k_{iA} \frac{l_A}{l_A} \quad (5.87)
\]
At receiver position $x_B$, the pin force is

$$\hat{F}_B(\omega) = (1 - \frac{k_{31} Y_0}{Y_e + (1 - k_{31}^2) Y_0}) k_{ib} \Delta u_B(\omega) = k_{ib} R(\omega) \Delta u_B(\omega) \quad (5.88)$$

And receiver elongation is

$$\Delta u_B(\omega) = \frac{k_{id} C_{AB}(\omega)}{k_{id} C_{AA}(\omega) + R(\omega) k_{ib} C_{BB}(\omega) - 1} u_{is\delta} \quad (5.89)$$

Substitution of Equation (5.89) into (5.88) yields

$$\hat{F}_B(\omega) = \frac{k_{id} C_{AB}(\omega)}{k_{id} C_{AA}(\omega) + R(\omega) k_{ib} C_{BB}(\omega) - 1} k_{ib} R(\omega) u_{is\delta} \quad (5.90)$$

Define FRF of pin-force at the PWAS B and applied voltage at the PWAS A as

$$FRF_{FVB}(\omega) = \frac{\hat{F}_B(\omega)}{V_A} = \frac{k_{id} C_{AB}(\omega)}{k_{id} C_{AA}(\omega) + R(\omega) k_{ib} C_{BB}(\omega) - 1} k_{ib} R(\omega) d_{31} \frac{l_A}{t_A} \quad (5.91)$$

### 5.3.1.2.2 Velocity FRF at Transmitter A edges

Recall the displacement under harmonic loading from Equation (4.51), i.e.,

$$u_p(t) = \frac{i \hat{F}_A}{2EA} \left\{ \left( -e^{-i\xi_y x_A} + e^{-i\xi_y (x_A + l_A)} \right) e^{i(\xi_F x - \omega t)} + 3 \left( -e^{-i\xi_y x_A} + e^{-i\xi_y (x_A + l_A)} \right) e^{i(\xi_F x - \omega t)} \right\}$$

$$+ \frac{i \hat{F}_B}{2EA} \left\{ \left( -e^{-i\xi_y x_B} + e^{-i\xi_y (x_B + l_B)} \right) e^{i(\xi_F x - \omega t)} + 3 \left( -e^{-i\xi_y x_B} + e^{-i\xi_y (x_B + l_B)} \right) e^{i(\xi_F x - \omega t)} \right\} \quad (5.92)$$

At $A = x_A$, the displacement is
\[
\begin{align*}
    u_A(x_A,t) &= \frac{i\hat{F}_A}{2EA} \left\{ \frac{(-e^{-i\xi_0 x_A} + e^{-i\xi_0 (x_A+t_b)})}{\xi_0} e^{i(\xi_0 x_A - \omega t)} + \frac{3(-e^{-i\xi_0 x_A} + e^{-i\xi_0 (x_A+t_b)})}{\xi_F} e^{i(\xi_0 x_A - \omega t)} \right\} \\
    &+ \frac{i\hat{F}_B}{2EA} \left\{ \frac{(-e^{-i\xi_0 x_B} + e^{-i\xi_0 (x_B+t_b)})}{\xi_0} e^{i(\xi_0 x_B - \omega t)} + \frac{3(-e^{-i\xi_0 x_B} + e^{-i\xi_0 (x_B+t_b)})}{\xi_F} e^{i(\xi_0 x_B - \omega t)} \right\} \\
    &= e^{-i\omega t} \left\{ \frac{i\hat{F}_A}{2EA} \left[ -1 + e^{-i\xi_0 l_A} \right] e^{i(\xi_0 x_A - \omega t)} + \frac{3+1+e^{-i\xi_0 l_A}}{\xi_F} e^{i(\xi_0 x_A - \omega t)} \right\} \\
    &+ \frac{i\hat{F}_B}{2EA} \left[ e^{-i\xi_0 (x_B-x_A)} \frac{-1+e^{-i\xi_0 l_B}}{\xi_0} + \frac{3e^{-i\xi_0 (x_B-x_A)} - 1+e^{-i\xi_0 l_B}}{\xi_F} \right] \\
    &= e^{-i\omega t} \left\{ \frac{i\hat{F}_A}{2EA} \left[ -1 + e^{-i\xi_0 l_A} \right] e^{i(\xi_0 x_A - \omega t)} + \frac{3+1+e^{-i\xi_0 l_A}}{\xi_F} e^{i(\xi_0 x_A - \omega t)} \right\} \\
    &+ \frac{i\hat{F}_B}{2EA} \left[ e^{-i\xi_0 (x_B-x_A)} \frac{-1+e^{-i\xi_0 l_B}}{\xi_0} + \frac{3e^{-i\xi_0 (x_B-x_A)} - 1+e^{-i\xi_0 l_B}}{\xi_F} \right] \\
    &= e^{-i\omega t} \left\{ \frac{i\hat{F}_A}{2EA} \left[ -1 + e^{-i\xi_0 l_A} \right] e^{i(\xi_0 x_A - \omega t)} + \frac{3+1+e^{-i\xi_0 l_A}}{\xi_F} e^{i(\xi_0 x_A - \omega t)} \right\} \\
    &+ \frac{i\hat{F}_B}{2EA} \left[ e^{-i\xi_0 (x_B-x_A)} \frac{-1+e^{-i\xi_0 l_B}}{\xi_0} + \frac{3e^{-i\xi_0 (x_B-x_A)} - 1+e^{-i\xi_0 l_B}}{\xi_F} \right] \\
    &= e^{-i\omega t} \left\{ \frac{i\hat{F}_A}{2EA} \left[ -1 + e^{-i\xi_0 l_A} \right] e^{i(\xi_0 x_A - \omega t)} + \frac{3+1+e^{-i\xi_0 l_A}}{\xi_F} e^{i(\xi_0 x_A - \omega t)} \right\} \\
    &+ \frac{i\hat{F}_B}{2EA} \left[ e^{-i\xi_0 (x_B-x_A)} \frac{-1+e^{-i\xi_0 l_B}}{\xi_0} + \frac{3e^{-i\xi_0 (x_B-x_A)} - 1+e^{-i\xi_0 l_B}}{\xi_F} \right] \\
    &= e^{-i\omega t} \left\{ \frac{i\hat{F}_A}{2EA} \left[ -1 + e^{-i\xi_0 l_A} \right] e^{i(\xi_0 x_A - \omega t)} + \frac{3+1+e^{-i\xi_0 l_A}}{\xi_F} e^{i(\xi_0 x_A - \omega t)} \right\} \\
    &+ \frac{i\hat{F}_B}{2EA} \left[ e^{-i\xi_0 (x_B-x_A)} \frac{-1+e^{-i\xi_0 l_B}}{\xi_0} + \frac{3e^{-i\xi_0 (x_B-x_A)} - 1+e^{-i\xi_0 l_B}}{\xi_F} \right] \\
    &= e^{-i\omega t} \left\{ \frac{i\hat{F}_A}{2EA} \left[ -1 + e^{-i\xi_0 l_A} \right] e^{i(\xi_0 x_A - \omega t)} + \frac{3+1+e^{-i\xi_0 l_A}}{\xi_F} e^{i(\xi_0 x_A - \omega t)} \right\} \\
    &+ \frac{i\hat{F}_B}{2EA} \left[ e^{-i\xi_0 (x_B-x_A)} \frac{-1+e^{-i\xi_0 l_B}}{\xi_0} + \frac{3e^{-i\xi_0 (x_B-x_A)} - 1+e^{-i\xi_0 l_B}}{\xi_F} \right] \\
    &= e^{-i\omega t} \left\{ \frac{i\hat{F}_A}{2EA} \left[ -1 + e^{-i\xi_0 l_A} \right] e^{i(\xi_0 x_A - \omega t)} + \frac{3+1+e^{-i\xi_0 l_A}}{\xi_F} e^{i(\xi_0 x_A - \omega t)} \right\} \\
    &+ \frac{i\hat{F}_B}{2EA} \left[ e^{-i\xi_0 (x_B-x_A)} \frac{-1+e^{-i\xi_0 l_B}}{\xi_0} + \frac{3e^{-i\xi_0 (x_B-x_A)} - 1+e^{-i\xi_0 l_B}}{\xi_F} \right] \\ 
\end{align*}
\]
\[ F_{RV} (\omega) = \frac{\hat{u}_A}{V_A} = -i \frac{1}{2EA} \left[ \begin{array}{c} 1 - e^{-i\xi_d^A} \\ \xi_0 \\ 3 - e^{-i\xi_f^A} \\ \xi_F \end{array} \right] F_{RF} (\omega) \]

\[ \text{(5.96)} \]

\[ F_{RV} (\omega) = \frac{\hat{v}_A}{V_A} = -i \frac{\omega}{2EA} \left[ \begin{array}{c} 1 - e^{-i\xi_d^A} \\ \xi_0 \\ 3 - e^{-i\xi_f^A} \\ \xi_F \end{array} \right] F_{RF} (\omega) \]

\[ \text{(5.97)} \]

At \( A' = x_A + l_A \), we get

\[ u_A (x_A + l_A, t) = \frac{i\hat{F}_d}{2EA} \left\{ \begin{array}{c} (e^{-i\xi_d^A} + e^{-i\xi_f^A}) \\ \xi_0 \\ 3 + (e^{-i\xi_f^A} + e^{-i\xi_d^A}) \\ \xi_F \end{array} \right\} e^{i(\xi_d^A)(x_A + l_A) - \alpha t} \]

\[ + \frac{i\hat{F}_b}{2EA} \left\{ \begin{array}{c} (e^{-i\xi_d^B} + e^{-i\xi_f^B}) \\ \xi_0 \\ 3 + (e^{-i\xi_f^B} + e^{-i\xi_d^B}) \\ \xi_F \end{array} \right\} e^{i(\xi_f^B)(x_A + l_A) - \alpha t} \]

\[ = e^{-i\alpha t} \frac{\hat{F}_d}{2EA} \left[ \begin{array}{c} 1 - e^{-i\xi_d^A} \\ \xi_0 \\ 3 - e^{-i\xi_f^A} \\ \xi_F \end{array} \right] \]

\[ + \frac{i\hat{F}_b}{2EA} \left[ e^{-i\xi_b^A} (x_A + l_A) - (x_A + l_A) + 3 \left( e^{-i\xi_f^B} (x_A + l_A) - (x_A + l_A) \right) \right] \]

\[ \text{(5.98)} \]

The velocity at \( x_A = x_A + l_A \) is
\[ v_{A'} = \dot{u}_{A'}(x_{A'},t) = e^{-ia} \frac{\omega}{2EA} \left\{ \frac{1-e^{i\xi_{0}l_{A}}}{\xi_{0}} + 3 \frac{1-e^{i\xi_{F}l_{A}}}{\xi_{F}} \right\} \hat{F}_{A}(\omega) \]
\[ + \left\{ e^{-i\xi_{0}((x_{A}+l_{A})-(x_{A}+l_{A}))} \frac{1-e^{i\xi_{0}l_{b}}}{\xi_{0}} + 3e^{-i\xi_{F}((x_{A}+l_{A})-(x_{A}+l_{A}))} \frac{1-e^{i\xi_{F}l_{b}}}{\xi_{F}} \right\} \hat{F}_{B}(\omega) \] 
\[ (5.99) \]

Define FRF of the velocity at \( A' = x_{A} + l_{A} \) of the PWAS A and applied voltage at the PWAS A as

\[ FRF_{u_{A'}}(\omega) = \frac{\hat{u}_{A'}}{V_{A}} \quad FRF_{v_{A'}}(\omega) = \frac{\hat{v}_{A'}}{V_{A}} \] 
\[ (5.100) \]

Hence

\[ FRF_{u_{A'}}(\omega) = \frac{\hat{u}_{A'}}{V_{A}} = \frac{i}{2EA} \left\{ \frac{1-e^{i\xi_{0}l_{A}}}{\xi_{0}} + 3 \frac{1-e^{i\xi_{F}l_{A}}}{\xi_{F}} \right\} FRF_{F_{Y_{A}}}(\omega) \]
\[ + \left\{ e^{-i\xi_{0}((x_{A}+l_{A})-(x_{A}+l_{A}))} \frac{1-e^{i\xi_{0}l_{b}}}{\xi_{0}} + 3e^{-i\xi_{F}((x_{A}+l_{A})-(x_{A}+l_{A}))} \frac{1-e^{i\xi_{F}l_{b}}}{\xi_{F}} \right\} FRF_{F_{Y_{B}}}(\omega) \] 
\[ (5.101) \]

\[ FRF_{v_{A'}}(\omega) = \frac{\hat{v}_{A'}}{V_{A}} = \frac{\omega}{2EA} \left\{ \frac{1-e^{i\xi_{0}l_{A}}}{\xi_{0}} + 3 \frac{1-e^{i\xi_{F}l_{A}}}{\xi_{F}} \right\} FRF_{F_{Y_{A}}}(\omega) \]
\[ + \left\{ e^{-i\xi_{0}((x_{A}+l_{A})-(x_{A}+l_{A}))} \frac{1-e^{i\xi_{0}l_{b}}}{\xi_{0}} + 3e^{-i\xi_{F}((x_{A}+l_{A})-(x_{A}+l_{A}))} \frac{1-e^{i\xi_{F}l_{b}}}{\xi_{F}} \right\} FRF_{F_{Y_{B}}}(\omega) \] 
\[ (5.102) \]

5.3.1.2.3 Mechanical power generated by Transmitter A

The time-averaged product of two harmonic variables is one half the product of one variable times the conjugate of the other.
The time averaged mechanical power under harmonic excitation for PWAS transmitter A is

\[ \langle P_A \rangle = \frac{1}{T} \int_0^T p(t) \, dt = \frac{1}{T} \int_0^T -F(t) v(t) \, dt = -\frac{1}{2} \tilde{F}_A(\omega) \tilde{v}_A(\omega) \]  

(5.103)

\[ \langle P_A \rangle = \frac{1}{T} \int_0^T p(t) \, dt = \frac{1}{T} \int_0^T F(t) v_A(t) \, dt = \frac{1}{2} \tilde{F}_A(\omega) \tilde{v}_A(\omega) \]  

(5.104)

### 5.3.1.3 Power and Energy of Propagation Waves

The propagation waves contains axial and flexural waves. The power and energy of axial and flexural wave are derived next.

#### 5.3.1.3.1 Axial Waves

Recall the total energy density per unit length of a propagating axial wave in a bar as

\[ e(x,t) = k(x,t) + v(x,t) = mc^2 \left[ u'(x-ct) \right]^2 \]  

(5.105)

The wave power the product of energy and wave speed, i.e.,

\[ P(x,t) = ce(x,t) \]  

(5.106)

For pitch-catch setup, the displacement of axial waves is

\[ u(x) = \frac{i}{2E\alpha_0} \left\{ \hat{F}_A(-e^{-i\xi_0 x} + e^{-i\xi_0 (x+l_A)}) + \hat{F}_B(-e^{-i\xi_0 x} + e^{-i\xi_0 (x+l_B)} \right\} e^{i\xi_0 x-\omega t} \]  

(5.107)

For pitch-catch setup in a bar, the time-average axial energy is

\[ \langle e \rangle = \frac{1}{2} m \omega^2 \text{Re}(u \bar{u}) \]  

(5.108)

For pitch-catch setup in a bar, the time-average axial power is

114
\[ \langle P_{\text{axial}} \rangle = c \langle e \rangle = \frac{1}{2} cm \omega^2 \Re \{ u \bar{u} \} \quad (5.109) \]

### 5.3.1.3.2 Flexural Waves

Recall the time-averaged total energy is

\[ \langle e \rangle = \langle k \rangle + \langle v \rangle = \frac{1}{2} m \omega^2 \Re \{ w \bar{w} \} \quad (5.110) \]

Here \( w \) is

\[ w(x) = -\frac{1}{\xi^2} \frac{3}{E A h} \left[ \hat{F}_d \left( e^{-i \xi \xi} + e^{-i \xi (\xi + \xi)} \right) + \hat{F}_b \left( e^{-i \xi \xi} + e^{-i \xi (\xi + \xi)} \right) \right] e^{i (\xi x + \xi k)} \quad (5.111) \]

Recall the flexural wave power and energy relation, i.e.,

\[ \langle P \rangle = c_F \langle e \rangle \quad (5.112) \]

The time-averaged flexural wave power generated by PWAS is

\[ \langle P_{\text{flexural}} \rangle = c_F \langle e \rangle = \frac{1}{2} c_F m \omega^2 \Re \{ w \bar{w} \} \quad (5.113) \]

### 5.3.1.4 Power and Energy at Receiver B Structure Interface

The force FRF, velocity FRF under harmonic excitation for receiver are derived next.

The time-average mechanical power is derived based on force and velocity FRF.

#### 5.3.1.4.1 Velocity of Receiver B Edges

Recall the displacement under harmonic loading from Equation (4.51), i.e.,
$$u_p(t) = \frac{i\hat{F}_a}{2EA} \left\{ \frac{\left( -e^{-i\xi_0 x_p} + e^{-i\xi_0 (x_p + l_A)} \right)}{\xi_0} e^{i(\xi_p x_p - \omega t)} + 3 \frac{\left( -e^{-i\xi_0 x_p} + e^{-i\xi_0 (x_p + l_A)} \right)}{\xi_F} e^{i(\xi_F x_p - \omega t)} \right\}$$

$$+ \frac{i\hat{F}_b}{2EA} \left\{ \frac{\left( -e^{-i\xi_0 x_p} + e^{-i\xi_0 (x_p + l_B)} \right)}{\xi_0} e^{i(\xi_p x_p - \omega t)} + 3 \frac{\left( -e^{-i\xi_0 x_p} + e^{-i\xi_0 (x_p + l_B)} \right)}{\xi_F} e^{i(\xi_F x_p - \omega t)} \right\}$$

(5.114)

At $x_B$, the displacement is

$$u_B(x_B,t) = \frac{i\hat{F}_a}{2EA} \left\{ \frac{\left( -e^{-i\xi_0 x_B} + e^{-i\xi_0 (x_B + l_A)} \right)}{\xi_0} e^{i(\xi_p x_B - \omega t)} + 3 \frac{\left( -e^{-i\xi_0 x_B} + e^{-i\xi_0 (x_B + l_A)} \right)}{\xi_F} e^{i(\xi_F x_B - \omega t)} \right\}$$

$$+ \frac{i\hat{F}_b}{2EA} \left\{ \frac{\left( -e^{-i\xi_0 x_B} + e^{-i\xi_0 (x_B + l_B)} \right)}{\xi_0} e^{i(\xi_p x_B - \omega t)} + 3 \frac{\left( -e^{-i\xi_0 x_B} + e^{-i\xi_0 (x_B + l_B)} \right)}{\xi_F} e^{i(\xi_F x_B - \omega t)} \right\}$$

(5.115)

The velocity is

$$v_B = \dot{u}_B(x_B,t) = -\frac{\omega}{2EA} e^{-i\omega t} \left\{ \frac{e^{i\xi_0 (x_B - x_A)} 1 - e^{-i\xi_0 l_A}}{\xi_0} \frac{\xi}{\xi} \hat{F}_A(\omega) + 3e^{i\xi_0 (x_B - x_A)} \frac{1 - e^{-i\xi_0 l_A}}{\xi_F} \hat{F}_A(\omega) \right\}$$

$$+ \left[ \frac{1 - e^{-i\xi_0 l_A}}{\xi_0} + 3 \frac{1 - e^{-i\xi_0 l_A}}{\xi_F} \right] \hat{F}_B(\omega)$$

(5.116)

Define FRF of the velocity at $x_B$ of the PWAS B and applied voltage at the PWAS A as
\[
FRF_{u_B} (\omega) = \frac{\hat{u}_B}{V_A} \quad FRF_{v_B} (\omega) = \frac{\hat{v}_B}{V_A}
\]

Hence,

\[
FRF_{u_B} (\omega) = \frac{\hat{u}_B}{V_A} = -\frac{i}{2EA} \left[ \begin{array}{c}
e^{i\xi_0(x_B-x_a)} \frac{1-e^{-i\xi_F l_A}}{\xi_0} \\
+3e^{i\xi_F(x_B-x_a)} \frac{1-e^{-i\xi_F l_A}}{\xi_F} \\
+ \frac{1-e^{-i\xi_F l_B}}{\xi_0} + 3 \frac{1-e^{-i\xi_F l_B}}{\xi_F}
\end{array} \right] FRF_{FVA} (\omega)
\]

\[
FRF_{v_B} (\omega) = \frac{\hat{v}_B}{V_A} = -\frac{\omega}{2EA} \left[ \begin{array}{c}
e^{i\xi_0(x_B-x_a)} \frac{1-e^{-i\xi_F l_A}}{\xi_0} \\
+3e^{i\xi_F(x_B-x_a)} \frac{1-e^{-i\xi_F l_A}}{\xi_F} \\
+ \frac{1-e^{-i\xi_F l_B}}{\xi_0} + 3 \frac{1-e^{-i\xi_F l_B}}{\xi_F}
\end{array} \right] FRF_{FVB} (\omega)
\]

At \( B' = x_B + l_B \), the displacement is

\[
u_B(x_B + l_B, t) = \frac{i\hat{F}_A}{2EA} \left\{ \begin{array}{c}
\left( -e^{-i\xi_0 x_B} + e^{-i\xi_F l_A} \right) e^{i\xi_0(x_B+l_B)-\omega t} \\
+3 \left( -e^{-i\xi_F x_B} + e^{-i\xi_F l_A} \right) e^{i\xi_F(x_B+l_B)-\omega t}
\end{array} \right\}
\]

\[
= e^{-i\alpha t} \left\{ \begin{array}{c}
\left( -e^{-i\xi_0 x_B} + e^{-i\xi_F l_A} \right) e^{i\xi_0(x_B+l_B)-\omega t} \\
+3 \left( -e^{-i\xi_F x_B} + e^{-i\xi_F l_A} \right) e^{i\xi_F(x_B+l_B)-\omega t}
\end{array} \right\}
\]

\[
= \left\{ \begin{array}{c}
i\hat{F}_A \left[ e^{i\xi_0((x_B+l_B)-(x_a+l_A))} \frac{1-e^{-i\xi_F l_A}}{\xi_0} + 3e^{i\xi_F((x_B+l_B)-(x_a+l_A))} \frac{1-e^{-i\xi_F l_A}}{\xi_F} \right]
+ \frac{1-e^{-i\xi_F l_B}}{\xi_0} + 3 \frac{1-e^{-i\xi_F l_B}}{\xi_F}
\end{array} \right\}
\]

\[
= \left\{ \begin{array}{c}
i\hat{F}_B \left[ e^{i\xi_0((x_B+l_B)-(x_a+l_A))} \frac{1-e^{-i\xi_F l_A}}{\xi_0} + 3e^{i\xi_F((x_B+l_B)-(x_a+l_A))} \frac{1-e^{-i\xi_F l_A}}{\xi_F} \right]
+ \frac{1-e^{-i\xi_F l_B}}{\xi_0} + 3 \frac{1-e^{-i\xi_F l_B}}{\xi_F}
\end{array} \right\}
\]
The velocity at \( B' = x_B + l_B \) is

\[
v_{B'} = \hat{u}_{B'}(x_B', t) = e^{-i\omega t} \frac{\omega}{2EA} \left\{ \begin{array}{l}
e^{i\xi_B((x_B + l_B) - (x_A + l_A))} \frac{1 - e^{i\xi_A l_A}}{\xi_0} \\
+ 3e^{i\xi_B((x_B + l_B) - (x_A + l_A))} \frac{1 - e^{i\xi_A l_A}}{\xi_F} \\
+ \left[ \frac{1 - e^{i\xi_A l_B}}{\xi_0} + 3 \frac{1 - e^{i\xi_A l_B}}{\xi_F} \right] \hat{F}_B(\omega) \end{array} \right\} \tag{5.121}
\]

Define FRF of the velocity at \( B' = x_B + l_B \) of the PWAS B and applied voltage on the PWAS A as

\[
FRF_{uB'}(\omega) = \frac{\hat{u}_{B'}}{V_A} \quad FRF_{vB'}(\omega) = \frac{\hat{v}_{B'}}{V_A} \tag{5.122}
\]

Hence

\[
FRF_{uB'}(\omega) = \frac{\hat{u}_{B'}}{V_A} = \frac{i}{2EA} \left\{ \begin{array}{l}
e^{i\xi_B((x_B + l_B) - (x_A + l_A))} \frac{1 - e^{i\xi_A l_A}}{\xi_0} \\
+ 3e^{i\xi_B((x_B + l_B) - (x_A + l_A))} \frac{1 - e^{i\xi_A l_A}}{\xi_F} \\
+ \left[ \frac{1 - e^{i\xi_A l_B}}{\xi_0} + 3 \frac{1 - e^{i\xi_A l_B}}{\xi_F} \right] FRF_{FVA}(\omega) \end{array} \right\} \tag{5.123}
\]

\[
FRF_{vB'}(\omega) = \frac{\hat{v}_{B'}}{V_A} = \frac{\omega}{2EA} \left\{ \begin{array}{l}
e^{i\xi_B((x_B + l_B) - (x_A + l_A))} \frac{1 - e^{i\xi_A l_A}}{\xi_0} \\
+ 3e^{i\xi_B((x_B + l_B) - (x_A + l_A))} \frac{1 - e^{i\xi_A l_A}}{\xi_F} \\
+ \left[ \frac{1 - e^{i\xi_A l_B}}{\xi_0} + 3 \frac{1 - e^{i\xi_A l_B}}{\xi_F} \right] FRF_{FVB}(\omega) \end{array} \right\} \tag{5.124}
\]
5.3.1.4.2 Mechanical Power Applied on Receiver

The time-averaged product of two harmonic variables is one half the product of one variable times the conjugate of the other. The time averaged mechanical power under harmonic excitation for PWAS receiver B is

\[
\langle P_B \rangle = \frac{1}{T} \int_0^T p(t) \, dt = \frac{1}{T} \int_0^T -F(t) v(t) \, dt = -\frac{1}{2} \bar{F}_B(\omega) \hat{v}_B(\omega) \quad (5.125)
\]

\[
\langle P_B' \rangle = \frac{1}{T} \int_0^T p(t) \, dt = \frac{1}{T} \int_0^T F(t) v(t) \, dt = \frac{1}{2} \bar{F}_B(\omega) \hat{v}_B'(\omega) \quad (5.126)
\]

5.3.1.5 Receiver Output Electrical Power

The Receiver electrical response under harmonic excitation can derived from FRF. Recall the FRF equation, i.e.,

\[
FRF(\omega) = \frac{k_{31}^2}{r_y + (1-k_{31}^2) k_{ii} C_{AB}(\omega) + jR(\omega)k_{ib} C_{BB}(\omega) - 1} \quad (5.127)
\]

The receiver output voltage is

\[
V_B(\omega) = FRF(\omega) V_A(\omega) \quad (5.128)
\]

The current is

\[
I_B(\omega) = Y_f FRF(\omega) V_A(\omega) \quad (5.129)
\]

The time-average electrical output power for receiver B is

\[
\langle P_{B \text{elec}}(\omega) \rangle = \frac{1}{2} \text{Re}\{\bar{F}_B(\omega) I_B(\omega)\} \quad (5.130)
\]
5.3.2 Power and Energy under Non-Harmonic Excitation

Under non-harmonic excitation, the power and energy is calculated using the FRF technique.

5.3.2.1 Instantaneous Input Power of Transmitter

For transmitter, the electrical instantaneous power is

\[ p(t) = v(t)i(t) \]  \hspace{1cm} (5.131)

Substitution of Equation (4.209) into (4.212) yields

\[ p(t) = v(t)[y(t)*v(t)] = v(t)\mathcal{F}^{-1}[Y(\omega)V(\omega)] \]  \hspace{1cm} (5.132)

Taking Fourier transform of equation (4.213) gives the complex power, i.e.,

\[ P(\omega) = \mathcal{F}\{v(t)[y(t)*v(t)]\} = \mathcal{F}\{v(t)\} * \mathcal{F}\{y(t)\} * v(t) = V(\omega) * F^{-1}[Y(\omega)V(\omega)] \]  \hspace{1cm} (5.133)

The transmitter A electrical response under non-harmonic excitation is

\[ p_{\text{elec}}^A(t) = v_A(t)i_A(t) = v_A(t)\mathcal{F}^{-1}[Y(\omega)V_A(\omega)] \]  \hspace{1cm} (5.134)

5.3.2.2 Instantaneous Mechanical Power at Transmitter Structure Interface

Mechanical power by definition is

\[ p(t) = f(t)v(t) \]  \hspace{1cm} (5.135)

Recall the FRF function of pin-force at the PWAS A and applied voltage at the PWAS A, i.e.,
\[ FRF_{FVA}(\omega) = \frac{\hat{F}_A(\omega)}{V_A(\omega)} \]  

(5.136)

Rearrangement yields

\[ \hat{F}_A(\omega) = FRF_{FVA}(\omega)\hat{V}_A(\omega) \]  

(5.137)

Inverse Fourier transform on both side yields

\[ f_A(t) = \mathcal{F}^{-1}\left[ \hat{F}_A(\omega) \right] = \mathcal{F}^{-1}\left[ FRF_{FVA}(\omega)\hat{V}_A(\omega) \right] \]  

(5.138)

For instantaneous velocity, we get

\[ v_A(t) = \mathcal{F}^{-1}\left[ \hat{v}_A(\omega) \right] = \mathcal{F}^{-1}\left[ FRF_{vVA}(\omega)\hat{V}_A(\omega) \right] \]  

(5.139)

The instantaneous power at \( A = x_A \) is

\[ p_A(t) = f_A(t)v_A(t) = \mathcal{F}^{-1}\left[ FRF_{FVA}(\omega)\hat{V}_A(\omega) \right] \mathcal{F}^{-1}\left[ FRF_{vVA}(\omega)\hat{V}_A(\omega) \right] \]  

(5.140)

The instantaneous power at \( A' = x_A + l_A \) is

\[ p_{A'}(t) = f_A(t)v_{A'}(t) = \mathcal{F}^{-1}\left[ FRF_{FVA}(\omega)\hat{V}_A(\omega) \right] \mathcal{F}^{-1}\left[ FRF_{vVA}(\omega)\hat{V}_A(\omega) \right] \]  

(5.141)

5.3.2.3 Instantaneous Wave Propagation Power

For the axial waves, the instantaneous energy is

\[ e_{axial}(x,t) = k(x,t) + v(x,t) = m\omega^2\dot{u}^2 \sin^2(\gamma_0 x - \omega t) \]  

(5.142)

For the axial waves, the instantaneous energy is

\[ e_{flexural}(x,t) = k(x,t) + v(x,t) = m\omega^2\dot{u}^2 \sin^2(\gamma_F x - \omega t) \]  

(5.143)
The power is the product of energy and wave speed, i.e.,

\[ p(x,t) = c_{axial}e_{axial}(x,t) + c_{flexural}e_{flexural}(x,t) \quad (5.144) \]

5.3.2.4 Instantaneous Mechanical Power at Receiver Structural Interface

Recall the FRF of pin-force at the PWAS B and applied voltage at the PWAS A, i.e.,

\[ FRF_{FVB}(\omega) = \hat{F}_B(\omega) \frac{V_A(\omega)}{V_{\omega}} \quad (5.145) \]

Rearrangement to get

\[ \hat{F}_B(\omega) = FRF_{FVB}(\omega)\hat{V}_A(\omega) \quad (5.146) \]

Inverse Fourier transform on both side yields

\[ f_B(t) = F^{-1}\left[ \hat{F}_B(\omega) \right] = F^{-1}\left[ FRF_{FVB}(\omega)\hat{V}_A(\omega) \right] \quad (5.147) \]

For instantaneous velocity, we get

\[ v_B(t) = F^{-1}\left[ \hat{v}_B(\omega) \right] = F^{-1}\left[ FRF_{v_{VB}}(\omega)\hat{V}_A(\omega) \right] \quad (5.148) \]

The instantaneous power at \( B = x_B \) is

\[ p_B(t) = f_B(t)v_B(t) = F^{-1}\left[ FRF_{v_{VB}}(\omega)\hat{V}_A(\omega) \right] F^{-1}\left[ FRF_{v_{VB}}(\omega)\hat{V}_A(\omega) \right] \quad (5.149) \]

The instantaneous power at \( B' = x_B + l_B \) is

\[ p_{B'}(t) = f_B(t)v_{B'}(t) = F^{-1}\left[ FRF_{FVB}(\omega)\hat{V}_A(\omega) \right] F^{-1}\left[ FRF_{FVB}(\omega)\hat{V}_A(\omega) \right] \quad (5.150) \]
5.3.2.5 Instantaneous Output Electrical Power

The Receiver B electrical response under non-harmonic excitation is

\[ p_{B}^{\text{elec}}(t) = v_{B}(t)i_{B}(t) = \mathcal{F}^{-1}\left[\hat{V}_{B}(\omega)\right] \mathcal{F}^{-1}\left[I_{B}(\omega)\right] \]
\[ = \mathcal{F}^{-1}\left[\text{FRF}(\omega)\hat{V}_{A}(\omega)\right] \mathcal{F}^{-1}\left[\text{FRF}(\omega)\hat{V}_{A}(\omega)/Z(\omega)\right] \]  

(5.151)

5.4 Numerical Simulation

The analytical model was used to perform numerical simulations that directly predicts PWAS output voltage in pitch-catch setup. The specimen is an infinite aluminum alloy 2024 bar with: with \( b = 40\text{mm} \) width and \( t = 1\text{mm} \) thickness. The PWAS has dimension is \( l = 5\text{mm} \) length, \( b = 40\text{mm} \) width and \( t = 0.2\text{mm} \) thickness. The transmitter PWAS location A is 200mm from and receiver locates at B. For harmonic excitation, the excitation amplitude is 10V. For instantaneous excitation, the excitation signal used in this application is a 10V 3-count 100kHz Hanning windowed tone burst.

Table 5.1 Pitch-catch simulation parameters

<table>
<thead>
<tr>
<th>Beam</th>
<th>Transmitter</th>
<th>Receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>( \infty )</td>
<td>5 mm</td>
</tr>
<tr>
<td>Height</td>
<td>1 mm</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>Width</td>
<td>40 mm</td>
<td>40 mm</td>
</tr>
<tr>
<td>Transmitter Receiver Distance</td>
<td></td>
<td>200 mm</td>
</tr>
<tr>
<td>Frequency</td>
<td>Frequency sweep 0-600 kHz</td>
<td></td>
</tr>
<tr>
<td>Measurement Instrument Resistance</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.4.1 Pitch-catch Power and Energy

In pitch-catch setup, PWAS transmitter power rating is dominant by the reactive power.

The active power is of the point of interest because it converts to the wave power.
Consider the electro-acoustic transduction at transmitter, the active power of transmitter provides the mechanical power at PWAS edge. The active power of transmitter is shown in Figure 5.3.

![Figure 5.3 Transmitter electrical active power and mechanical power.](image)

The mechanical power is shown in Figure 5.3. The mechanical power at both end of transmitter is not equal due to the reflection of PWAS B. The total mechanical power is not the same as the transmitter active power. Instead it is the transmitter active power plus the reflection mechanical power from PWAS B. The reflection of PWAS B provides more mechanical power at PWAS A edges (Figure 5.4).

![Figure 5.4 Comparison of active electrical and mechanical power at PWAS edges.](image)
Figure 5.5 Comparison of active electrical power and mechanical power at PWAS edges.

The axial wave, flexural wave power that propagates in the bar was simulated. The total wave power is summation of the axial and flexural wave power. The total wave power is close to the mechanical power provided by the PWAS transmitter from one edge (Figure 5.). Due to the reflection of PWAS B, the power curve has ripples.

Figure 5.6 Receiver mechanical and electrical output power.

The receiver PWAS edges mechanical power is shown in Figure 5.6. The power at receiver B both edges are equal. The mechanical power at receiver shows the tuning effects. The receiver acoustic transduction to electrical power is shown here. The wave
propagates through PWAS receiver will generate voltage. When the receiver connects to a measurement instrument or harvested power equipment, the output electrical power is shown in Figure 5.6. To make it clear, the electrical power in the picture is magnified by 20 times. Under high resistance load, the output electrical power is quite small.

5.4.2 Instantaneous Response

The instantaneous response here focuses on the pitch-catch voltage output under high resistive impedance. The comparison of NME and wave propagation in FRF is shown in Figure 5.7.

![Figure 5.7 Frequency response function of a pitch-catch simulation.](image)

A 100-kHz central frequency 3-count Hanning window tone-burst signal is applied to the transmitter. The receiver instantaneous voltage response is shown in Figure 5.8. The fast axial wave is separated from the low speed flexural wave. The axial wave is non-dispersive and keep the shape of excitation signal. The flexural wave spread out due to the dispersive nature.
Comparison of flexural and A0 velocity for a 1-mm thick aluminum beam are shown in Figure 5.9. At low frequency, the phase velocity of A0 and flexural wave are close. Up to 220 kHz, the difference of flexural and A0 phase velocity is less than 5%. For frequency less than 370 kHz, the difference is less than 10%. To simulate the flexural vibration at high frequency, it is better to use the S0 and A0 wave speed to replace the axial and flexural wave speed.

Figure 5.9 Comparison of flexural and A0 wave speed for 1mm thick aluminum.
6 SINGLE PWAS ANALYSIS USING NORMAL MODE EXPANSION MODEL

In previous chapters, we consider the wave propagation in an infinite bar. An alternative way to consider the boundary condition of a finite beam is using the normal mode expansion (NME) method. In the next two chapters, we use the NME method to analyze the vibration of PWAS attached to a finite length beam. The FRF functions for a single PWAS setup and a pitch-catch setup are developed next.

6.1 VIBRATION ANALYSIS USING NME

Consider forced vibration of a beam of length $L$, width $b$ and thickness $h$; the beam undergoes simultaneous axial and flexural vibrations. PWAS is considered mounted on the beam at $x_0$, as shown in Figure 6.1.

![Figure 6.1 Schematic of a single PWAS attached to a beam.](image)

The equation of motion for axial vibrations is
\[ \rho A \ddot{u}(x,t) - E A \dot{u}'(x,t) = N'_t(x,t) \]  \hspace{1cm} (6.1)

where \( A = bh \) is the beam cross-section area, \( \rho \) is material density and \( E \) is the Young's modules of the beam.

Recall the pin-force model Equation (3.9) and substitute into Equation (6.1) yields

\[ \rho A \ddot{u}(x,t) - E A \dot{u}'(x,t) = \left\{ \hat{F} \left[ -\delta(x-x_0) + \delta(x-x_0-l) \right] \right\} e^{-i\omega t} \]  \hspace{1cm} (6.2)

where \( \delta \) is Dirac's function and \( \delta = H' \).

Assume axial normal mode expansion, i.e.,

\[ u(x,t) = \sum_{n=0}^{\infty} C_n U_n(x) e^{-i\omega t} \]  \hspace{1cm} (6.3)

where \( U_n(x) \) are orthonormal mode shapes, i.e.,

\[ \int U_n U_m dx = \delta_{mn} \]  \hspace{1cm} (6.4)

with \( \delta_{mn} = 1 \) for \( m = n \) and 0 otherwise. \( C_n \) are the mode amplitudes.

Substitute Equation (6.3) into Equation (6.2), multiply by \( U_m \) and integrate Equation (6.2) to get

\[ -\int_0^l U_m \omega^2 \rho A \sum_{n=0}^{\infty} C_n U_n dx - \int_0^l U_m E A \sum_{n=0}^{\infty} C_n U'_n dx \]

\[ = \int_0^l \left\{ \hat{F} \left[ -\delta(x-x_0) + \delta(x-x_0-l) \right] \right\} U_m dx \]  \hspace{1cm} (6.5)

Upon rearrangement, we get

\[ -\omega^2 A \sum_{n=0}^{\infty} C_n \int_0^l U_n U_m dx - \sum_{n=0}^{\infty} C_n \int_0^l U_m U'_n dx \]

\[ = \hat{F} \left[ -U_m(x_0) + U_m(x_0 + l) \right] \]  \hspace{1cm} (6.6)
The mode shapes satisfy the free-vibration differential equation, i.e.,

\[ EAU_n'' + \omega_n^2 \rho A U_n = 0 \]  \hspace{1cm} (6.7)

Multiply Equation (6.7) by \( U_m \) and integrate to get

\[ \int_0^l EAU_m U_n^* dx + \omega_n^2 \rho A \int_0^l U_m U_n dx = 0 \]  \hspace{1cm} (6.8)

If Equation (6.4) and (6.8) are substituted into Equation (6.6), then only the \( m = n \) term remains in the sum, i.e.,

\[ C_n \omega_n^2 \rho A - C_n \omega_n^2 \rho A = \hat{F} \left[ -U_n(x_0) + U_n(x_0 + l) \right] \]  \hspace{1cm} (6.9)

Rearrangement to get

\[ C_n = \frac{1}{\omega_n^2 - \omega^2} \frac{\hat{F}}{\rho A} \left[ -U_n(x_0) + U_n(x_0 + l) \right] \]  \hspace{1cm} (6.10)

Define

\[ \Delta U(x, l) = -U(x) + U(x + l) \]  \hspace{1cm} (6.11)

The mode amplitude becomes

\[ C_n = \frac{1}{\omega_n^2 - \omega^2} \frac{1}{\rho A} \left[ \hat{F} \Delta U_n(x_0, l) \right] \]  \hspace{1cm} (6.12)

Hence, Equation (6.3) becomes

\[ u(x, t) = \sum_{n=0}^\infty \frac{1}{\omega_n^2 - \omega^2} \frac{1}{\rho A} \left[ \hat{F} \Delta U_n(x_0, l) \right] U_n(x) e^{-iat} \]  \hspace{1cm} (6.13)

### 6.1.2 Flexural Vibrations

For Euler-Bernoulli beams, the equation of motion under moment excitation is
\[ \rho A \ddot{w}(x,t) + EI \omega''(x,t) = -M''_c(x,t) \]  
(6.14)

Substitution of Equation (3.10) into Equation (6.14) gives

\[
\rho A \ddot{w}(x,t) + EI \omega''(x,t) = \frac{h}{2} \hat{F} \left[ -\delta'(x-x_0) + \delta'(x-x_0 - l) \right] e^{-i\omega x} \]  
(6.15)

where \( \delta' \) is the first derivative of Dirac's function \( \delta = H^* \).

Assume the flexural normal mode expansion, i.e.,

\[
w(x,t) = \sum_{n=0}^{\infty} C_n \omega_n(x) e^{-i\omega x} \]  
(6.16)

where \( \omega_n(x) \) are the orthonormal bending mode shapes.

Substitute Equation (6.16) into (6.15), multiply by \( m \omega_n \) and integrate to get

\[
\int_0^l W_m \omega' e \rho A \sum_{n=0}^{\infty} C_n \omega_n dx + \int_0^l W_m EI \sum_{n=0}^{\infty} C_n \omega'' dx \]  
\[
= \int_0^l \left( \frac{h}{2} \hat{F} \left[ -\delta'(x-x_0) + \delta'(x-x_0 - l) \right] W_m dx \right) \]  
(6.17)

Rearrangement yields

\[
\sum_{n=0}^{\infty} C_n \int_0^l EI W_m \omega'' dx = \sum_{n=0}^{\infty} C_n \int_0^l W_m \omega' e \rho A dx = \frac{h}{2} \hat{F} \left[ -W'_m(x_0) + W'_m(x_0 + l) \right] \]  
(6.18)

The mode shapes satisfy the free-vibration differential equation, i.e.,

\[
EI \omega'' = \omega_n^2 \rho A \omega_n \]  
(6.19)

Multiplication of (6.19) by \( X_m \) and integration yields

\[
\int_0^l W_m EI \omega'' dx = \omega_n^2 \rho A \int_0^l W_m \omega dx = \omega_n^2 \rho A \delta_{mn} \]  
(6.20)
If Equation (6.20) is used into (6.17), then only \( m = n \) terms survive, i.e.

\[
C_n \omega^2 \rho A - C_n \alpha^2 \rho A = \frac{h}{2} \hat{F} \left[ -W_n'(x) + W_n'(x + l) \right]
\]  
(6.21)

Define the notation

\[
\Delta W'(x, l) = -W'(x) + W'(x + l)
\]  
(6.22)

The flexural mode amplitude becomes

\[
C_n = -\frac{1}{\omega_n^2 - \omega^2} \frac{1}{\rho A} \left[ \frac{h}{2} \hat{F} \Delta W_n'(x_0, l) \right]
\]  
(6.23)

Hence, Equation (6.16) becomes

\[
w(x, t) = \sum_{n=0}^{\infty} -\frac{1}{\omega_n^2 - \omega^2} \frac{1}{\rho A} \left[ \frac{h}{2} \hat{F} \Delta W_n'(x_0, l) \right] W_n(x) e^{-i\omega t}
\]  
(6.24)

### 6.1.3 Kinematic Analysis

Kinematic analysis gives the horizontal displacement of a generic point \( P \) on the beam surface (Figure 6.2) in terms of the axial and flexural displacement as

\[
u_p(t) = u(x) - \frac{h}{2} w'(x)
\]  
(6.25)

where \( u \) and \( w' \) are the axial and bending displacements measured at the neutral axis.

Given by Equation (6.13), (6.24), the \( u \) and \( w' \) becomes

\[
u(x, t) = \left[ \frac{\hat{F}}{\rho A} \sum_n \frac{\Delta U_n(x_0, l)}{\omega_n^2 - \omega^2} \right] U_n(x) e^{-i\omega t}
\]  
(6.26)

\[
w'(x, t) = -\left[ \frac{1}{\rho A} \frac{h}{2} \hat{F} \sum_n \frac{\Delta W_n'(x_0, l)}{\omega_n^2 - \omega^2} \right] W_n'(x) e^{-i\omega t}
\]  
(6.27)
Figure 6.2  The total horizontal displacement, \( u_p \), results from the superposition of axial displacement \( u \) and the rotation \( w' \).

The horizontal displacement of a generic point P becomes

\[
\begin{align*}
\hat{v}(x,t) &= \hat{v}_0(x,t) + v(x,t) \\
&= \frac{\hat{\theta}}{\rho A} \left[ \sum_{n_u} \Delta U_{n_u}(x_0, l) U_{n_u}(x) + \left( \frac{h}{2} \right)^2 \sum_{n_w} \Delta W_{n_w}(x_0, l) W_{n_w}(x) \right] e^{-i \alpha x}
\end{align*}
\]

(6.28)

where \( n_u \) is the axial mode number and \( n_w \) is the flexural mode number.

For free-free beams, the axial and flexural mode shapes can be calculated with the formulae,

\[
\begin{align*}
U_{n_u}(x) &= A_{n_u} \cos(\gamma_{n_u} x), \quad A_{n_u} = \frac{2}{L}, \quad \gamma_{n_u} = \frac{n_u \pi}{L}, \quad \omega_{n_u} = \gamma_{n_u} c, \quad c = \sqrt{\frac{E}{\rho}}, \quad n_u = 1, 2, \ldots
\end{align*}
\]

(6.29)

\[
W_{n_w}(x) = A_{n_w} \left[ \cosh \gamma_{n_w} x + \cosh \gamma_{w} x - \sigma_{n_w} (\sinh \gamma_{n_w} x + \sin \gamma_{w} x) \right]
\]

(6.30)

\[
\omega_{n_w} = \gamma_{n_w} a, \quad a = \sqrt{\frac{EI}{\rho A}}, \quad I = \frac{bh^3}{12}, \quad A_{n_w} = \frac{1}{\sqrt{\int_0^L W_{n_w}^2(x) dx}}, \quad n_w = 1, 2, \ldots
\]

(6.31)

The proof of \( \int_0^L W_{n_w}^2(x) dx = L \) is shown in Appendix C.

Define the elongation of the PWAS as

\[
\Delta u(x_0, t) = u(x_0 + l) - u(x_0)
\]

(6.32)
Substitution Equation (6.28) into Equation (6.32) and use of Equation (6.11) and (6.22) gives

\[
\Delta u = \frac{\hat{F}}{\rho A} \left[ \sum_{n_v} \left( \frac{\Delta U_{n_v}(x_0, l)}{\omega_{n_v}^2 - \omega^2} \right)^2 \left( \frac{h}{2} \right)^2 + \left( \frac{h}{2} \right)^2 \sum_{n_v} \left( \frac{\Delta W'_{n_v}(x_0, l)}{\omega_{n_v}^2 - \omega^2} \right)^2 \right] e^{-i\omega t} \quad (6.33)
\]

The mode damping ratio, \(\zeta\), is introduced to account for the inherent dissipation loss encountered in practical experiments. Considering the damping, the horizontal displacement becomes

\[
\Delta u = \frac{\hat{F}}{\rho A} \left[ \sum_{n_v} \frac{\left( \Delta U_{n_v}(x_0, l) \right)^2}{\omega_{n_v}^2 + 2i\zeta_{n_v} \omega_{n_v} \omega - \omega^2} + \left( \frac{h}{2} \right)^2 \sum_{n_v} \frac{\left( \Delta W'_{n_v}(x_0, l) \right)^2}{\omega_{n_v}^2 + 2i\zeta_{n_v} \omega_{n_v} \omega - \omega^2} \right] e^{-i\omega t} \quad (6.34)
\]

Define the dynamic structural compliance as

\[
C_{AA}(\omega) = \frac{\Delta u}{\hat{F}} = \frac{1}{\rho A} \left[ \sum_{n_v} \frac{\left( \Delta U_{n_v}(x_0, l) \right)^2}{\omega_{n_v}^2 + 2i\zeta_{n_v} \omega_{n_v} \omega - \omega^2} + \left( \frac{h}{2} \right)^2 \sum_{n_v} \frac{\left( \Delta W'_{n_v}(x_0, l) \right)^2}{\omega_{n_v}^2 + 2i\zeta_{n_v} \omega_{n_v} \omega - \omega^2} \right] \quad (6.35)
\]

The dynamic structural stiffness is the inverse of the structural compliance.

\[
k_{str}(\omega) = \frac{1}{\Delta u} = \frac{1}{C_{AA}(\omega)}
= \rho A \left[ \sum_{n_v} \frac{\left( \Delta U_{n_v}(x_0, l) \right)^2}{\omega_{n_v}^2 + 2i\zeta_{n_v} \omega_{n_v} \omega - \omega^2} + \left( \frac{h}{2} \right)^2 \sum_{n_v} \frac{\left( \Delta W'_{n_v}(x_0, l) \right)^2}{\omega_{n_v}^2 + 2i\zeta_{n_v} \omega_{n_v} \omega - \omega^2} \right]^{-1} \quad (6.36)
\]

The expression of \(\Delta u\) as function of \(\hat{F}\) and \(\omega\) is

\[
\Delta u(\hat{F}, \omega) = \hat{F} C_{AA}(\omega) e^{-i\omega t} \quad (6.37)
\]
The E/M impedance measured at the PWAS is influenced by the dynamic stiffness $k_{str}(\omega)$ represented by the structure to the PWAS. In fact, the dynamic stiffness $k_{str}(\omega)$ is constraining the PWAS in its oscillation. The frequency–dependent stiffness ratio $r(\omega)$ is a function of frequency and is defined as

$$r(\omega) = \frac{k_{str}(\omega)}{k_i}$$  \hspace{1cm} (6.38)

where $k_i$ is the PWAS stiffness given by (3.20).

The constrained PWAS admittance is assumed of the form (Giurgiutiu, 2008, pp. 285)

$$Y(\omega) = i\omega C_0 \left[ 1 - k_{31}^2 \frac{r(\omega)}{1 + r(\omega)} \right]$$  \hspace{1cm} (6.39)

Hence, the constrained PWAS impedance is

$$Z(\omega) = \left[ Y(\omega) \right]^{-1} = \frac{1}{i\omega C_0} \left[ 1 - k_{31}^2 \frac{r(\omega)}{1 + r(\omega)} \right]^{-1}$$  \hspace{1cm} (6.40)

### 6.2 Frequency Response Function Using NME

In a finite beam, $\hat{F}$ can be considered as sum of two parts: two parts: $\hat{F}_1$ and $\hat{F}_2$. The first part, $\hat{F}_1$, is generated in reaction to the input voltage when the PWAS acts as an actuator. The second part, $\hat{F}_2$, is generated by the wave reflections coming to the PWAS when it acts as a sensor. In fact, the PWAS simultaneously acts as both actuator and sensor, hence

$$\hat{F} = \hat{F}_1 + \hat{F}_2$$  \hspace{1cm} (6.41)
Substitute Equation (6.41) into (6.37) yields

\[ \Delta u = \left( \hat{F}_1 + \hat{F}_2 \right) C_{AA}(\omega) e^{-i\alpha x} = \left[ \hat{F}_1 C_{AA}(\omega) + \hat{F}_2 C_{AA}(\omega) \right] e^{-i\alpha x} \quad (6.42) \]

Substitution of the pin force of transmitter Equation (3.23) and that of receiver Equation (3.49) into (6.42) gives

\[ \Delta u(\omega) = k_i(\Delta u(\omega) - u_{IS4})C_{AA}(\omega) + R(\omega)k_i\Delta u(\omega)C_{AA}(\omega) \quad (6.43) \]

Upon rearrangement, we get

\[ (k_iC_{AA}(\omega) - 1 + R(\omega)k_iC_{AA}(\omega))\Delta u(\omega) = k_iC_{AA}(\omega)u_{IS4} \quad (6.44) \]

Equation (6.44) yields the solution for \( \Delta u(\omega) \) in the form

\[ \Delta u(\omega) = \frac{k_iC_{AA}(\omega)}{(R(\omega) + 1)k_iC_{AA}(\omega) - 1} u_{IS4} \quad (6.45) \]

The reflection waves generate voltage \( \hat{V}_2 \). Recall Equation (3.40)

\[ \hat{V}_2(\omega) = i\omega \frac{1}{Y_e + (1 - k_{31}^2)Y_0} \frac{bd_{d1}}{s_{11}^d} \Delta u(\omega) \quad (6.46) \]

Substitution of Equation (6.45) into (6.46) yields

\[ \hat{V}_2(\omega) = \frac{bd_{d1}}{s_{11}^d} \frac{1}{Y_e + (1 - k_{31}^2)Y_0} \frac{k_iC_{AA}(\omega)}{(R(\omega) + 1)k_iC_{AA}(\omega) - 1} u_{IS4} \quad (6.47) \]

Recall Equation (3.44) to simplify Equation (6.47)

\[ \hat{V}_2(\omega) = \frac{k_{31}^2 Y_0}{Y_e + (1 - k_{31}^2)Y_0} \frac{k_iC_{AA}(\omega)}{(R(\omega) + 1)k_iC_{AA}(\omega) - 1} \hat{V}(\omega) \quad (6.48) \]
Recall Equation (3.47) \( r_y(\omega) = \frac{Y_e}{Y_0} \), the FRF of voltage generated from the reflection waves is

\[
FRF(\omega) = \frac{V_2(\omega)}{V(\omega)} = \frac{k_s^2}{r_y + (1-k_s^2) k_r C_{AA}(\omega)[1 + R(\omega)]^{-1}}
\] (6.49)

### 6.3 Numerical Simulation

The analytical model is used to perform numerical simulations that directly predict the E/M impedance signature and FRF at the PWAS terminals. We consider a 100 mm long aluminum beam.

The numerical simulation was performed using the NME method. Numerically exact expressions for the axial and flexural frequencies and mode shapes were used. The simulation was performed over a modal subspace that incorporates all modal frequencies in the frequency bandwidth of interest.

A beam with 100 mm long, 7 mm wide and 1-mm thick was used in the simulation. The PWAS dimension is 7 mm long, 7 mm wide and 0.2-mm thick. The PWAS is in the center of the beam. The damping coefficient was assumed \( \zeta = 1\% \). The real part of impedance is shown in Figure 6.3(a). The FRF of reflection output voltage vs. input voltage is shown in Figure 6.3(b).
Figure 6.3  A 100-mm aluminum beam (a) Real part of Impedance; (b) FRF.
7 PITCH-CATCH ANALYSIS USING NORMAL MODE EXPANSION MODEL

In this chapter, we use the NME method to analyze the vibrations of a pitch-catch setup. The FRF for receiver PWAS voltage is derived. Under non-harmonic excitation, the receiver output is simulated using the FRF method.

7.1 PITCH-CATCH VIBRATION ANALYSIS

The pitch-catch vibration analysis is based on axial and flexural vibration on a finite length beam. The 1-D pitch-catch is shown in Figure 7.1. PWAS transmitter A with length $l_A$ locates at location $x_A$, PWAS receiver B with length $l_B$ locates at location $x_B$. Consider both PWAS pin forces applied on the beam, we get both the axial and flexural vibrations. The beam length is $L$.

Figure 7.1 PWAS pitch-catch setup on a beam.
7.1.1 Axial Vibrations

The equation of motion for axial vibrations is

\[ \rho A \ddot{u}(x,t) - E A u'(x,t) = N'_c(x,t) \]  (7.1)

Recall the pin-force Equation (3.52) for pitch-catch model, i.e.,

\[ N_c(x,t) = \left\{ \hat{F}_A \left[-H(x-x_A) + H(x-x_A-l_A) \right] \right\} e^{-i\omega t} \]  (7.2)

Substitute Equation (7.2) into (7.1) to get

\[ \rho A \ddot{u}(x,t) - E A u'(x,t) = \left\{ \hat{F}_A \left[-\delta(x-x_A) + \delta(x-x_A-l_A) \right] \right\} e^{-i\omega t} \]  (7.3)

Using the same procedure in previous analysis of single PWAS axial vibration, the axial
NME is

\[ u(x,t) = \sum_{n=0}^{\infty} \frac{1}{\omega_n^2 - \omega^2} \frac{1}{\rho A} \left[ \hat{F}_A \Delta U_n(x_A,l_A) + \hat{F}_B \Delta U_n(x_B,l_B) \right] U_n(x)e^{-i\omega t} \]  (7.4)

7.1.2 Flexural Vibrations

The equation of motion for flexural vibration is

\[ \rho A \ddot{w}(x,t) + E I w''(x,t) = -M'_c(x,t) \]  (7.5)

Recall the moment of Equation(3.53), i.e.,

\[ M_c(x,t) = -\left\{ \frac{h}{2} \hat{F}_A \left[-H(x-x_A) + H(x-x_A-l_A) \right] \right\} e^{-i\omega t} \]  (7.6)
Substitute Equation (7.6) into Equation (7.5) to get

\[
\rho A \ddot{w}(x,t) + EI w''''(x,t) = \frac{h}{2} \left[ \hat{F}_A \left[ -\delta'(x-x_A) + \delta'(x-x_A-l_A) \right] + \hat{F}_B \left[ -\delta'(x-x_B) + \delta'(x-x_B-l_B) \right] \right] e^{-i\omega t} \tag{7.7}
\]

Using the same procedure in previous single PWAS flexural vibration, the flexural NME is

\[
w(x,t) = \sum_{n=0}^{\infty} \frac{1}{\omega_n^2 - \omega^2} \frac{1}{\rho A} \left[ \frac{h}{2} \hat{F}_A \Delta W'_n(x_A,l_A) + \frac{h}{2} \hat{F}_B \Delta W'_n(x_B,l_B) \right] W_n(x) e^{-i\omega t} \tag{7.8}
\]

### 7.1.3 Kinematic Analysis

Kinematic analysis gives the horizontal displacement of a generic point P on the beam surface in terms of the axial and flexural displacement as

\[
u_p(x,t) = e^{-i\omega t} \left[ \sum_{n_a} \frac{\Delta U_{n_a}(x_A,l_A)}{\omega_{n_a}^2 - \omega^2} U_{n_a}(x) \right] + \frac{h}{2} \sum_{n_a} \frac{\Delta W''_{n_a}(x_A,l_A)}{\omega_{n_a}^2 - \omega^2} W_{n_a}(x) \left[ \hat{F}_A \right]
\]

\[
+ \frac{h}{2} \sum_{n_B} \frac{\Delta W''_{n_B}(x_B,l_B)}{\omega_{n_B}^2 - \omega^2} W_{n_B}(x) \left[ \hat{F}_B \right]
\]

Considering the damping, the horizontal displacement becomes
Define the elongation of the PWAS A and B as

$$\Delta u_A = u(x_A + l_A) - u(x_A)$$
$$\Delta u_B = u(x_B + l_B) - u(x_B)$$  \hspace{1cm} (7.11)$$

Substitution of Equation (7.10) into Equation (7.11) and use of Equation (6.11) and (6.22) gives

$$\Delta u_A = \left\{ \frac{\hat{F}_A}{\rho A} \left[ \sum_{n_x} \frac{\Delta U_{n_x}(x_A, l_A)^2}{\omega_{n_x}^2 + 2i\zeta_{n_x} \omega_{n_x} - \omega^2} \right] + \left( \frac{h}{2} \right)^2 \sum_{n_x} \frac{\Delta W'_{n_x}(x_A, l_A)^2}{\omega_{n_x}^2 + 2i\zeta_{n_x} \omega_{n_x} - \omega^2} \right\} e^{-i\alpha x}$$

$$\Delta u_B = \left\{ \frac{\hat{F}_B}{\rho A} \left[ \sum_{n_x} \frac{\Delta U_{n_x}(x_B, l_B) \Delta U_{n_x}(x_A, l_A)}{\omega_{n_x}^2 + 2i\zeta_{n_x} \omega_{n_x} - \omega^2} \right] \right\} e^{-i\alpha x}$$

$$\Delta u_A = \left\{ \frac{\hat{F}_A}{\rho A} \left[ \sum_{n_x} \frac{\Delta U_{n_x}(x_B, l_B) \Delta U_{n_x}(x_A, l_A)}{\omega_{n_x}^2 + 2i\zeta_{n_x} \omega_{n_x} - \omega^2} \right] \right\} e^{-i\alpha x}$$

$$\Delta u_B = \left\{ \frac{\hat{F}_B}{\rho A} \left[ \sum_{n_x} \frac{\Delta W'_{n_x}(x_B, l_B) \Delta W'_{n_x}(x_A, l_A)}{\omega_{n_x}^2 + 2i\zeta_{n_x} \omega_{n_x} - \omega^2} \right] \right\} e^{-i\alpha x}$$
Define the following coefficients

\[
C_{AA}(\omega) = \frac{1}{\rho A} \left[ \sum_{n} \left( \frac{\Delta U_{n_0}(x_A, l_A) \Delta U_{n_0}(x_B, l_B)}{\omega_{n_0}^2 + 2i\zeta_{n_0} \omega_{n_0} - \omega^2} \right)^2 + \left( \frac{h}{2} \right)^2 \sum_{n_u} \left( \frac{\Delta W'_{n_u}(x_A, l_A) \Delta W'_{n_u}(x_B, l_B)}{\omega_{n_u}^2 + 2i\zeta_{n_u} \omega_{n_u} - \omega^2} \right)^2 \right]
\]  
(7.14)

\[
C_{AB}(\omega) = \frac{1}{\rho A} \left[ \sum_{n} \left( \frac{\Delta U_{n_0}(x_A, l_A) \Delta U_{n_0}(x_B, l_B)}{\omega_{n_0}^2 + 2i\zeta_{n_0} \omega_{n_0} - \omega^2} \right)^2 + \left( \frac{h}{2} \right)^2 \sum_{n_u} \left( \frac{\Delta W'_{n_u}(x_A, l_A) \Delta W'_{n_u}(x_B, l_B)}{\omega_{n_u}^2 + 2i\zeta_{n_u} \omega_{n_u} - \omega^2} \right)^2 \right]
\]  
(7.15)

\[
C_{BB}(\omega) = \frac{1}{\rho A} \left[ \sum_{n} \left( \frac{\Delta U_{n_0}(x_B, l_B) \Delta U_{n_0}(x_B, l_B)}{\omega_{n_0}^2 + 2i\zeta_{n_0} \omega_{n_0} - \omega^2} \right)^2 + \left( \frac{h}{2} \right)^2 \sum_{n_u} \left( \frac{\Delta W'_{n_u}(x_B, l_B) \Delta W'_{n_u}(x_B, l_B)}{\omega_{n_u}^2 + 2i\zeta_{n_u} \omega_{n_u} - \omega^2} \right)^2 \right]
\]  
(7.16)

The expression of \( \Delta u_A \) and \( \Delta u_B \) as function of \( \hat{F}_A, \hat{F}_B, \omega \) is

\[
\Delta u_A(\hat{F}_A, \hat{F}_B, \omega) = \left[ \hat{F}_A C_{AA}(\omega) + \hat{F}_B C_{AB}(\omega) \right] e^{-i\omega t}
\]  
(7.17)

\[
\Delta u_B(\hat{F}_A, \hat{F}_B, \omega) = \left[ \hat{F}_A C_{AB}(\omega) + \hat{F}_B C_{BB}(\omega) \right] e^{-i\omega t}
\]  
(7.18)
7.2 Pitch-catch Frequency Response Function Using NME

Substitute transmitter pin-force Equation (3.55) and receiver pin-force Equation (3.57) into (7.17) and (7.18) to get

\[ \hat{\Delta}u_A(\omega) = k_{ia}(\Delta\hat{u}_A(\omega) - u_{ISA})C_{AA}(\omega) + R(\omega)k_{ib}\Delta\hat{u}_B(\omega)C_{AB}(\omega) \]  
\[ (7.19) \]

\[ \hat{\Delta}u_B(\omega) = k_{ib}(\Delta\hat{u}_B(\omega) - u_{ISA})C_{BB}(\omega) + R(\omega)k_{ib}\Delta\hat{u}_B(\omega)C_{BB}(\omega) \]  
\[ (7.20) \]

Rearrangement to get

\[ (k_{ia}C_{AA}(\omega) - 1)\Delta\hat{u}_A(\omega) + R(\omega)k_{ib}C_{AB}(\omega)\Delta\hat{u}_B(\omega) = k_{ia}C_{AA}(\omega)u_{ISA} \]  
\[ (7.21) \]

\[ k_{ia}C_{AB}(\omega)\Delta\hat{u}_A(\omega) + (R(\omega)k_{ib}C_{BB}(\omega) - 1)\Delta\hat{u}_B(\omega) = k_{ia}C_{AB}(\omega)u_{ISA} \]  
\[ (7.22) \]

We have two unknown variables \( \Delta\hat{u}_A(\omega) \) and \( \Delta\hat{u}_B(\omega) \) and the analytical solution can be solved by the linear equations. Define the following coefficient

\[ a_1 = k_{ia}C_{AA}(\omega) - 1 \]  
\[ (7.23) \]

\[ b_1 = R(\omega)k_{ib}C_{AB}(\omega) \]  
\[ (7.24) \]

\[ c_1 = k_{ia}C_{AA}(\omega)u_{ISA} \]  
\[ (7.25) \]

\[ a_2 = k_{ia}C_{AB}(\omega) \]  
\[ (7.26) \]

\[ b_2 = R(\omega)k_{ib}C_{BB}(\omega) - 1 \]  
\[ (7.27) \]

\[ c_2 = k_{ia}C_{AB}(\omega)u_{ISA} \]  
\[ (7.28) \]

Substitution of Equation (7.23) - (7.28) into (7.21) and (7.22) yields

\[ a_1\Delta\hat{u}_A + b_1\Delta\hat{u}_B = c_1 \]  
\[ (7.29) \]

\[ a_2\Delta\hat{u}_A + b_2\Delta\hat{u}_B = c_2 \]
The solution of \( \Delta \hat{u}_A (\omega) \) is

\[
\Delta \hat{u}_A (\omega) = \frac{b_c c_1 - b_c c_2}{b_a a_1 - b_a a_2} = \frac{(R(\omega)k_{ib} C_{BB} (\omega) - 1)k_{ia} C_{AA} (\omega) - R(\omega)k_{ib} C_{AB} (\omega)k_{ia} C_{AB} (\omega)}{(R(\omega)k_{ib} C_{BB} (\omega) - 1)(k_{ia} C_{AA} (\omega) - 1) - R(\omega)k_{ib} C_{AB} (\omega)k_{ia} C_{AB} (\omega)} u_{ISA}
\]

(7.30)

Rearrangement of Equation (7.30) yields

\[
\Delta \hat{u}_A (\omega) = \frac{R(\omega)k_{ia} k_{ib} \left[ C_{AA} (\omega) C_{BB} (\omega) \right] - k_{ia} C_{AA} (\omega)}{R(\omega)k_{ia} k_{ib} \left[ -C_{AB}^2 (\omega) \right] - k_{ia} C_{AA} (\omega) - R(\omega)k_{ib} C_{BB} (\omega) + 1} u_{ISA}
\]

(7.31)

The solution of \( \Delta \hat{u}_B (\omega) \) is

\[
\Delta \hat{u}_B = \frac{a_c c_1 - a_c c_2}{a_a b_1 - a_a b_2} = \frac{k_{ia} C_{AB} (\omega)k_{ia} C_{AA} (\omega) - (k_{ia} C_{AA} (\omega) - 1)k_{ia} C_{AB} (\omega)}{k_{ia} C_{AB} (\omega)R(\omega)k_{ib} C_{AB} (\omega) - (k_{ia} C_{AA} (\omega) - 1)(R(\omega)k_{ib} C_{BB} (\omega) - 1)} u_{ISA}
\]

(7.32)

Rearrangement of \( \Delta \hat{u}_B (\omega) \) yields

\[
\Delta \hat{u}_B (\omega) = \frac{k_{ia} C_{AB} (\omega)}{R(\omega)k_{ia} k_{ib} \left[ C_{AB}^2 (\omega) \right] - k_{ia} C_{AA} (\omega) C_{BB} (\omega) + k_{ia} C_{AA} (\omega) + R(\omega)k_{ib} C_{BB} (\omega) - 1} u_{ISA}
\]

(7.33)

The receiver output voltage is related to \( \Delta \hat{u}_B (\omega) \). Recall receiver output voltage Equation (3.40), i.e.,

\[
\hat{V}_B (\omega) = i\omega \frac{1}{Y_e + (1 - k_{_3}^2)Y_{BB}} \frac{b_d d_{31}}{s_{11}} \Delta \hat{u}_B (\omega)
\]

(7.34)

Substitution of Equation (7.33) into (7.34) gives
\[
\hat{V}_B(\omega) = i\omega \frac{b_0 d_{31}}{s_{11}} \frac{k_{ia} C_{AA}(\omega)}{Y_e + (1 - k_{31}^2) Y_{0B}} \left( R(\omega) k_{ia} k_{ib} \left[ C_{AB}^2(\omega) - C_{AA}(\omega) C_{BB}(\omega) \right] \right) \hat{V}_A(\omega)
\]

Recall Equation (3.44), the receiver output voltage in relation of transmitter input voltage is

\[
\hat{V}_B(\omega) = \frac{k_{31}^2 Y_{0B}}{Y_e + (1 - k_{31}^2) Y_{0B}} \frac{k_{ia} C_{AB}(\omega)}{R(\omega) k_{ia} k_{ib} \left[ C_{AB}(\omega) - C_{AA}(\omega) C_{BB}(\omega) \right]} \hat{V}_A(\omega)
\]

Recall Equation (3.47), \( r_Y = \frac{Y_e}{Y_{0B}} \), the frequency response function is

\[
FRF(\omega) = \frac{V_B(\omega)}{V_A(\omega)} = \frac{k_{ia} C_{AB}(\omega)}{r_Y + (1 - k_{31}^2) R(\omega) k_{ia} k_{ib} \left[ C_{AB}^2(\omega) - C_{AA}(\omega) C_{BB}(\omega) \right] + k_{ia} C_{AA}(\omega) + R(\omega) k_{ib} C_{BB}(\omega) - 1}
\]

### 7.3 Numerical Simulation

The simulation uses a free-free aluminum alloy 2024 beam. It is 1220 mm long, 40 mm wide and 1 mm thick. The PWAS dimension is 5 mm long, 40 mm wide and 0.2 mm thick. The transmitter PWAS location is \( A = 500 \) mm from the left edge and receiver locates at \( B = 700 \) mm from the left edge of beam. The sketch of the simulation is shown in Figure 7.2.
The pitch-catch simulation used a long beam to separate the axial and flexural wave in time domain. The mode shapes of long beam have many resonances due to the beam length. The FRF shows a lot of resonances in the frequency range (Figure 7.3). The numerical simulation directly predicts PWAS output voltage in pitch-catch setup.

The wave propagation is best illustrated through the study of the propagation of single-frequency wave packets (tone bursts). A tone burst consists of a single-frequency carrier wave of frequency, whose amplitude is modulated such as to generate a burst-like behavior. The excitation signal used in this application is a 3-count 10V amplitude 50kHz Hanning windowed tone burst. The excitation voltage signal is shown in Figure 7.4.

The receiver output voltage is shown in Figure 7.5. The first wave pack is axial wave that is not dispersive. The second wave pack is flexural wave that shows the dispersive nature. The third wave pack is the axial wave reflected from the boundary and received by the receiver.
Figure 7.3  Simulation results of FRF of a pitch-catch setup on a 1220-mm long beam.

Figure 7.4  Transmitter non-harmonic excitation signal (3-count 100-kHz Hanning window tone-burst)
Figure 7.5 Simulation of receiver output voltage in a pitch-catch setup.
8 COUPLED-FIELD FINITE ELEMENT ANALYSIS OF PWAS BEHAVIOR

Commercially available FEM packages have offered the option of coupled-field element. In this chapter, coupled-field finite element method (CF-FEM) for PWAS applications was presented. The analytical, FEM and experimental results are compared.

8.1 INTRODUCTION

To perform a coupled-field analysis of a PWAS, coupled-field elements in ANSYS were used to deal with both mechanical and electrical fields. For the coupled-field piezoelectric analysis, the stress field and the electric field are coupled to each other such that change in one field will induce change in the other field based on the piezoelectric constitutive equation. In our piezoelectric analysis, each element consists with several nodes. Each node has up to six degrees of freedom (dof’s) for displacement and one additional dof of the electrical voltage. Reaction force of each node FX, FY, FZ corresponds to the X, Y, Z displacement dof’s, respectively. The electric charge Q or electric displacement D (electric flux density, ANSYS term) is the electrical reaction corresponding to the voltage dof. Available analysis types are static, modal, harmonic and transient analysis. All were used to demonstrate the normal PWAS used in SHM.

In this section, our work on CF-FEM approach was modeled for PWAS and the CF-FEM results were compared with analytical prediction and experimental measurements. The
finite element analysis allows the accurate calculation of resonance modes and frequencies as well as impedance spectra. However, the limitation of memory and computing time limits its use for small structures.

PWAS impedance was modeled with the CF-FEM approach. Voltage constrains were applied to the PWAS electrodes. A time harmonic analysis was performed on the FEM models and the variation of the electrical charge with frequency were monitored. From the electrical charge, we can calculate the electric current and then the E/M impedance of the PWAS. The PWAS E/M impedance was calculated at each frequency value as the ratio of V/I, where V is the voltage and I is the electric current. In this calculation, the voltage V was the actual constrain applied to the PWAS surface electrodes. The current I was calculated from the electric charge dynamically accumulated on the surface electrodes of the PWAS. Harmonic analysis provided an alternating electric voltage at each frequency.

The commercial FEM package ANSYS was used for the modeling described in this work. The 3-dimensional models were built by coupled-field elements. There are four 3-D coupled-field brick elements and two 2-D coupled field elements. SOLID5 is one of 3-dimensional coupled-field brick elements that has eight nodes and six degrees of freedom (3 displacements, voltage, scalar magnetic potential and temperature). Here, only the electromechanical coupling was employed, scalar magnetic potential and temperature are not used. In each element, nodes and degrees of freedom (3 displacements, voltage) are coupled. The stress field and the electric field are coupled to each other such that change in one field induces change in the other field.
For the modeling we applied the material parameters of APC-850 provided by APC International Ltd. To convert the material properties that ANSYS can be recognized, check Appendix A for more details. The nodes of the bottom and top planes were defined as electrodes by a common electrical potential equal to the driving voltage and the ground potential, respectively. The impedance spectra of PWAS were modeled with the harmonic response analysis of ANSYS.

8.2 **FEM Mesh Size Effects**

Mesh size is an important factor. Small size can give better approximation of real situation but it needs more computation time and power. The general guideline for appropriate mesh size is given below.

8.2.1 **Time step**

The accuracy of the transient dynamic solution depends on the integration time step: the smaller the time step, the higher the accuracy. A time step that is too large introduces an error that affects the response of the higher modes (and hence the overall response). Too small time step wastes computer resources. To calculate an optimum time step, adhere to the following guidelines:

Resolve the response frequency. The time step should be small enough to resolve the motion (response) of the structure. Since the dynamic response of a structure can be thought of as a combination of modes, the time step should be able to resolve the highest mode that contributes to the response. For the Newark time integration scheme, it has been found that using approximately twenty points per cycle of the highest frequency of
interest results in a reasonably accurate solution. That is, if \( f \) is the frequency (in cycles/time), the integration time step (ITS) is given by \( \text{ITS} = 1/(20f) \).

Smaller ITS values may be required if acceleration results are needed. The effect of ITS on the period elongation in a single-DOF spring-mass system showed that 20 or more points per cycle result in a period elongation of less than 1 percent. For flexural wave less than 200kHz, the ITS should be 0.25 \( \mu s \). A 50 \( \mu s \) total time need 200 load steps (the sampling frequency is 4MHz).

Resolve the applied load-versus-time curve(s). The time step should be small enough to "follow" the loading function. The response tends to lag the applied loads, especially for stepped loads. Stepped loads require a small ITS at the time of the step change so that the step change can be closely followed. ITS values as small as \( 1/180f \) may be needed to follow stepped loads.

8.2.2 Wavelength

Resolve the wave propagation. If you are interested in wave propagation effects, the time step should be small enough to capture the wave as it travels through the elements. The mesh should be fine enough to resolve the wave. A general guideline is to have at least 20 elements per wavelength along the direction of the wave.

8.2.3 Model Mesh Size

Figure 8.1(a) shows the phase velocity below 1Mhz on a 1mm thick Aluminum plate. To analyze the symmetric mode with a mesh size of 1mm in plate (20mm wavelength, wave speed is 5400m/s), the frequency of the S0 can be well monitored up to 270 kHz. Plate wave generated by lead break in the top surface of the plate is mainly flexural waves. The
flexural wave is dispersive. Based on the guideline, at 100 kHz, the wave speed is 1000m/s. The wavelength is 10mm, the mesh size should be 0.5mm. We need this 0.5mm mesh size to model flexural wave below 100 kHz. Above 100kHz, we need even smaller mesh size. At 200kHz, $c=1300m/s$, the wavelength is 6.5mm, the mesh size should be 0.325mm.

![Lamb wave phase velocity of Aluminum-2024-T3](image1)

![Lamb wave group velocity of Aluminum-2024-T3](image2)

(a) (b)

Figure 8.1 1mm thick Aluminum plate (a) phase velocity (b) group velocity.

Figure 8.1(b) is the group velocity for 1mm Aluminum plate. In simulation, we have 50 $\mu$s total travel time with 50 mm distance. We can simulate the group wave speed which is high than 1000m/s. That should be able to cover all the flexure waves above 20 kHz.

8.3 PWAS IMPEDANCE

8.3.1 Impedance of Free PWAS Resonator

When excited by an alternating electric voltage, the free PWAS acts as an electromechanical resonator. The modeling of a free PWAS is useful for understanding the electromechanical coupling between the mechanical vibration response and the complex electrical coupling between the mechanical vibration response and the complex electrical response of the sensors. The analytical model equation can be used to predict
the 1-D admittance and impedance frequency response. As the excitation frequency varies, resonance and anti-resonance frequencies are encountered, the admittance and impedance go through $+\infty$ to $-\infty$ transitions. In practice, admittance and impedance magnitudes always display some limited values due to the effects of internal losses inside piezoelectric material.

Harmonic CF-FEM analysis was performed on a circular PWAS and on a square PWAS. The circular PWAS considered in this analysis was 7mm diameter and 0.2mm thick, APC 850 piezoceramic. The square PWAS was 7mm square and 0.2mm thick, APC 850 piezoceramic. The FEM mesh for PWAS is shown in Figure 8.2. The simulation of PWAS was simplified by using the symmetry condition. Only one quarter of the PWAS was modeled. The nodes at the top and bottom surfaces had their VOLTAGE dof coupled to a common master node in order to simulate the existence of electrodes on these PWAS surfaces. This approach simplified the solution process and yields a faster solution.

For rectangular PWAS, both length and width contributed to impedance resonance frequency. Considering the analytical PWAS 1-dimensions bar model, the impedance of length and width can be superposed together to approximately predict the impedance spectrum with analytical model. Using this concept, for a 25 mm x 5 mm x 0.15 mm rectangular PWAS, the approximately analytical impedance after superposition of the length and width impedance, the CF-FEM impedance and the experimental impedance spectra are shown in Figure 8.2(c). The superposition analytical can predict the impedance of rectangle PWAS at low frequency range. The FEM analysis gives more accurate results.
Figure 8.2 Comparison of impedance from theoretical calculation, coupled-field finite element simulation and experimental result, (a) 7 mm diameter, 0.2 mm thickness round APC 850 PWAS; (b) 7mm square, 0.2mm thickness square PWAS; (c) 25 mm x 5 mm x 0.15 mm rectangular PWAS.
When a free PWAS is excited either electrically or mechanically, resonance may happen when the response is very large. These resonances can be of two types: (a) electromechanical resonances (b) mechanical resonances.

Mechanical resonances take place in the same condition as in a conventional elastic structure whereas electromechanical resonances are specific to piezoelectric materials. Electromechanical resonances reflect the coupling between mechanical and electrical variables, happen under electric excitation, which produces electromechanical response. At resonance, the PWAS is drawing very large currents; the admittance becomes very large where impedance goes to zero. The mechanical response at electrical resonance also becomes very large. This happens because the electromechanical coupling of the piezoelectric materials transfers energy from electrical input into the mechanical response. Figure 8.2 shows the log-scale plot of the real part of the electromechanical impedance calculated with the couple-field FEM analysis in the 200 kHz-500 kHz frequency range. To verify these E/M impedance results, we acquired experimental data on an actual PWAS with the HP 4194A Impedance Analyzer. Comparison of theoretical, FEM and experimental impedance of free circular PWAS are shown in Figure 8.2. The analytical and FEM methods can predict the resonance frequency very well. For square PWAS, FEM method impedance spectra agreed with experimental result well.

8.3.2 Impedance of PWAS Modal Sensor

Giurgiutiu and Zagrai (2002) considered a thin, isotropic circular plate with a PWAS surface-mounted at its center. Under PWAS excitation, both axial and flexural vibrations are set in motion. The structural dynamics affects the PWAS response modifies its electromechanical impedance. They use 2-D analysis of PWAS and structure impedance
to model the interaction between the PWAS and the structure and to predict the impedance spectrum that would be measured at the PWAS terminals during the structural process. The experimental specimen is shown in Figure 8.3. The analytical calculation predicted the resonance impedance very well (Figure 8.4 (a)). However, this analytical model is only suit for round disk with a round PWAS perfectly mounted in the center.

![Figure 8.3](image)

**Figure 8.3** Experimental setup (a) illustration of PWAS bonded on a circular plate; (b) test specimen.

![Figure 8.4](image)

**Figure 8.4** Comparison of experimental and analytical impedance of PWAS bonded on a circular plate (Giurgiutiu et al., 2002).
Figure 8.5  Comparison of experimental and FEM impedance of the same structure.

The FEM model gave more control ability to predict the impedance spectra. Figure 8.5 showed FEM model with and without considering the adhesive layer. The adhesive layer reduce the amplitude of impedance and change the resonance frequency as expected. The model with the adhesive layer is more accurate and close to the experimental data.

PWAS can detect subtle changes in the high-frequency structural dynamics at local scales. The local changes in the high-frequency structural dynamics are associated with the presence of incipient damage. Giurgiutiu and Zagrai tested impedance of the simulated crack on circular plates (Figure 8.6). The analytical model is not applicable for structure with crack. The FEM model can be use to model the incipient damage in this situation. The FEM model impedance with crack is compared with the undamaged FEM impedance (Figure 8.7).
Figure 8.6  (a) PWAS sample bonded on a circular plate with crack at 25mm from center  
(b) detail mesh in crack area.

Figure 8.7  Comparison of the experimental results with FEM model impedance with and  
without incipient crack.
8.3.3 Impedance Model of PWAS Array

In damage detection experiment on thin-plate structure by using guided waves, Giurgiutiu, Bao and Zagrai (2006, US patent 6,996,480 B2)) utilized guided Lamb wave embedded ultrasonic structure radar (EUSR) with PWAS. They used square PWAS bonded onto the structure. The sensor layout is shown in Figure 8.8. The consistence of PWAS bond is essential to damage detection. We used the impedance method to check the bond consistence.

\[
\begin{array}{cccccc}
\square & \square & \square & \square & \square & \square \\
\end{array}
\]

\[d = 9\text{mm} \quad 8\text{ PWAS, 7-mm sq}
\]

(a) \hspace{1cm} (0.36 \text{ in}) \hspace{1cm} (0.276 \text{ in sq.})

Figure 8.8 PWAS array (a) 9-element PWAS array with 7-mm square PWAS.

PWAS array installed one by one (Figure 8.8) has its advantage; the nearby PWAS in the array will not affect each other directly. But it is hard to install them consistently. An alternative way is to install a whole PZT with deposited electrode pattern as PWAS arrays. Figure 8.9 showed the 2-D PWAS array with thirty-two 7-mm square electrodes on top. The 2-D PWAS array has its advantage for easy installation and consistent performance. It is more complicated because the nearby PWAS constrained each other.
and affected the impedance. To study the constrain affect to the impedance, the FEM models of a single PWAS (no constrain), PWAS electrode at a corner (2-side constrain) and PWAS electrode at center (4-side constrain) were made to generate the impedance spectra. It is shown in Figure 8.10(a). The impedance spectra of these models are shown in Figure 8.10(b).

![Figure 8.10 Model and impedance spectra of the free PWAS array without bonding on plate (a) model of free PWAS, PWAS electrode at a corner and PWAS electrode in the center of array (b) impedance spectrum.](image)
8.4 PWAS Resonator Mode Shapes

PWAS consists of PZT material (APC 850) with its polarization direction aligned along the same direction. Electrodes are placed on the two surfaces orthogonal to the polarization axis. A modal analysis can determine the coupled-mode (breathing-type deformation) natural frequencies for the short circuit (resonance) case and the open circuit (anti-resonance) case. The electrode regions represent equal potential surfaces and are not modeled explicitly. For the short-circuit case the top and bottom electrodes are grounded (voltages are set equal to zero). For the open-circuit case only the bottom electrode is grounded. The short-circuit case represents excitation by potential while the open-circuit case represents excitation by charge.

A one-quarter symmetry sector is modeled with symmetry boundary conditions applied. The mesh density selected for analysis along the axes (X, Y, Z) are (20, 20, 2) elements respectively. All active displacement degrees of freedom are selected as master degrees of freedom. All non-specified voltage degrees of freedom are condensed out during matrix reduction to allow for electro-elastic coupling.

The modes that produce a breathing-type deformation pattern indicate the desired results. The modal analysis gave both mechanical and electromechanical resonances. The mechanical resonance and antiresonance occurred at the same frequency. However, the electromechanical resonance and antiresonance are not at the same frequency.
Figure 8.11 PWAS resonator electro-mechanical resonance mode shapes and admittance.
Figure 8.12 PWAS electro-mechanical anti-resonance mode shapes and impedance.
Table 8.1 shows the natural frequencies of a 7mm square 0.2mm thick PWAS resonance and anti-resonance frequency in the range from 200 kHz to 500 kHz. The bold numbers indicates that they are electromechanically resonances and anti-resonances.

At each electromechanical resonance of 7mm square PWAS between 100 kHz and 900 kHz, the deformed shape is shown in Figure 8.11. The anti-resonance deformed mode shape is shown in Figure 8.12. Although the admittance spectrum peaks are not in the same amplitude, they are indeed independent modes because the mode shape patterns are different.

### 8.5 PWAS Electric Field

Consider an infinite continuous plate dielectric medium with thickness \( t \), with a continuous electrode on one surface, and a circular electrode with a finite radius \( r_s \) on the other surface. When a voltage difference is created between the two electrodes, the material that receives the highest electrical field flux will be located near the axis of symmetry between the two electrodes. As \( r_s \) becomes large relative to \( t \) the percentage of the flux which flows normal and between the two electrodes increases. A simplified drawing of the electric field flux profile is given in Figure 8.13.
An interdigital transducer (IDT), or interdigitated transducer, is a device which consists of two interlocking comb-shaped metallic coatings (in the fashion of a zipper) which are applied to a piezoelectric substrate (Figure 8.14(a)). IDTs are primarily used to convert surface acoustic waves (SAW) applications. When IDT are applied to piezoelectric substrate, the electric field study between two adjacent fingers is needed for the relationship among finger width, gap and substrate thickness. Figure 8.14 (b) demonstrate the cross-section display of electric field generated by adjacent fingers with coupled field finite analysis.

Figure 8.14 (a) Typical IDT pattern (top view), (b) Multi-physics FEM simulation of the cross-section electric field lines and remnant polarization after poling.
A PZT ceramic is a mass of perovskite crystals consisting of a small, tetravalent metal ion, titanium and zirconium in a lattice of larger, divalent metal lead ions and oxygen ions. Above a critical temperature, the Curie point, each perovskite crystal element exhibits a simple cubic symmetry with no dipole moment. At temperatures below the Curie point, however, each crystal has tetragonal symmetry and a dipole moment. Adjoining dipoles form regions of local alignment called domains. The alignment gives a net dipole moment to the domain, and thus a net polarization. The direction of polarization among neighboring domains is random, however, so the ceramic element has no overall polarization.

The domains in a PZT ceramic element are aligned by exposing the element to a strong, direct current electric field, usually at a temperature slightly below the Curie point. Through this polarizing (poling) treatment, domains most nearly aligned with the electric
field expand at the expense of domains that are not aligned with the field, and the element lengthens in the direction of the field. When the electric field is removed most of the dipoles are locked into a configuration of near alignment. The element now has a permanent polarization, the remanent polarization, and is permanently elongated.

The poling process with SAW-type array was modeled using the coupled-field finite element method. The thin-film poling process with SAW-type array is different from the traditional ceramic disc that has a uniform polarization after poled by uniform electric field. SAW-type array has the small interdigitated electrodes on one side of substrate. A 2D model of the cross-section of the substrate was considered to analyze the effects of electrode width and film thickness. The cross-section was divided to small elements and each element was considered as a domain. Before poling, the net polarization in each domain had random direction. After adding DC voltage on the electrodes, the electric field was calculated based on the PZT material with random polarization. Assuming the polarization of material was properly aligned with the calculated electric field, the new electric field based on aligned polarization material was calculated. After several iterations, the electric field and material polarization got the same direction. The remnant polarization is permanently set after the electric field is removed. Figure 8.14 showed the remnant polarization of FEM simulation. Two 1 \( \mu \text{m} \) width electrodes with 1 \( \mu \text{m} \) space were on surface of a 2 \( \mu \text{m} \) thick piezo ceramic. The left electrode was connected to ground and a 3V DC voltage was applied to the right electrode.

The PWAS poling process was performed with the experimental setup show in Figure 8.16. The PWAS sat in the silicon oil bath. The high voltage DC amplifier was connected to a DC power supply to give the necessary poling voltage. The first experiment is to
measure how capacitance changes with temperature. The Curie temperature of PWAS is 325°C. The PWAS was heated to its Curie temperature with 50°C increment each step and the capacitance was recorded. After it reached the Curie temperature, PWAS was cooled down to room temperature. The capacitance at each mark point was also measured.

Figure 8.16 Experimental setup for PWAS poling.

Figure 8.17 showed how capacitance is related to the temperature. Group 1 indicated that the free PWAS with manufacture poling was heated from room temperature to its Curie temperature. After it reached Curie temperature, the capacitance meter had no reading anymore. Group 2 showed how the capacitance changed when domains’ remnant polarization was random. All the reading was from the free PWAS in the silicon oil bath. A noticeable change was found at room temperature, the free PWAS capacitance reduced from 3nF to 2.5nF after heating over the Curie temperature.
Poling process was made to a single PWAS in the silicon bath as well. Before each poling, the PWAS was heated to the Curie temperature and cooled down to neutral the piezoelectricity. Several poling conditions with different poling voltage, poling temperature and time are tested. It is shown in the Table 8.2. Voltage was applied to the neutral PWAS and heated to the designed temperature. After it reaches the designed temperature, the heat plate was shut down and let it cool with constant poling voltage. The voltage supplier maintained until it dropped to 50C All the readings were recorded after it cooled down to room temperature.

The impedance reading is below. It looks like all the poling condition can get back the piezoelectricity. Somehow 100V at Curie temperature looks like the best poling condition based on these trials. It was close to the original manufacture baseline. A replicated test for 100V confirmed it.
<table>
<thead>
<tr>
<th>Poling voltage (V)</th>
<th>Electric field (kV/mm)</th>
<th>Temperature (C)</th>
<th>Poling Time (h)</th>
<th>Capacitance (nF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>2.79</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>0.5</td>
<td>325 (Curie)</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>300</td>
<td>1.5</td>
<td>325 (Curie)</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>2.5</td>
<td>100</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 8.18 Measured PWAS impedance after different poling conditions. (a) 100 kHz to 900 kHz; (b) 1 Mhz to 15 MHz
8.7 PITCH-CATCH FEM ANALYSIS

The pitch-catch was modeled using FEM in two parts: 2-D model and 3-D model. In 2-D model, the beam and PWAS is z-invariant and can be considered as infinite in the z-direction. The PWAS and beam are along the x-axis. The PZT polarization is along the y-axis. According to the model size effect, the mesh size used here is 0.5 mm. A mesh plot with 7-mm PWAS on a 1-mm thick plate is shown in Figure 8.19.

![Mesh plot of PWAS attached to a bar.](image)

On a 1220-mm long, 1-mm thick aluminum beam, PWAS transmitter is at 500 mm from left edge and receiver is 700 mm from the left edge. A harmonic analysis in ANSYS was evaluated to show the FRF of receiver output voltage vs. transmitter input voltage. The FRF is shown in Figure 8.20. Due to the large length of beam, the FRF shows a lot of resonance peak in the range.

![FRF in 2D FEM model.](image)
Using the transient analysis in ANSYS, a 3-count tone-burst voltage signal was applied to the transmitter. We monitored the receiver voltage output. It is clear that S0 and A0 wave are separated. S0 is not dispersive and A0 is dispersive.

![Receiver output](image)

**Figure 8.21** Receiver voltage response under a 3-count 100 kHz tone-burst excitation.

In 3-D model, the PZT polarization is along the z-axis. The free-free aluminum alloy 2024 beam dimension is 1220 mm long, 40 mm wide, and 1 mm thick. The PWAS dimension is 5 mm long, 40 mm wide and 0.3 mm thick. The PWAS location are: transmitter 500mm from the left edge; receiver 700mm from the left edge of beam. An experimental setup is shown in Figure 8.22

![Experimental setup](image)

**Figure 8.22** Pitch-catch experimental setup. In comparison with FEM results.
Figure 8.23 FRF in 3-D FEM model.

Figure 8.24 Comparison of FEM and experimental result of receiver output signal.
Both harmonic and transient analysis was done in ANSYS. Harmonic analysis gives the FRF of output voltage vs. input voltage in Figure 8.23. Figure 8.24 shows the comparison of FEM and experimental receiver output voltage. The FEM result and experimental result are very close that indicates the FEM analysis is similar to the real experiment. In comparison of 2-D and 3-D model, we notice that in 3-D model shows the reflection from the beam width edge in the receiver signal.
PART II

PWAS Installation and Durability Study
9 PWAS INSTALLATION AND QUALITY CHECK

Sensor installation on the health-monitored structure is an important step that may have significant bearing on the success of the health monitoring process. The development of a reliable and repeatable installation method, which would provide consistent results, was one of the major objectives of PWAS development.

In the installation process, the adhesive used to bond the sensor to the structure plays a crucial role. The thickness and stiffness of the adhesive layer can significantly influence the sensor’s capability to excite the structure and may affect the quality and repeatability of the E/M impedance signatures. Adhesives are appropriate and convenient for short-term experiments, though their performance degrades under prolonged environmental exposure which has been proved by the durability test below. For long-term environmental exposure, other adhesive types (e.g., epoxy) may be more appropriate.

The durability and survivability issues associated with various environmental conditions on PWAS were explored for structural health monitoring (SHM). The durability and survivability of the PWAS transducers under various exposures (cryogenic and high temperature, temperature cycling, outdoor environment, operational fluids, large strains, fatigue load cycling) were considered over a long period time. Both free PWAS and bonded PWAS on metallic structural substrates were used. Different adhesives and protective coatings were compared to find the candidate for PWAS application in SHM.
In most cases, PWAS survived the tests successfully. The cases when PWAS did not survive the tests were closely examined and possible causes of failure were discussed.

### 9.1 State of the Art

Piezoelectric wafer active sensors can send and receive ultrasonic Lamb waves and determine the presence of cracks, delaminations, disbands, and corrosion. In recent years investigators (Chang (1995,1998), Wang and Chang (2000), Lin and Yuan (2001), Ihn and Chang (2002), Giurgiutiu (2008), and others) have explored the generation and detection of structural waves with PWAS. These successful experiments have positioned PWAS as an enabling technology for the development and implementation of active SHM systems. Crawley and de Luis (1987), Sirohi and Chopra (2000), Giurgiutiu (2008) studied the shear lag effect caused by a bond layer. A modified E/M impedance model by incorporating the effect of bond layers was proposed by Xu and Liu (2002). The durability and survivability of PWAS in different aspects were investigated by many researchers. Park et al. investigated the degradation or failure of PWAS and its bond with structure may affect the impedance readings and the Lamb wave propagations (2006). Isogai have done the study of piezoelectric actuator responsive to environmental humidity (1994). Wang have studied the piezoelectric ceramics stress corrosion cracking in water, methanol and formamide at a constant load test (2003). Qing et al determine the survivability and functionality of a PZT SMART Tape at cryogenic temperature (2006,.2008) Sakai have found out that the coercive field and mechanical strength of PZT can be improved by doping various additives to improve the durability of PZT ceramics(1995). Another study of the effect of thermal cycling on the performance of PWAS bonded to aluminum plates have been done by Blackshire et al. (2005). It was
observed that the amplitude of the excited elastic wave reduced after thermal cycling. However, durability and survivability of PWAS and its bond with structure has not been well established.

9.2 RELIABLE PWAS INSTALLATION AND PWAS QUALITY STUDY

In the installation process, the adhesive used to bond the sensor to the structure plays a crucial role. The thickness and stiffness of the adhesive layer can significantly influence the sensor’s capability to excite the structure and may affect the quality and repeatability of the E/M impedance signatures. Adhesives are appropriate and convenient for short-term experiments, though their performance degrades under prolonged environmental exposure which has been proved by the durability test below. For long-term environmental exposure, other adhesive types (e.g., epoxy) may be more appropriate. The use of conductive epoxy can ignore its contribution to sensor-structure interaction but it shorts the electrode Cyanoacrylate unexpected. The cyanoacrylate, epoxy and conductive adhesive used in this study was M-Bond 200, AE-10 and conductive epoxy CW2400 from Measurements Group, Inc and ITW Chemtronics.

9.2.1 Installation Procedure

The sensor installation procedure followed the general steps mentioned in the strain gage installation instruction bulletin, with some modifications introduced to account for the rigidity and fragility of the PZT material. The complete procedure that was elaborated for bonding piezoelectric active sensors is followed below.
Figure 9.1 Installation procedure for piezoelectric active sensors.

For developing a chemically clean surface, the degreasing is performed to remove oils, organic contaminants, etc. The CSM – 1 degreaser was used for this purpose. The surface abrading is needed to remove oxides, coatings, etc. In the case of metallic specimens used in the experiments, the roughness of the surface was not the critical issue and skipping the abrading in the installation procedure did not introduce any noticeable changes in PWAS readings. However, for each particular application the need for surface abrading should be considered. Depending on surface roughness the silicon-carbide paper of a suitable grit can be used. In many cases, it is desirable to designate the sensor location and orientation with layout lines. 309D Instruction Bulletin recommends that the reference or layout lines should be burnished, rather than scored or scribed, on the surface. For the metallic specimens a drafting pencil gave satisfactory results. The
purpose of surface conditioning is to remove the residue of layout lines drafting. This can be done with Conditioner A that is available from Measurements Group Inc. and cotton-tipped applicators or a gauze sponge. The surface preparation is finished with neutralizing, which provides optimum alkalinity for M-Bond 200 adhesive. M-Prep Neutralizer 5A was applied on the surface, which then was scrubbed with cotton-tipped applicators. Finally, the surface was dried by wiping it with a gauze sponge. The detailed description of surface preparation is given in Instruction Bulletin 309D (Measurements Group Inc., 1992).

Figure 9.2 The installation kit for strain gages (Measurements Group, Inc) was used in the bonding of piezoelectric active sensors.

9.2.2 Adhesives

During bonding, particular attention should be paid to the handling of PWAS. PZT material is very brittle and the sensor can be easily broken. This is especially true for thin
PWAS larger than 10mm. It is recommended that sensors should always remain on the supporting foam until they are bonded. The cyanoacrylate adhesive M-Bond 200 requires application of a catalyst. The catalyst is applied in a thin uniform coat to cover the surface of the sensor, which will be bonded to the host structure. After the catalyst coat is dried out (usually 1-5 minutes), the adhesive drop is placed on a surface of the structure where the PWAS location is designated with layout lines. Using a cotton-tipped applicator, the sensor is aligned to the layout lines. This should be done very quickly before adhesive is solidified. When the sensor is properly aligned, the thin (2-5mm) layer of rubber is used for a safe interface between the sensor and a small mechanical clip. The clip provides appropriate pressure to ensure a good bond and a thin adhesive layer between the PWAS and the host structure. For the metallic specimens, the thickness of the cyanoacrylate adhesive layer is much smaller than the thickness of the PWAS itself and its effect was not considered in the analysis. It was observed that adequate bonding is developing in 3 hours at room temperature. However, the best results were obtained after 24 hours. The excess of the solidified adhesive can be removed mechanically with fine tools and chemically with acetone. Acetone cleaning should be done carefully to prevent its leakage under the edges of the sensor, which sometimes causes disbonding. After installation, the sensor capacitance was measured again and checked for consistency against the free-sensor capacitance on file.

The epoxy adhesive M-Bond AE-10 has two-component, 100%-solid epoxy system. Surface preparation for AE-10 is the same as before. Particular attention should be paid to get maximum performance. From instruction bulletin B-137, a cure time as low as six hours at +75°F (+24°C) may be used to get 6% elongation capability. Elongation
capability of approximately 10% can be obtained by extending the cure time to 24 to 48 hours at +75°F. To mix, fill one of the calibrated droppers with Curing Agent 10 exactly to the number 10 and dispense the contents into the center of the jar of Resin AE. Mix thoroughly for 5 minutes, using the plastic stirring rod. The pot life or working time after mixing is 15 to 20 minutes. Discard the dropper after use.

Circuitworks conductive epoxy has two parts: A and B. Surface preparation for conductive epoxy is the same as before. After cleaning the surface, mix equal amounts of Part ‘A’ and Part ‘B’. Mix thoroughly for at least 2 minutes and apply to clean surface within 5 minutes. Room temperature curing can be achieved in 4 hours at or above 75°F. For faster curing times, maximum conductivity and adhesion, heat the bond to between 150°F -250 °F for 10 minutes and allow to cool.

9.3 Quality Check

It is well known that PWAS affixed to, or embedded into, the structure play a major role in the successful operation of the health monitoring and damage detection system. Integrity of the sensor and consistency of the sensor/structure interface are essential elements. The general expectation is that, once PWAS have been placed on or into the structure, they behave consistently throughout the duration of the health monitoring process. For real structures, the duration of the health monitoring activity is extensive and can span several decades. It also encompasses various service conditions and several loading cases. Therefore, in-situ self-diagnostics methods are mandatory. The PWAS array should be scanned periodically as well as prior to any damage detection cycle.
Self diagnostic methods and quality check for assessing active sensor integrity are readily available with the E/M impedance technique. The piezoelectric active sensor is predominantly a capacitive device that is dominated by its reactive impedance $1/i\omega C$. It was found experimentally that the reactive (imaginary) part of the impedance ($\text{Im } Z$) can be a good indication of active sensor integrity. Base-line signatures taken at the beginning of endurance experiments when compared with recent reading could successfully identify defective active sensors. The $\text{Im } Z$ spectrum of a well-bonded PZT sensor with that of a disbond (free) sensor was compared. The appearance of sensor free-vibration resonance and the disappearance of structural resonances constitute un-ambiguous features that can be used for automated sensor self-diagnostics.

We have developed a quality-controlled PWAS fabrication method that ensures consistent results with narrow statistical spread. PWAS fabrication starts with small size piezoelectric wafers, which are first equipped with minute terminals, and then applied to the structure through a rigorously controlled procedure. PWAS quality control consists in a very careful measurement and recording of the PWAS geometrical, mechanical, electrical, and electromechanical properties; the sorting of PWAS in tightly controlled classes of intrinsic properties; and the monitoring of these properties through the fabrication process and the installation procedure. Initial efforts by Zagrai have indicated that a rigorous fabrication and quality control methodology is needed to assure that PWAS present a consistent intrinsic behavior. In order to reduce uncertainties, we identified vendors of small-size piezoelectric wafers, produced with precision technologies. The PZT wafers produced by American Piezo Ceramics, Inc. and Steminc were selected based on their low cost and good manufacturing tolerances (Table 9.1).
The PWAS quality check procedure consists of measuring the sensor’s geometrical dimensions, capacitance, and dynamic characteristics (i.e. E/M admittance or impedance spectrum in the resonance neighborhood). If the information about the host structure will be used later, the desirable characteristics are measured.

<table>
<thead>
<tr>
<th>Manufacture</th>
<th>APC</th>
<th>STEMINC</th>
<th>Piezo systems</th>
<th>Staveley</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material type</td>
<td>850</td>
<td>SM-D</td>
<td>SM-K</td>
<td>SM-QA</td>
</tr>
<tr>
<td>d31</td>
<td>-175</td>
<td>-320</td>
<td>-210</td>
<td>-150</td>
</tr>
<tr>
<td>d33</td>
<td>400</td>
<td>650</td>
<td>450</td>
<td>320</td>
</tr>
<tr>
<td>price round</td>
<td>$1.74</td>
<td>$0.3</td>
<td>$2.09</td>
<td>$5.7</td>
</tr>
<tr>
<td>price square</td>
<td>$1.88</td>
<td>$0.3</td>
<td>$1.44</td>
<td>$7</td>
</tr>
</tbody>
</table>

### 9.3.1 Free PWAS Quality Check

Quality check of free APC and Steminc PWAS were done by Dr. Zagrai, James Kendall and Bin Lin. Twenty-five APC-850 piezoelectric wafers and twenty-five Steminc piezoelectric wafers were acquired and subjected to statistical evaluation. The mechanical and electrical properties declared by the vendor were compared with our own measurements to verify the vendor data and evaluate its veridicality. Standardized test methods for in process quality assurance of active sensor fabrication process were developed based on the following measurements, geometrical dimensions, electrical capacitance and electromechanical (E/M) impedance and admittance spectra.

The intrinsic E/M impedance and admittance spectra of the PWAS, before being attached to the structure, were measured with the HP 4194A Impedance Phase-Gain Analyzer. Center clamping conditions were simulated and the wafer could perform free vibrations while being tested. To obtain the intrinsic E/M impedance and admittance spectra, the
PWAS were tested in the 100Hz — 12MHz frequency range using the HP 4194A Impedance Analyzer. The data was collected through the GPIB interface and processed in MS Excel.

Figure 9.3  Impedance characteristic of a free 7-mm square PWAS: (a) real part of impedance; (b) imaginary part of impedance.
9.3.2 EUSR Specimen Quality Study

9.3.2.1 EUSR Experimental Setup

EUSR can detect the damage and located it graphically. The basic experimental setup is shown in Figure 9.4. The sensor layout is shown in Figure 9.5. For EUSR application, each PWAS need to be used as actuator and receiver. The consistence of PWAS bond is essential to the correct damage detection.

![Diagram of EUSR experimental setup]

Figure 9.4 EUSR experimental setup.
9.3.2.2 Capacitance Check

We first checked the capacitance of each sensor on plate 1 and compared with theoretical free capacitance.

Table 9.2 shows the PWAS thickness, diameter, length, area, relative dielectric constant $\varepsilon_r$, permittivity of free space $\varepsilon_0$, and the capacitance that can be estimated using equation $C = \varepsilon_r \varepsilon_0 A / t$.

<table>
<thead>
<tr>
<th>Thickness (mm)</th>
<th>Dia/length (mm)</th>
<th>Area (mm²)</th>
<th>$\varepsilon_r$</th>
<th>Capacitance (nF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Round</td>
<td>0.2</td>
<td>7</td>
<td>38.4</td>
<td>1750</td>
</tr>
<tr>
<td>Square</td>
<td>0.2</td>
<td>7</td>
<td>49.0</td>
<td>1750</td>
</tr>
</tbody>
</table>
Table 9.3 shows the capacitance for the 7 mm square PWAS array on Plate #1 and 7 mm round PWAS array on Plate #2. There are a total of eight PWAS in the array on each plate. The experimental bonded PWAS capacitance is lower than the experimental free PWAS capacitance. The adhesive layer forms a new capacitance and serially connects with PWAS that reduces the capacitance.

Table 9.3  PWAS capacitance on plate #1 and #2

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>C (nF) (Square)</td>
<td>2.85</td>
<td>2.88</td>
<td>2.91</td>
<td>2.84</td>
<td>2.69</td>
<td>2.76</td>
<td>2.8</td>
<td>2.7</td>
<td>2.80</td>
</tr>
<tr>
<td>C (nF) (Round)</td>
<td>2.67</td>
<td>2.71</td>
<td>2.68</td>
<td>2.65</td>
<td>2.63</td>
<td>2.68</td>
<td>2.71</td>
<td>2.71</td>
<td>2.68</td>
</tr>
</tbody>
</table>

Table 9.4 Max and Min values for the magnitude of the impedance and admittance of the 7 mm square PWAS on plate #1

| PWAS # | |Z| (Ohms) | |Y| (Siemens) |
|--------|-------------|-------------|
|        | Max | Min | Max | Min | Max | Min |
| 0      | 5583.9 | 9.97 | 0.100 | 0 |       |
| 1      | 5543.7 | 16.87 | 0.058 | 0 |       |
| 2      | 5480.8 | 17.92 | 0.055 | 0 |       |
| 3      | 5587.5 | 42.89 | 0.025 | 0 |       |
| 4      | 5972.8 | 11.25 | 0.088 | 0 |       |
| 5      | 5742.5 | 16.18 | 0.061 | 0 |       |
| 6      | 5697.1 | 12.39 | 0.080 | 0 |       |
| 7      | 5759.3 | 14.28 | 0.070 | 0 |       |

9.3.2.3 Impedance Spectrum Check

The E/M impedance and admittance spectra of the PWAS, after being attached to the structure, were measured with the HP 4194A Impedance Phase-Gain Analyzer as baseline data. The wafer could perform in-plate vibrations while being tested. To obtain the E/M impedance and admittance spectra, the PWAS were tested in the 100Hz —
2MHz frequency range using the HP 4194A Impedance Analyzer. The data was collected through the GPIB interface and processed in MS Excel.

Figure 9.6 Real and imaginary impedance of 7-mm Square PWAS in EUSR array on Plate 1.
Figure 9.7  Real and imaginary Admittance of 7 mm Square PWAS in EUSR array on Plate 1.
PWAS need to be permanently installed onto the monitored structure. Sensor installation on the health-monitored structure is an important step that may have significant bearing on the success of the health monitoring process. In the installation process, the adhesive used to bond the sensor to the structure plays a crucial role. The thickness and stiffness of the adhesive layer can significantly influence the sensor’s capability to excite the structure and may affect the quality and repeatability of the E/M impedance signatures.

![PWAS attached to a substrate structure through the adhesive bond layer.](image)

The installation procedure for PWAS has been developed in previous chapter, but there are still some uncertainties in the current procedure. The development of a reliable, repeatable and optimum installation method, which would provide consistent results, was one of the major objectives of this experiment.
The design of experiments method is used next to evaluate the PWAS installation factors and find the reliable PWAS installation procedure.

10.1 Introduction of Design of Experiment

Design of Experiments (DOE) is a systematic approach to investigation of a system or process. A series of structured tests are designed in which planned changes are made to the input variables of a process or system. The effects of these changes on a pre-defined output are then assessed. DOE is important as a formal way of maximizing information gained while resources required. It has more to offer than 'one change at a time' experimental methods, because it allows a judgment on the significance to the output of input variables acting alone, as well input variables acting in combination with one another.

DOE can be used to find answers in problem solving, parameter design and robustness study. In each case, DOE is used to find the answer. DOE technique starts with identifying the input variables and the response (output) that is to be measured. For each input variable, a number of levels are defined that represent the range for which the effect of that variable is desired to be known. An experimental plan is produced which tells the experimenter where to set each test parameter for each run of the test. The response is then measured for each run. The method of analysis is to look for differences between response (output) readings for different groups of the input changes. These differences are then attributed to the input variables acting alone (called a single effect) or in combination with another input variable (called an interaction).
DOE techniques enable designers to determine simultaneously the individual and interactive effects of many factors that could affect the output results in any design. DOE also provides a full insight of interaction between design elements; therefore, it helps turn any standard design into a robust one. DOE helps to pinpoint the sensitive parts and sensitive areas in designs that cause problems in Yield. Designers are then able to fix these problems and produce robust and higher yield designs prior going into production.

In order to draw the maximum amount of information a full matrix is needed which contains all possible combinations of factors and levels. If this requires too many experimental runs to be practical, fractions of the matrix can be taken dependent on which effects are of particular interest. The fewer the runs in the experiment the less information is available.

In order to draw statistically conclusions from the experiment, it is necessary to integrate simple and powerful statistical methods into the experimental design methodology. Fisher (1935) illustrated the principles of experimental design such as comparison, randomization, replication and blocking, orthogonality and factorial design. These principles can be utilized to improve the efficiency of experimentation. These principles of experimental design are applied to reduce or even remove experimental bias. Here are some backgrounds of the statistical concepts used in this chapter.

- **Comparison**

In many fields of study it is hard to reproduce measured results exactly. Comparisons between treatments are much more reproducible and are usually preferable. Often one compares against a standard or traditional treatment that acts as baseline.
• Randomization

Randomization is a core principle in statistical theory. In the statistical theory of design of experiments, randomization involves randomly allocating the experimental units across the treatment groups. Randomization reduces bias by equalizing other factors that have not been explicitly accounted for in the experimental design (according to the law of large numbers).

• Replication

Measurements are usually subject to variation, both between repeated measurements and between replicated items or processes. Multiple measurements of replicated items are necessary so the variation can be estimated.

• Blocking

Blocking is the arrangement of experimental units into groups (blocks) that are similar to one another. Blocking reduces known but irrelevant sources of variation between units and thus allows greater precision in the estimation of the source of variation under study.

• Orthogonality

When performing statistical analysis, variables that affect a particular result are said to be orthogonal if they are uncorrelated. That is to say that by varying each separately, one can predict the combined effect of varying them jointly.

• Factorial design

The most commonly used experimental designs are full and fraction factorial designs at two-levels and three-levels. Factorial designs would enable an experimenter to study the
joint effect of the factors (or process / design parameters) on a response. A full factorial designed experiment consists of all possible combinations of levels for all factors. The total number of experiments for studying k factors at two-levels is $2^k$. The $2^k$ full factorial design is particularly useful especially when the number of process parameters (or factors) is less than or equal to 4.

- **Pareto methods**

Pareto analysis is a statistical technique in decision making that is used for selection of a limited number of tasks that produce significant overall effect. It uses the Pareto principle – the idea that by doing 20% of work you can generate 80% of the advantage of doing the entire job. Or in terms of quality improvement, a large majority of problems (80%) are produced by a few key causes (20%).

- **Normal probability plot**

The normal probability plot is a graphical technique for normality testing: assessing whether or not a data set is approximately normally distributed. The data are plotted against a theoretical normal distribution in such a way that the points should form an approximate straight line. Departures from this straight line indicate departures from normality.

- **Main effects plot**

A main effect plot is a plot of the mean response values at each level of a design parameter or process variable. One can use this plot to compare the relative strength of the effects of various factors. The sign of a main effect would tell us the direction of the
effect, i.e. if the average response value increases or decreases. The magnitude tells us of the strength of effect.

- Interactions plot

An interactions plot uses graphical tool which plots the mean response of two factors at all possible combinations of their settings. If the lines are parallel, then there is no interaction between the factors. Non-parallel lines is an indication of the presence of interaction between the factors.

10.2 DOE ANALYSIS OF PWAS INSTALLATION

This research is based on a study of the effects of four factors on the installation of PWAS in SHM. The adhesive type, PWAS polarization, surface preparation and curing method are considered as factors to compare the thickness of the adhesive layer, capacitance, and pitch-catch signal amplitude. This four-factor full factorial design will provide some basic knowledge about PWAS installation and guidance for future improvement of the PWAS installation procedure.

10.2.1 Design of Experiment and Experimental Setup

10.2.1.1 DOE Factors, Levels of Parameters

PWAS have a poled structure and remain polarized after manufacture curing. PWAS have top and bottom electrodes. According to the datasheet, 0.2mm PWAS can withstand -30 to 120 Volts without deteriorating performance. Normally I bond the PWAS with “+” polarization to afford high voltage. However, sometimes the PWAS polarization mark is
not clear or PWAS may be accidentally bonded on the reverse side. In this experiment, the sensor polarization is the first factor to be tested.

Two kinds of adhesive were tested. Vishay M-200 is a modified alkyl cyanoacrylate compound that has fast room-temperature cure and ease of application. It is weakened by humidity, and the normal operating temperature is -30 to 65°C. M-bond AE-15 is a 100%-solids epoxy adhesive that requires moderately elevated curing temperatures.

The curing process is important due to the requirements for AE-15. A vacuum bag with heating blanket can provide the pressure and temperature requirements for proper AE-15 curing.

Surface preparation normally will provide a reliable bonding. Basic cleaning just removes the dirt on the surface with degreaser. A deep cleaning procedure was followed by the instructions provided by the adhesive manufacture. Four main factors were studied at two levels (Table 10.1).

<table>
<thead>
<tr>
<th>Factor</th>
<th>Level 1</th>
<th>Level 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Sensor polarization</td>
<td>Reverse</td>
<td>Normal polarization</td>
</tr>
<tr>
<td>B: Adhesive type</td>
<td>M200</td>
<td>AE15</td>
</tr>
<tr>
<td>C: Curing procedure</td>
<td>No vacuum and heating</td>
<td>Vacuum and heating</td>
</tr>
<tr>
<td>D: Surface preparation</td>
<td>Basic cleaning</td>
<td>Deep cleaning</td>
</tr>
</tbody>
</table>

The responses of interest to the experiment were the thickness of the adhesive layer, the capacitance reading of installed PWAS, and the pitch and catch signal amplitude of PWAS. The details will be discussed in the experimental setup section.
10.2.1.2 Experimental Setup

A reference PWAS by APC international Inc. was bonded on the middle of a 1-mm thickness 560mm diameter round Aluminum plate. Sixteen PWAS were bonded randomly the same distance (268 mm) from the reference PWAS. A digital micrometer was used to measure the thickness of the adhesive layer. One capacitance meter and one multimeter were used to get the capacitance data, and they gave slightly different readings.

For the pitch-catch experimental setup, the center reference PWAS was first a transmitter (pitcher) with a smoothed 330 kHz tone-burst excitation with a 10 Hz repetition rate to generate the Lamb wave in the thin plate. The purpose is to test the PWAS Lamb wave sensing ability under different conditions. The center PWAS can be directly connected to an HP33120A function generator with 10V peak-to-peak excitation. The 16 PWAS were used as receivers (catcher) to collect the Lamb wave signals. They were directly connected to a Tektronix digital oscilloscope one at a time using the same probe to reduce the measurement errors.

Figure 10.2 Installation factors DOE experimental setup.
To test the PWAS Lamb wave generating ability as affected by different conditions, the 16 PWAS were used as transmitters one by one, and the center reference PWAS was used as a receiver. In this study, the transmitter were set at the same condition and only the amplitude of receivers was recorded. The peak-to-peak amplitude was recorded as the response.

**10.2.2 Data Collection**

The dimension and capacitance of free PWAS provided by APC international, Ltd. were measured before installation to test the consistency of sensors themselves, as shown in Table 10.2. After bonding with different conditions, the bonded PWAS’ thickness, capacitance and pitch-catch amplitude were recorded and shown in Table 10.3.

**Table 10.2 Measurement of free PWAS used in DOE experiment**

<table>
<thead>
<tr>
<th>Free Sensor</th>
<th>Diameter (mm)</th>
<th>Thickness (mm)</th>
<th>Capacitance (nF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.99</td>
<td>0.206</td>
<td>3.058</td>
</tr>
<tr>
<td>2</td>
<td>7.01</td>
<td>0.205</td>
<td>3.132</td>
</tr>
<tr>
<td>3</td>
<td>6.96</td>
<td>0.202</td>
<td>3.169</td>
</tr>
<tr>
<td>4</td>
<td>7.01</td>
<td>0.199</td>
<td>3.36</td>
</tr>
<tr>
<td>5</td>
<td>6.97</td>
<td>0.203</td>
<td>3.104</td>
</tr>
<tr>
<td>6</td>
<td>7.02</td>
<td>0.199</td>
<td>3.224</td>
</tr>
<tr>
<td>7</td>
<td>6.99</td>
<td>0.214</td>
<td>3.184</td>
</tr>
<tr>
<td>8</td>
<td>6.98</td>
<td>0.206</td>
<td>3.173</td>
</tr>
<tr>
<td>9</td>
<td>7.02</td>
<td>0.209</td>
<td>3.116</td>
</tr>
<tr>
<td>10</td>
<td>6.99</td>
<td>0.202</td>
<td>3.144</td>
</tr>
<tr>
<td>11</td>
<td>7.01</td>
<td>0.199</td>
<td>3.269</td>
</tr>
<tr>
<td>12</td>
<td>6.97</td>
<td>0.202</td>
<td>3.191</td>
</tr>
<tr>
<td>13</td>
<td>6.97</td>
<td>0.197</td>
<td>3.188</td>
</tr>
<tr>
<td>14</td>
<td>6.99</td>
<td>0.198</td>
<td>3.211</td>
</tr>
<tr>
<td>15</td>
<td>7.04</td>
<td>0.204</td>
<td>3.175</td>
</tr>
<tr>
<td>16</td>
<td>7</td>
<td>0.205</td>
<td>3.176</td>
</tr>
</tbody>
</table>

| Mean        | 6.995         | 0.203125       | 3.179625         |
| Std. Dev.   | 0.0222        | 0.00444        | 0.0693           |

201
10.2.3 Statistical Analysis of Free PWAS

The manufacturer states that the tolerances of thickness, diameter and capacitance are 0.2mm ± 0.025 mm, 7mm ± 0.25 mm, 3.0nF ± 20% respectively. Free PWAS in this test are all within the bounds of statistical quality control (Figure 10.3).

Table 10.3 List of factors and responses in factorial design experiment

<table>
<thead>
<tr>
<th>Std Order</th>
<th>Run Order</th>
<th>Sensor</th>
<th>Adhesive</th>
<th>Vacuum</th>
<th>Surface</th>
<th>Adhesive (mm)</th>
<th>C1 (nF)</th>
<th>C2 (nF)</th>
<th>Catch (mV)</th>
<th>Pitch (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>0.021</td>
<td>2.414</td>
<td>2.235</td>
<td>41.5</td>
<td>46.4</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0.009</td>
<td>2.555</td>
<td>2.335</td>
<td>38.5</td>
<td>45.4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>0.001</td>
<td>2.625</td>
<td>2.4</td>
<td>40.1</td>
<td>48.6</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>0.006</td>
<td>2.5215</td>
<td>2.31</td>
<td>41</td>
<td>47.7</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>0.020</td>
<td>2.786</td>
<td>2.6</td>
<td>42.76</td>
<td>46</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>0.003</td>
<td>2.6815</td>
<td>2.125</td>
<td>30</td>
<td>29.8</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0.004</td>
<td>2.6625</td>
<td>2.4</td>
<td>24.85</td>
<td>34.1</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.032</td>
<td>2.14</td>
<td>1.99</td>
<td>41.22</td>
<td>43.4</td>
</tr>
<tr>
<td>12</td>
<td>9</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>0.048</td>
<td>2.525</td>
<td>2.365</td>
<td>43.74</td>
<td>54.6</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>0.042</td>
<td>2.678</td>
<td>2.49</td>
<td>38.35</td>
<td>50.6</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>0.047</td>
<td>2.7085</td>
<td>2.63</td>
<td>36.5</td>
<td>47.9</td>
</tr>
<tr>
<td>14</td>
<td>12</td>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0.021</td>
<td>2.7055</td>
<td>2.415</td>
<td>35.6</td>
<td>42</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
<td>0.012</td>
<td>2.4315</td>
<td>2.21</td>
<td>40.5</td>
<td>35.1</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>0.075</td>
<td>2.7415</td>
<td>2.55</td>
<td>42.35</td>
<td>51.1</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.061</td>
<td>2.601</td>
<td>2.42</td>
<td>41.63</td>
<td>51.7</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>0.002</td>
<td>2.653</td>
<td>2.45</td>
<td>29.68</td>
<td>35.9</td>
</tr>
</tbody>
</table>
Figure 10.3 Statistics graphical summary (a) diameter, (b) thickness, (c) capacitance.
10.2.4 DOE Analysis of Bonded PWAS

The original design is a four-factor full factorial design. Three factors are directly related to the bonding procedure: adhesive type, curing procedure and surface preparation. The sensor polarization determines how much voltage you can apply to the sensor. From the manufacture’s datasheet, a voltage between -30 and 120V can be applied to PWAS. It will affect the pitch signal if we apply a high voltage to send out the wave. However, in this study, the peak-to-peak voltage is 18V without any voltage bias. It can be safely removed and the experiment can be analyzed as a replicated $2^3$ full factorial design. Both $2^4$ and replicated $2^3$ full factorial designs will be discussed below.

10.2.4.1 DOE Analysis of the Adhesive Thickness

Both $2^4$ and replicated $2^3$ full factorial designs give the same results here. Both Pareto analysis and the normal probability plot show adhesive type is a statistically significant factor for the thickness of adhesive layer. Vishay M-bond 200 gives a thinner adhesive layer of about 7.25 $\mu$m on average. Changing from M-200 to AE-15 will increase the adhesive layer’s thickness about 36 $\mu$m on average. Due to the physical properties of AE-15, it is reasonable that epoxy adhesive will provide a thicker interface than alkyl cyanoacrylate.
Figure 10.4 Adhesive thickness based on $2^4$ full factorial designs (a) Pareto analysis (b) Normal probability plot.
Figure 10.5 Adhesive thickness based on $2^3$ full factorial designs (a) Pareto analysis (b) Normal probability plot.
10.2.4.2 DOE Analysis of the Capacitance

After bonding to the Al plate, all the capacitance readings were reduced by about 0.5nF. Both $2^4$ and replicated $2^3$ full factorial designs’ Pareto and normal probability plot did not show any statistically significant effects. Reading from the capacitance meter and multimeter showed some differences between two measurement tools, but no significant effects were found by these two collected data.

10.2.4.3 DOE Analysis of the Pitch-Catch Signal

When testing whether the PWAS Lamb wave generating ability was affected by different conditions, the signal amplitude recorded by the center reference PWAS was used.

Based on $2^4$ full factorial designs, the Pareto analysis and normal probability plot (Figure 10.6) of bonded PWAS suggested that the adhesive type is the most important factor. The interaction among adhesive type, curing procedure and surface preparation also significantly affects the pitch-sending ability. AE-15 can increase PWAS sending signal by 9mV. It can be explained that the epoxy adhesive is thicker and stronger, therefore it can provide a better shear interface between the PWAS and structure compared to M-200. The three-way interaction among adhesive, curing and surface preparation is an interesting phenomenon. To get maximum sending ability we need to use AE-15 and follow the adhesive curing procedure (which may increase the long term durability) without preparing the surface. Alternatively, if we have cleaned the surface, we should not use the vacuum or heating. It is somewhat hard to explain from empirical experience. Separating vacuum and heating into two factors and more replicated DOE experiment will provide us with a better understanding of this aspect in the future.
Figure 10.6 Pitch (Sending from outside) and catch (receiving from center reference PWAS) signal based on $2^4$ full factorial designs. (a) Pareto analysis, (b) Normal probability plot.
Figure 10.7 Pitch (Sending from outside) and catch (receiving from center reference PWAS) signal based on $2^4$ full factorial designs. (a) Adhesive main effect, (b) Interaction plot.
Based on $2^3$ full factorial designs, the Pareto analysis and normal probability plot (Figure 10.8) of bonded PWAS suggested that the adhesive type is the most important factor. The two-way interaction between curing procedure and surface preparation and three-way interaction among adhesive type, curing procedure and surface preparation also significantly affect the pitch-sending ability.

Figure 10.8 Pitch (Sending from outside) and catch (receiving from center reference PWAS) signal based on $2^3$ full factorial designs. (a) Pareto analysis, (b) Normal probability plot, (c) Adhesive main effect, (d) Interaction plot.
Figure 10.9 Catch signal (receiving the pitch from center reference) of $2^4$ full factorial designs; (a) Pareto analysis (b) Normal probability plot.

For PWAS catch-receiving ability, there are no statistically significant factors based on $2^4$ full factorial designs. The Pareto analysis (Figure 10.9) of bonded PWAS suggested that the adhesive type and the interaction among adhesive type, curing procedure and surface preparation were still the most important two factors, but they are not statistically significant. The normal probability plot also cannot find any statistically significant effects for receiving ability.
For replicated $2^3$ full factorial designs, the Pareto analysis and normal probability plot (Figure 10.9) of bonded PWAS suggested that the adhesive type is the most important factor. The interaction among adhesive type, curing procedure and surface preparation also significantly affects the pitch-sending ability.

The explanation is that sending signals is more sensitive to those factors because generating waves have more shear displacement (normally 100 times) than that of receiving waves.
11 DURABILITY AND SURVIVABILITY OF PWAS TRANSDUCERS

PWAS used in this durability tests were circular PWAS with 7mm diameter and 0.2mm thick. The PWAS was made of APC-850 PZT material by APC International, Ltd. PZT APC-850 was chosen because of its material properties (Table 4.3) those are balanced for both actuating and sensing. The dimension and capacitance of free PWAS were measured before installation to test the consistency of sensors. Free PWAS dimensions and capacitances in this experimental setup were all within the bounds of statistical quality control for the tolerances that the manufacturer states.

11.1 OVERVIEW OF PWAS DURABILITY AND SURVIVABILITY STUDIES

A number of studies have been performed to assess PWAS durability and survivability of extreme temperature exposure, exposure to environmental factors (sun, rain, humidity, freeze-thaw, etc.), exposure to water and maintenance fluids, and large strains, and fatigue cyclic loads. The durability and survivability studies of the PWAS under various exposures were considered to verify the durability of PWAS over a long period time. The durability and survivability of the PWAS transducers under various environmental exposures was tested in several stages as follows:

1) Cryogenic temperature

2) High temperature

3) Temperature cycling
4) Outdoor environment

5) Operational fluids

6) Fatigue load cycling

Figure 11.1 Free PWAS and PWAS bonded to a metallic plate.

Both free PWAS and bonded PWAS were studied (Figure 11.1). Free PWAS were tested with two wires attached on both electrodes. The bonded PWAS specimens were made by attaching PWAS onto the center of a 100m diameter 1mm thick Aluminum plate. The PWAS behavior was monitored using the electromechanical (E/M) impedance method. Giurgiutiu et al (2008) established a complete analytical model of free and bonded PWAS on circular plates as function of material properties of PWAS and structures. The analytical model results have been verified by experimental results and finite element simulation. For a free PWAS, the real part of the E/M impedance reflects its free vibration spectrum. For a bonded PWAS, The real part of the E/M impedance
directly reflects the vibration spectrum of the tested specimen through the point-wise mechanical impedance at the point where PWAS is attached to the structure.

In the installation process, the adhesive plays a crucial role. The thickness and stiffness of the adhesive layer between the PWAS to the structure can significantly influence the sensor’s capability to excite the structure and may affect the quality and repeatability of the E/M impedance signatures. Cyanoacrylate adhesives are appropriate and convenient for short-term experiments, though their performance degrades under prolonged environmental exposure which has been proved by the durability test below. For long-term environmental exposure, other adhesive types (e.g., epoxy) may be more appropriate. The use of conductive epoxy can ignore its contribution to sensor-structure interaction but it shorts the electrode unexpected. The cyanoacrylate, epoxy used in this study was M-Bond 200, AE-15 from Measurements Group, Inc. Due to the wide temperature range involved in this experiment, extra care must be taken for the bonding of sensors. The short and long term of the adhesives' operating temperatures are listed in Table 11.1

<table>
<thead>
<tr>
<th></th>
<th>M-Bond 200</th>
<th>M-Bond 610</th>
<th>AE-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short term</td>
<td>-300° to +200°F</td>
<td>-452° to +700°F</td>
<td>NA</td>
</tr>
<tr>
<td>Long term</td>
<td>-25° to +150°F</td>
<td>-452° to +500°F</td>
<td>-452° to +200°F</td>
</tr>
</tbody>
</table>

**11.2 CRYOGENIC TEMPERATURE**

At atmospheric pressure, liquid nitrogen boils at $-321 \, ^\circ F \, (-196 \, ^\circ C)$ and is a cryogenic fluid. Free and bonded PWAS were submerged into the liquid nitrogen and kept for 10
minutes before measurements to test the survivability. The impedance signatures were measured by Hewlett Packard 4194A Impedance analyzer

![Free PZT PWAS](attachment:image.jpg)

**Figure 11.2** Indication of PWAS operability while submersed in liquid nitrogen.

The real part of impedance signature indicated that free PWAS, when subjected to liquid nitrogen at cryogenic temperatures, maintained their E/M impedance (Figure 11.2). The first resonance frequency increased from 344 kHz to 362 kHz. Initial impedance signature of free PWAS was recovered when PWAS was warmed to room temperature.

The short term operation temperature of M-bond 200 is from -300° to +200°F. Bonded PWAS with M-Bond 200 adhesive did not survive the cryogenic cycling. However, AE-15 maintained its ability during cryogenic cycling. Figure 11.3 shows the real part of impedance signatures taken from a bonded PWAS specimen with AE-15 adhesive that was submerged into liquid nitrogen for 10 times. The specimen was submerged in the liquid nitrogen for 10 minutes before each impedance measurement.
Figure 11.3. The real part of impedance indication of operability through retention of resonant properties while submersed in liquid nitrogen.

Figure 11.4 shows the real part of impedance signatures taken from the bonded PWAS specimen at room temperature after each submersion. The real part of the impedance retained its peaks and their relative frequency location. No major E/M impedance changes were recorded for both free and bonded PWAS working at cryogenic temperature. The reverberated response is significantly higher under submersion than in air (e.g. approximately 1800Ω @ 5 kHz under submersion vs. 400Ω @ 5 kHz in air), which indicates more damping when submerged as expected. However, the damping of the resonance peaks is more elaborate to deduce and would need a separate study.
Figure 11.4 The real part of impedance indication of survivability through resumption of resonant properties at room temperature after submersion in liquid nitrogen.

11.3 HIGH TEMPERATURE

Three requirements that must be considered for high temperature piezoelectric applications would be the Curie transition temperature, the pyroelectric properties, and the ferroelectric properties. The Curie transition temperature must be well above the operating temperature; or depolarization may occur under combined temperature and pressure conditions. The thermal energy facilitates displacement of domain walls, leading to the large power dissipation and hysteretic behavior especially closed to the Curie transition temperature. The temperature variation may produce pyroelectric charges, which may interfere with the piezoelectric effect. In addition, many ferroelectrics become conducting at high temperatures, leading to the charge drifts and partial loss of signal. The conductivity problem is aggravated during operation is atmosphere with low oxygen
content, in which many oxygen-containing ferroelectrics may rapidly lose oxygen and become semi-conducting.

A free PWAS was subjected to a series of high temperatures in an oven for 30 minutes time intervals. After each 30 minutes, the PWAS was cooled, and their impedance spectrums were measured in room temperature. Figure 11.5 (a) shows the real part impedance spectrum of a free PWAS measured in room temperature and the real part impedance spectrums of the PWAS after exposure to oven temperatures ranging from 100 °F to 700 °F with 100 °F increment. The Curie temperature for PZT PWAS is 625°F. As seen in the Figure 11.5 (a), under 500 °F, the real part impedance spectrums of PWAS possesses two strong anti-resonant peaks due to the intrinsic electro-mechanical impedance of the free PWAS. With the further increment of oven temperature, PWAS lost its piezoelectric effect, which is manifested by the loss of anti-resonance peaks in real part impedance spectra. Figure 11.5 (b) shows the free PWAS was able to maintain its anti-resonance peak amplitude at the room temperature after the short-term heating up to 500 °F. This suggested that free PWAS can survive after heating up to 500°F.
Figure 11.5 Real part impedance spectrum of a free PWAS: (a) Impedance at room temperature after heating. (b) Amplitude of impedance peak at room temperature after heating.
A bonded PWAS was subjected to high temperature in the oven with the similar setup as the free PWAS. Due to the high temperatures, extra care must be taken for the bonding of sensors. Vishay adhesive M-Bond 610 has a short term operating temperature range from -452° to +700°F and a long term operating temperature range from -452° to +500°F. The PWAS durability test for bonded PWAS at high temperature used M-Bond 610 adhesive. PWAS impedance spectrums were measured at the oven. Figure 11.6 (a) shows the real part impedance spectrum of the bonded PWAS after exposure to oven temperatures ranging from 100 °F to 600 °F with 100 °F increment. As seen in the figure, the real part impedance spectrums of bonded PWAS anti-resonant frequencies and amplitudes reduced after increasing the temperature. At temperature above 400 °F, The real part of impedance spectrum becomes very small. After reaching the Curie temperature, the bonded PWAS lost its piezoelectric effect. Figure 11.6 (b) shows the first anti-resonance peak amplitude remain the same when the environmental temperature was below 200 °F. The amplitude started to drop above 200 °F. However, this drop is not as abrupt as noticed for free PWAS above 500 °F. This suggest that the performance of PWAS bonded to aluminum plate with M-Bond 610 adhesive is constant up to 200 °F and starts to degrade thereof. The earlier onset of degradation for bonded PWAS in comparison to free PWAS may be attributed to (a) softening of organic adhesive due to temperature and (b) mismatch of thermal expansions between PWAS and structural substrate. Further studies would be necessary to quantify these outputs.
Figure 11.6 Real part impedance spectrum of a bonded PWAS: (a) Impedance at the elevated temperature, (b) Amplitude of impedance peak at the elevated temperature.
11.4 Temperature Cycling

The first temperature cycling set included one free PWAS-01 and one bonded PWAS-02. The Vishay strain gage M-Bond 200 adhesive was used and the strain gage installation procedure was followed to apply the APC-850 PWAS to the aluminum alloy plate.

![Temperature Cycling Graph](image)

Figure 11.7 Temperature cycling.

For temperature cyclic testing, free and bonded PWAS were placed in an oven and exposed to a temperature variation between 100 ºF and 175 ºF. The temperature cycle consisted of a slow rise from 100 ºF to 175 ºF followed by a 5-minute dwell at 175 ºF, then a slow descend from 175 ºF to 100 ºF followed by a 5-minute dwell at 100 ºF (Figure 11.7). The peak of test environmental temperature is 25 ºF higher than the adhesive normal operating temperature. The data taken at the beginning of these experiments showed a “settling in” effect, i.e., some amplitude reduction during the first few cycles, followed by a leveling off of the variations. The free PWAS survived the temperature cycling without any significant change in the E/M impedance spectrum. In total, 1700 cycles have been performed (eleven months of testing). The free PWAS survived 1700 oven cycles without significant changes in the E/M impedance spectrum.
Bonded PWAS survived 1400 cycles in the oven without significant changes in the E/M impedance spectrum. The spectrum taken after 1500 and 1600 cycles showed small changes. The spectrum taken after 1700 cycles showed marked changes, which were attributed to the failure of the boded PWAS.

Further investigation with a Quantum 350 Scanning Acoustic Microscope\(^1\) indicated that disbond between PWAS and substrate took place. Figure 11.8 (a) showed C-scan adhesive and PWAS interface images of the PWAS-02 specimen after 1700 cycles. We can easily identify some disbond areas. There are four distinct regions on the C-scan interface image, namely the black background (aluminum plate), white interface (probable disbond at the adhesive/PWAS interface), grey interface (good bond) and black interface (probable disbond at the aluminum/adhesive interface). In comparison, we put a C-scan image of good bond specimen in Figure 11.8(b). Hence, it was concluded that the failure was not due to the piezoelectric material failure, but due to the failure of the bonded interface between the PWAS and the substrate. The operating temperature is beyond the adhesive normal working range, this failure can be attributed to the repeated differential thermal expansion for the adhesive layer between the ceramic PWAS and the metallic substrate.

The second temperature cycling test set also included one free PWAS-03 and one bonded PWAS-04. Vishay strain gage M-Bond AE-10 adhesive was used in this set. The operating temperature range is from cryogenic region to 200 °F. M-Bond AE-10 is a 100%-solids epoxy system. For this set of temperature cyclic testing, free and bonded

\(^1\) This investigation was done in collaboration with Professor Peter Nagy of the University of Cincinnati, which has this facility.
PWAS were placed in an oven and exposed to the same temperature cycling described before. The data taken at the beginning of this experiment showed a similar “settling in” effect. The free PWAS and bonded PWAS survived the temperature cycling without any significant change in the E/M impedance spectrum after 1000 cycles.

Figure 11.8 C-Scan adhesive and PWAS interface images of (a) bad bond specimen; (b) good bond specimen.
Figure 11.9 E/M impedance spectrum of PWAS after exposure to temperature cycling: (a) free PWAS; (b) PWAS bonded to a metallic plate.
11.5 Environmental Outdoors Exposure

Free PWAS and bonded PWAS were exposed to the outdoors environment (Figure 11.10a) over a long time period. In this study, several adhesives and several protective coatings were examined (Table 11.2). The measured quantity was the E/M impedance spectrum. The data taken at the beginning of these experiments showed a “settling in” effect, i.e., some amplitude reduction during the first few cycles, followed by a leveling off of the variations. After this, the E/M impedance data stayed rather constant for the duration of the test. This test has been conducted for 120 weeks and average temperature was shown in Figure 11.10b. Minor repairs to the wire attachments were done. So far, significant changes of PWAS E/M impedance spectrum have been noticed in the bonded specimen. Free PWAS are still in good working condition. A brief description of all the PWAS condition is described in Table 11.2.

![Figure 11.10](image)

(a) Environmental testing of free and bonded PWAS: (a) outdoors test fixture: (b) temperature profile.

For sample PWAS-bond8, this sample used the cyanoacrylate fast adhesive and had no protective coating. The historical evolution of the E/M impedance spectrum for this sample is shown in Figure 11.10a: up to 42 weeks, no significant changes were recorded;
at 51 weeks, small changes were observed. The main peak dropped a little and more small peaks can noticed. At 54 weeks, more changes were observed. At 63 weeks, significant changes were observed. The main peak dropped a lot and more small peaks appeared. At this moment, the sample was sent to Dr. Blackshire at Air Force Research Laboratory, where it was subjected to laser scanning interferometer investigation. An asymmetry in the displacement field and large amplitude edge vibrations indicative of incipient disbonding were found (Figure 11.11b). In addition, a crack was detected in the piezoceramic PWAS under optical microscope investigation. The crack was located diagonally in the right lower corner. These effects may explain the changes noted in the E/M impedance spectrum. The PWAS-bond8 was sent back and continued the outdoors test. The impedance spectrum was still recorded until final reading in 84 weeks. After that, the specimen completely was dead and impedance spectrum didn’t reflect the structure information anymore.

Table 11.2 Circular plate specimens for outdoors test

<table>
<thead>
<tr>
<th>Protective coating</th>
<th>PWAS-22</th>
<th>PWAS-23</th>
<th>PWAS-27</th>
<th>PWAS-28</th>
<th>PWAS-33</th>
<th>PWAS-34</th>
<th>PWAS-35</th>
<th>PWAS-36</th>
</tr>
</thead>
<tbody>
<tr>
<td>No coating</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M-Coat A Polyurethane</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M-Coat C-Silicon</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>M-Coat D-Acrylic</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 11.11 (a) E/M impedance spectrum of specimen Bond-8; (b) asymmetric displacement field.
### Table 11.3 Summary of PWAS specimens (experiment started in Nov 2003)

<table>
<thead>
<tr>
<th>Specimen #</th>
<th>Adhesive</th>
<th>Coating</th>
<th>Lifetime (week)</th>
<th>Possible reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond1</td>
<td>AE-10</td>
<td>None</td>
<td>117</td>
<td>Adhesive</td>
</tr>
<tr>
<td>Bond2</td>
<td>AE-10</td>
<td>Acrylic</td>
<td>120</td>
<td>good</td>
</tr>
<tr>
<td>Bond3</td>
<td>AE-10</td>
<td>Silicon</td>
<td>117</td>
<td>Adhesive</td>
</tr>
<tr>
<td>Bond4</td>
<td>AE-10</td>
<td>Polyurethane</td>
<td>84</td>
<td>Adhesive</td>
</tr>
<tr>
<td>Bond5</td>
<td>M-Bond 200</td>
<td>Acrylic</td>
<td>107</td>
<td>Adhesive</td>
</tr>
<tr>
<td>Bond6</td>
<td>M-Bond 200</td>
<td>Silicon</td>
<td>138</td>
<td>Adhesive</td>
</tr>
<tr>
<td>Bond7</td>
<td>M-Bond 200</td>
<td>Polyurethane</td>
<td>91</td>
<td>adhesive</td>
</tr>
<tr>
<td>Bond8</td>
<td>M-Bond 200</td>
<td>None</td>
<td>107</td>
<td>PWAS cracked</td>
</tr>
<tr>
<td>Free1</td>
<td>N/A</td>
<td>None</td>
<td>91</td>
<td>Lost</td>
</tr>
<tr>
<td>Free2</td>
<td>N/A</td>
<td>Polyurethane</td>
<td>120</td>
<td>good</td>
</tr>
<tr>
<td>Free3</td>
<td>N/A</td>
<td>Silicon</td>
<td>120</td>
<td>good</td>
</tr>
<tr>
<td>Free4</td>
<td>N/A</td>
<td>None</td>
<td>91</td>
<td>Electrode defects</td>
</tr>
<tr>
<td>Free5</td>
<td>N/A</td>
<td>Polyurethane</td>
<td>120</td>
<td>good</td>
</tr>
<tr>
<td>Free6</td>
<td>N/A</td>
<td>Acrylic</td>
<td>84</td>
<td>Lost</td>
</tr>
<tr>
<td>Free7</td>
<td>N/A</td>
<td>Silicon</td>
<td>120</td>
<td>good</td>
</tr>
<tr>
<td>Free8</td>
<td>N/A</td>
<td>Acrylic</td>
<td>84</td>
<td>Lost</td>
</tr>
</tbody>
</table>

#### 11.6 Submersion Exposure

The purpose of these tests was to determine how PWAS behaves when exposed to water and various maintenance fluids. The specimens used in this experiment were free 5-mm PWAS with two connecting wires soldered by the manufacturer. The PWAS were submerged in plastic bottles containing the fluids (Figure 11.12). The fluids used in the submersion test were:

- **(0) Distilled water**
- **(1) Saline solution**
- **(2) Hydraulic fluid MIL-PRF- 83282 Synthetic hydrocarbon**
(3) Hydraulic fluid MIL-PRF- 87257 Synthetic hydrocarbon

(4) Hydraulic fluid MIL-PRF- 5606 Mineral

(5) Aircraft cube oil MIL-PRF-7808L Grade 3 Turbine engine synthetic

(6) Kerosene

Figure 11.12 PWAS submersion test: (a) test containers; (b) 5-mm diameter free PWAS specimen.

This test has been conducted for more than 425 days (60 weeks). So far, no significant changes have been noticed in the PWAS E/M impedance spectrum except for the PWAS submerged in saline solution. The PWAS submerged in saline solution survived only a little over 85 days (15 weeks). The E/M impedance reading taken at 85 days exposure showed marked differences from the previous readings. The failure of this PWAS was traced to the detachment of the soldered connection. This can be attributed to the corrosive effect of the saline solution.

In a separate test, Blackshire et al. (2008) subjected PWAS bonded to an aluminum plate to electrochemical corrosion exposure using a corrosion cell and 3.5% NaCl solution. Two specimens, one unprotected, the other covered with an enamel protective coating, were used. No significant change was reported after exposure to eight corrosion cycles.
Figure 11.13 E/M impedance spectrum of PWAS after submersion exposure: (a) PWAS in saline water; (b) PWAS in distilled water.
11.7 LARGE STRAINS AND FATIGUE CYCLIC LOADING

Doane and Giurgiutiu (2005) studied the behavior of PWAS transducers under large strains and under fatigue cyclic loading. To test the characteristics of the PWAS under large strain conditions, a PWAS was bonded to aircraft grade 2024-T3 aluminum alloy test specimen and subjected to tensile loading.

![Figure 11.14](image)

Figure 11.14 Large strains and fatigue cyclic loading tests: (a) large-strain test specimen; (b) fatigue cycling loading test specimen; (c) experimental setup.

In the large strain experiments, the specimen shown in Figure 11.14(a) was used. The specimens were fabricated from 2024 T3 aluminum with a nominal thickness of 1 mm. The specimens were loaded in tension under strain control. Two specimens were used: the first one was loaded up to 5000 micro-strain, the other up to failure. Measurements were taken at 200 micro-strain intervals. The baseline impedance was recorded at zero strain and additional readings were recorded until failure of the PWAS occurred. Minimal changes occurred to the impedance signature until the value of 3000 micro-strain was
exceeded. Significant changes begin to happen after 3000 micro-strain. After 6000 micro-strain, the changes in the E/M impedance were very strong (Figure 11.15a). Eventually the PWAS failed in tension at approximately 7200 micro-strain. The PWAS failure was in the form of a transverse crack (Figure 11.15b).

![Figure 11.15 Large strain tests: (a) Impedance signatures up to 6000 με (b) micrograph of the cracked PWAS at 7200 με.]

In the fatigue cyclic loading experiments, the specimen shown in Figure 11.14b was used. The specimens were fabricated from 2024 T3 aluminum with a nominal thickness of 1 mm. A 7-mm square PWAS was bonded to the specimen with M-Bond 200 cyanoacrylate adhesive. A 1-mm hole was drilled in the specimen to act as stress concentration and localize the fatigue failure. Five specimens were used (Table 11.4). The specimens were loaded in fatigue cyclic loading with the mean loads and amplitudes adjusted such as to cause failure of the aluminum substrate at various values between 100 thousand and 10 million cycles (Figure 11.16 and Table 11.4). The baseline impedance reading was taken with the mean load applied at the beginning of the tests and at
predetermined cyclic intervals. Small settle-in changes occurred in the impedance readings in the first 30 to 40 thousand cycles. Beyond this the PWAS readings were relatively unchanged until the metallic specimen finally broke under fatigue cyclic loading. The specimen failure always occurred at the 1-mm stress-concentration hole. The PWAS survived the fatigue failure of all the metallic specimens.

| Table 11.4 Fatigue specimens overview (R = 0.1) |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Max load | 2104 N | 1560 N | 1335 N | 1156 N | 1067 N |
| Min load | 210 N | 156 N | 134 N | 116 N | 107 N |
| Mean load | 1157 N | 858 N | 734 N | 636 N | 587 N |
| Cycles to specimen failure | 178 kc | 670 kc | 1.3 Mc | 6.25 Mc | 12.2 Mc |

Figure 11.16 (a) Stress concentration factor for flat plate with hole; (b) S-N curve determined during reported work.
PART III

Novel PWAS Configurations
12 IN-SITU FABRICATED PWAS

PWAS can be made from piezoceramics, piezocomposites, or piezopolymers. The most common PWAS are made of piezoceramics (e.g. lead zirconate titanate, a.k.a. PZT). Piezoceramics are typically made of simple perovskites and solid solution perovskite alloys. Mechanical compression or tension on a poled piezoelectric ceramic element changes the dipole moment, creating a voltage. Compression along the direction of polarization or tension perpendicular to the direction of polarization generates voltage of the same polarity as the poling voltage.

Brittle PZT-PWAS can withstand very little bending. This brittleness imposes difficulties in handling and bonding of the PWAS onto the structure being monitored. In addition, they have poor conformability to curved surfaces and local straightening of the structural surface is required for PWAS installation.

12.1 INTRODUCTION

Composite PWAS are made by dispersing electroactive, magnetoactive, or piezoelectric particles (powders) in polymeric matrix materials. The resulting particulate composite displays effective piezoelectric or piezomagnetic properties that are somehow dependent on the volume ratio. However, the connectivity and the interfacial bond between the phases also play a considerable role. Recent research interest has been shown in the combination of electroactive and magnetoactive effects into a more complex composite that displays both electroactive and magnetoactive responses. These composites, which
have a three-way coupling between the electric, magnetic, and elastic fields, are known as magnetoelectric or multiferroic composites.

Piezopolymers (e.g. polyvinylidene fluoride, a.k.a. PVDF) are a class of piezoelectric materials that display piezoelectric properties similar to those of quartz and piezoceramics. PVDF are supplied in the form of thin films that are flexible and show large compliance. They are less expensive and easier to fabricate than piezoceramics. The flexibility of PVDF-PWAS overcomes some of the drawbacks associated with the brittleness of the piezoelectric ceramics.

In this chapter, we will analyze the PWAS model and experimentally verify the model with vibration and impact detections using Composite-PWAS, PZT-PWAS and PVDF-PWAS. The focus of this chapter is to find a suitable in-situ piezoelectric active sensor for sending and receiving Lamb wave to be used in SHM of structures with curved surface. The motivation of our research was to explore the use of piezoeelastic polymers PVDF. However PVDF stiffness is orders of magnitude lower than the PZT stiffness, and hence PVDF Lamb wave transmitters are much weaker than PZT transmitters. Thus, our research proceeded in two main directions: (a) to model and understand how piezoelectric material properties affect the behavior of PWAS; and (b) to perform experiments to test the capabilities of the flexible PVDF PWAS in comparison with those of stiffer but brittle PZT PWAS. We have shown that with appropriate signal amplification, PVDF PWAS can perform the same Lamb wave transmission and reception functions currently performed by PZT PWAS.
12.2 State of the Art for In-situ Fabricated Smart Material Active Sensors

Investigation of the state of the art in electroactive and magnetoactive composites reveals that considerable work has already been done in this direction. One common approach is to disperse particles of electroactive or magnetoactive particles (powders) in polymeric matrix materials. The resulting particulate composite displays effective piezoelectric or piezomagnetic properties that are somehow dependent on the volume ratio. However, the connectivity and the interfacial bond between the phases also play a considerable role. Particular recent interest has been the combination of electroactive and magnetoactive effects into a more complex composite that displays both electroactive and magnetoactive responses. These composites, which have a three-way coupling between the electric, magnetic, and elastic fields, are known as magnetoelectric or multiferroic composites. Details on previous work on piezomagnetic, piezoelectric, and magnetoelectric composites are given next.

12.2.1 Piezomagnetic Composites

Terfenol-D is a magnetic anisotropy-compensated alloy \( \text{Tb}_x\text{Dy}_{1-x}\text{Fe}_2 \) which shows a strong magnetostrictive behavior. When subjected to mechanical strain, Terfenol-D produces magnetic field. White reviewed the magnetostrictive tagging methodology of composites for structural health monitoring and measured the response of magnetostrictive-tagged composites under axial loading. Neat resin beams tagged 2.24% by volume with magnetostrictive Terfenol-D powder were used. Trovillion et al. studied the magnetic characteristics of neat resin and glass-fiber-reinforced magnetostrictive composites subjected to axial load. The fiber reinforced polymer composite (FRP) beams
consisted of 4 layers of continuous strand glass mat fibers embedded in a polyester resin. The top lamina of the composite was impregnated with Terfenol-D powder at a volume fraction of 2.24% for that lamina. The beams were subjected to uniaxial loading under load control at a rate of 0.02 kN/s. Rearrangement of the magnetic dipoles chains of the magnetostrictive molecules through application of strong magnetic field was performed by applying a magnetic field through the thickness of the beam using a pair of magnets to apply a field of 800 Gauss.

Armstrong presented a new model of non-linear magneto-elastic behavior of magnetostrictive particulate composite. The analysis assumed uniform external magnetic field that is operating on a large number of well-distributed ellipsoidal magnetostrictive particles encased in an elastic, nonmagnetic composite matrix. Nersessian and Carman studied five different volume-fraction of magnetostrictive particulate composites which were tested under two different conditions: (a) constant magnetic field and varying mechanical load; and (b) constant mechanical load and varying magnetic field. The results presented for the constant load indicated a strong dependence of strain output on applied pre-stress.

Krishnamurthy et al. considered health-monitoring detection of delaminations in composite materials using an excitation coil and a sensing coil. The open-circuit voltage induced in the sensing coil is proportional to the stress generated in the magnetostrictive layer by the presence of the delamination.

12.2.2 Piezoelectric Composites

The combination of polymer and piezoelectric ceramic to form composite PWAS offers the unique blending of the high electro-active properties of piezoelectric ceramics and the
mechanical flexibility and formability of organic synthetic polymers. Recently, active composite PWAS have been developed, namely 1–3 composites by Smart Materials Corp, active fiber composite developed by MIT, and macrofiber composite (MFC) actuators at NASA Langley Center. These composite PWAS are capable of being repeatedly manufactured at low cost, are tolerant to damage, capable of conforming to complex or curved surfaces, and embeddable into structures. The uses of composite PWAS need to be investigated, and would provide the advantages of being robust, reliable, and easily adaptable for impedance-based health monitoring. Composites PWAS can be classified according to the connectivity of piezoelectric ceramics and matrix phases. The composite PWAS described in this paper has a 0-3 connectivity pattern. The “0-3” means that the ceramic particles are randomly dispersed in a polymer matrix. 0-3 composites can be more easily fabricated in complex shapes than other forms of composites. Various approaches have been tried for producing 0-3 composites.

12.2.3 Magnetoelectric Composites

Composite aggregates that consist of an electroactive phase and a magnetoactive phase exhibit a magnetoelectric effect, which is not present in the constituents. The magnetoelectric effect consists in coupling the magnetic and electric responses through the elastic interaction between the magnetoactive and electroactive phases. For example, if an applied electric field activates the electroactive phase, the resulting geometric change will be felt by the magnetoactive phase and, through the magnetoactive effect, will result in an output magnetic field. Conversely, an input magnetic field will result in an output electric field.
Several magnetoelectric composite configurations have been considered. One configuration comes in the form of multilayer magnetoelectric composites consist of alternating layers of electroactive and magnetoactive materials. Another configuration is that of particulate composites with multi-phase aggregates of electroactive and magnetoactive phases. For example, a magnetoactive phase may appear as particulate inclusions into an electroactive phase. If the aspect ratio of the particulates is large, the particulate may appear as elongated inclusions that can be idealized as cylindrical microfibers. A third configuration is that of self-assembled nanofilms having magnetoactive nanopillars in an electroactive matrix that are grown vertically on a substrate.

The most common magnetoelectric composite is the combination of electroactive BaTiO$_3$ with the magnetoactive CoFe$_2$O$_4$. In the electroactive phase, the perovskite crystal structure of BaTiO$_3$ imparts ferroelectric properties that result in the piezoelectric effect. In the magnetoactive phase, the spinel structure of CoFe$_2$O$_4$ imparts magnetostrictive response that, upon biased linearization, results in the piezomagnetic effect. Nan developed a theoretic framework for the analysis of magnetoelectric composites using a modified Green’s function approach. In their derivation, they started from the coupled magnetic-electric-elastic constitutive equations in the form

$$
\begin{align*}
\sigma &= C_s - e^T E - q^T H \\
D &= e s + \varepsilon E + \alpha H \\
B &= q s + \alpha^T E + \mu H
\end{align*}
$$

(7.1)

where the notation $(...)^T$ signifies the transpose operation and $\sigma, s, D, E, B, H, C, \varepsilon, \mu$ are tensors representing stress, strain, electric displacement, electric field, magnetic
induction, magnetic field, stiffness (at constant electric and magnetic fields), dielectric and magnetic permeabilities (at constant strain), respectively. The interaction between the electric, magnetic, and elastic is represented by the tensors \( e, q, \alpha \) which contain the piezoelectric, piezomagnetic, and magnetoelastic coefficients. The magnetoelectric property tensor \( \alpha \) is a new property that was not present in the individual piezoelectric and piezomagnetic phases. In compressed Voigt notations, the tensors \( C, e, q, \varepsilon, \mu, \alpha \) are \((6\times6), (6\times6), (3\times6), (3\times3), (3\times3), (3\times3)\) matrices, such that Equation (7.1) can be rewritten as.

\[
\begin{bmatrix}
\sigma \\
D \\
B
\end{bmatrix}
= 
\begin{bmatrix}
C & -e^T & -q^T \\
e & \varepsilon & \alpha \\
q & \alpha^T & \mu
\end{bmatrix}
\begin{bmatrix}
s \\
E \\
H
\end{bmatrix}
\tag{7.2}
\]

For the composite, coefficients in Equation (7.2) take local values depending on the special position \( x \). However, one can devise effective constitutive coefficients in terms of the average fields, namely,

\[
\begin{bmatrix}
<\sigma> \\
<D> \\
<B>
\end{bmatrix}
= 
\begin{bmatrix}
C^* & -e^{*T} & -q^{*T} \\
e^* & \varepsilon^* & \alpha^* \\
q^* & \alpha^{*T} & \mu^*
\end{bmatrix}
\begin{bmatrix}
<s> \\
<E> \\
<H>
\end{bmatrix}
\tag{7.3}
\]

where the notation \(<\ldots>\) signifies space average. Using numerical simulation performed on 0-3 and 1-3 composites of BaTiO\(_3\)-CoFe\(_2\)O\(_4\), Nan showed that the effective magnetoelectric coefficients of the composite can be strongly influenced by the connectivity, the volume fraction, and the aspect ratio of the particles. For example, low
volume fractions of the piezoelectric phase result in large values of the magnetoelectric voltage coefficient of the composite results.

Pan considered the exact solution of a multilayer sandwich plate made of piezoelectric BaTiO$_3$ and magnetostrictive CoFe$_2$O$_4$. Simply supported boundary conditions were considered. It was found that the response of the electric and magnetic variables depends strongly on the stacking sequence. Echigoya *et al.* studied experimentally the directional solidification of the BaTiO$_3$-CoFe$_2$O$_4$ eutectic fabricated by the floating zone melting method. It was observed that the structure of the eutectic consists of grains having lamellar or fibrous morphology. Intercalated elongated grains of BaTiO$_3$ and CoFe$_2$O$_4$ were observed (Figure 12.1).

![Figure 12.1 Optical photograph of directionally solidified BaTiO$_3$-CoFe$_2$O$_4$ eutectic grown in air at 10 mm/h showing lamellar structure of alternating BaTiO$_3$ and CoFe$_2$O$_4$ elongated grains (after Echigoya et al.)](image-url)
Misfit dislocations due to the accommodation of the lattice mismatch between the 
BaTiO	extsubscript{3} and CoFe	extsubscript{2}O	extsubscript{4} were observed at the grain interfaces. Zheng et al. considered the 
nanoscale magnetoelectric composites consisting of multiferroic BaTiO	extsubscript{3}-CoFe	extsubscript{2}O	extsubscript{4} 
structures. Several configurations were considered, such as multilayer films and self-
assembled nanostructured rods (Figure 12.2a). Self-assembled BaTiO	extsubscript{3}-CoFe	extsubscript{2}O	extsubscript{4} 
nanocomposites were formed from a 0.65BaTiO	extsubscript{3}-0.35CoFe	extsubscript{2}O	extsubscript{4} target by pulsed laser 
deposition. (SrRuO	extsubscript{3} was chosen as the lattice-matched bottom electrode to enable 
heteroepitaxy as well as to facilitate electric measurements.) The resulting samples 
displayed nano-structured pillars of CoFe	extsubscript{2}O	extsubscript{4} with ~20 nm diameter embedded in a 
BaTiO	extsubscript{3} matrix (Figure 12.2b). Cai et al. considered 3-component magnetoelectric 
composites consisting of a piezoelectric ceramic (PZT), a magnetostrictive alloy 
(Terfenol-D) and a piezoelectric polymer (PVDF). In one experiment, three-phase 
particulate composites of PZT and Terfenol-D in a PVDF matrix were considered. In a 
different experiment, three-layer laminates of two-phase particulate composites were 
investigated. A Terfenol-D/PVDF particulate composite layer was sandwiched between 
two PZT/PVDF particulate composite layers. The fabrication was achieved through 
lamination and hot-pressing. Both methods gave good magnetoelectric response, but the 
latter approach achieved better values than the former.
Figure 12.2 Nanoscale magnetoelectric composites consisting of multiferroic BaTiO$_3$-CoFe$_2$O$_4$ structures: (a) (A) Superlattice of a spinel structure (top) and a perovskite structure (middle) on a perovskite substrate (bottom). (B) resulting multilayer configuration; (C) Epitaxial alignment of a spinel (top left) and a perovskite (top right) on a perovskite substrate (bottom). (D) resulting pillars substrate; (b) TEM picture of an experimental sample showing self-assembled nanostructured CoFe$_2$O$_4$ pillars with ~20 nm diameter embedded in a BaTiO$_3$ matrix (after Zheng et al.)
12.3 COMPOSITE PWAS

One approach considered in our research was that of piezoelectric composite PWAS. The piezoelectric composite PWAS have the advantage that electric signals can be collected directly, whereas in the case of piezomagnetic composites magnetic flux probes had to be used. Our purpose in pursuing the fabrication of piezoelectric composites was to eventually be able to achieve in-situ fabrication of PWAS of a quality comparable with that of ceramic PWAS. By fabricating the PWAS directly onto the structure a seamless bond would be achieved between the PWAS and the structure that would be impervious to environmental attacks can be created. Thus, we eliminated the “weak link” in the present use of bonded PWAS and achieved a long time durability of the embedded sensory system.

12.3.1 Preparation of Composite PWAS

The epoxy resin used in the fabrication of the piezoelectric composite PWAS was EPO-TEK 301-2 from Epoxy Technology, Inc.. This epoxy has a good handling property, high dielectric strength and a low viscosity. The mixture ratio for the epoxy part “A” and part “B” (hardener) is specified as 1: 0.35 by the supplier and was followed in this study.

The PZT particles in the composite PWAS were PZT-5B powders from Morgan Electro Ceramics. PZT-5B powders have a high sensitivity and high time stability (Figure 12.3). The silver paint used for creating the electrodes was acquired from SPI Supplies and Structure Probe, Inc. The poling workstation consists of an oven, a constant DC power supply, a high voltage power supply, a multimeter and a thermometer.
12.3.2 Composite PWAS Fabrication

Clean the surface of host structure and put a mask. The mask has a 7mm diameter, 0.2mm thickness hole and its center is in the desired position. PZT powder with a weight fraction of 85% was added to the epoxy matrix phase and was stirred thoroughly. The paste was then spread into the mask and let it cure at an elevated temperature (50 degree C) until hard. Remove excess and sand it down to final thickness. A small piece of copper foil was applied to one side of the cured composite PWAS to form an electrode. After
curing and electroding had been completed, poling of the composite PWAS was subsequently carried out to activate its piezoelectric effect. A constant DC voltage of 1 kV was applied to the composite PWAS to pole the sample for four hours at 80 degree C.

![Figure 12.4 Composite PWAS sample fabricated on a thin aluminum plate was used in measuring the impact waves.](image)

12.3.3 Characterization of Composite PWAS

We measured the final size of the composite PWAS. It is 7mm diameter and 0.2mm thickness which is same as the traditional PWAS. The electricity of the composite PWAS was measured before and after poling. The resistance is greater than 40 $M\Omega$ before and after poling. The resistivity is higher than 40 $M\Omega m$. The capacitance of composite is only 0.02nF which is lower than the traditional PWAS value 3nF.
12.3.3.1 Impact Test on Composite PWAS

The effectiveness of the composite PWAS for impact was investigated based on an impact test setup show in Figure 12.4. Impact point showed in the Figure 12.4 and it is 20mm away from the center of the composite PWAS. The output of the composite PWAS was measured simply as a voltage signal using a digital oscilloscope. We measured the voltage signal before and after poling. Before poling, the voltage signal damped out very soon. After poling, we can find the voltage amplitude is much higher than that before poling. And it also shows some vibration signal after the impact. From this figure, it is seen that upon hit, the beam vibrated first and then slowly damped out. The composite PWAS have a good repeatability in its output signal when subjected to similar impact forces.

Figure 12.5 The electric voltage signal measured by the impact hammer, (a) before poling (b) after poling.
12.3.3.2 Composite PWAS Vibration Test

The effectiveness of the composite PWAS for dynamic measurement was investigated based on a vibration test setup shown in Figure 12.6. The beam was mounted as a cantilever in a calibrated cantilever fixture. A composite PWAS was placed on the top of the beam and a traditional PWAS was placed at the other side of the beam. The tip of the beam was displaced to a certain value (approx. 10 mm) and then suddenly released such as the beam entered in free vibration.

An oscilloscope connected directly to the PWAS was used to record the electric signals induced in both PWAS by the piezoelectric coupling between the mechanical vibration and the electric field (Figure 12.7). In Figure 12.7 (a), it is traditional PWAS voltage signal. We can easily see it reflected the beam vibration very well. But before and after poling, we still can’t find the signal from the composite PWAS. Figure 12.7 (b) only showed the noise.
Overall, the technique for fabricating composite PWAS has been established. The effectiveness of composite PWAS has been preliminarily characterized through a series of tests, which include impact test and vibration test. The experiment results of the composite PWAS have been compared with the traditional PWAS. The experimental results in this study gave the basic demonstration of the piezoelectricity of composite PWAS. The composite PWAS are capable of use as an active sensor.

Our future work is focused on perfecting the composite technology methods for the in-situ fabrication of composite PWAS. The composite fabrication methods considered for investigation are: (a) mask; (b) poling; (c) powders technology; (d) coatings. The ultimate vision is to create a methodology to create the composite PWAS directly onto the structural substrate through an easy technique followed by electroding and poling.

12.4 PVDF PWAS

Piezopolymers (e.g. polyvinylidene fluoride, a.k.a. PVDF) are a class of piezoelectric materials that display piezoelectric properties similar to those of quartz and piezoceramics. PVDF are supplied in the form of thin films that are flexible and show large compliance. They are less expensive and easier to fabricate than piezoceramics. PVDF-PWAS can be cut from the thin film sheet to desired dimension of sensors. The flexibility of PVDF-PWAS overcomes some of the drawbacks associated with the brittleness of the piezoelectric ceramics. In our research, the PVDF film is provided by Measurement Specialties, Inc. The PVDF-PWAS are at the same size (7 mm square) as traditional PZT-PWAS. The comparison of the PZT-PWAS and PVDF-PWAS for dynamic measurement was investigated based on a serial of tests.
Figure 12.7 Measured electric signals during the plucked beam experiments: (a) conventional piezoceramic PWAS; (b) piezoelectric composite PWAS.

Table 12.1 Comparison of different materials’ parameters

<table>
<thead>
<tr>
<th>Property</th>
<th>Units</th>
<th>PZT</th>
<th>BaTiO$_3$</th>
<th>PVDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$10^3$ kg/m$^3$</td>
<td>7.5</td>
<td>5.7</td>
<td>1.78</td>
</tr>
<tr>
<td>Relative Permittivity</td>
<td>$e/e_0$</td>
<td>1,200</td>
<td>1,700</td>
<td>12</td>
</tr>
<tr>
<td>$d_{31}$ constant</td>
<td>$(10^{-12})$ C/N</td>
<td>110</td>
<td>78</td>
<td>23</td>
</tr>
<tr>
<td>$g_{31}$ constant</td>
<td>$(10^{-3})$ Vm/N</td>
<td>10</td>
<td>5</td>
<td>216</td>
</tr>
<tr>
<td>$k_{31}$ constant</td>
<td>% at 1 kHz</td>
<td>30</td>
<td>21</td>
<td>12</td>
</tr>
<tr>
<td>Acoustic Impedance</td>
<td>$(10^6)\text{kg/m}^2\text{sec}$</td>
<td>30</td>
<td>30</td>
<td>27</td>
</tr>
</tbody>
</table>
12.4.1 Cantilever Beam Free Vibration

12.4.1.1 Experimental Setup

Cantilever beams were well studied by Voltera. A typical cantilever beam, used in this study, is L = 300 mm long, w = 19.2 mm wide, and t = 3.23 mm thick. When the force $P$ is removed from a displaced beam, the beam will return to its original shape. However, inertia of the beam will cause the beam to vibrate around that initial location. According to the vibration and boundary equation, we can calculate the resonance frequency. The beam material is stainless steel 304 with density 8030 kg/m$^3$, and Young’s modulus $E =$ 195 Gpa. Theoretically, the first three nature frequencies are $f_1 = 28.6$ Hz, $f_2 = 179$ Hz, and $f_3 = 501.3$ Hz.

![Figure 12.8 PZT-PWAS, PVDF-PWAS, and strain gauge on a cantilever beam: (a) experimental setup; (b) close-up view of the bottom surface showing the 200 $\mu$m PZT-PWAS and strain gauge; (c) close-up view of the top surface showing the 28 $\mu$m and 110 $\mu$m PVDF-PWAS.](image)
The PZT-PWAS, PVDF-PWAS and strain gauge for dynamic measurement were investigated based on a vibration test setup shown in Figure 12.8. The beam was mounted as a cantilever in a calibrated cantilever fixture. Two 7-mm square PVDF-PWAS with different thicknesses were placed on the top of the beam; a 7-mm square PZT-PWAS and a strain gauge were placed at the bottom of the beam (Figure 12.8).

![Vibration signal recorded by strain gauge, PVDF-PWAS and PZT-PWAS.](image)

**Figure 12.9** Vibration signal recorded by strain gauge, PVDF-PWAS and PZT-PWAS.

The tip of the beam was displaced to a certain value (approx. 10 mm) and then suddenly released as the beam entered in free vibration. A 4-channel Tektronix TDS5030B oscilloscope was used to collect the vibration response from different kinds of sensors. Channel one was connected to the Vishay P3 strain indicator to record the electric signal generated by the resistance change in the strain gauge due to strain elongation. The other three channels were directly connected to the PZT-PWAS and PVDF-PWAS to record the electric signals generated through the piezoelectric coupling between the mechanical
vibration and the electric field. The recorded traces are shown in Figure 12.9. The Fourier transform was used to analyze the frequency contents of the signals, which should correspond to the natural free vibration frequencies of the cantilever beam. The resulting spectra are shown in Figure 12.10. The first three natural frequencies are shown to be $f_1 = 29.7$ Hz, $f_2 = 181$ Hz, and $f_3 = 501$ Hz. The PZT-PWAS was found to give the highest voltage but it was less responsive to the higher frequencies. The PVDF-PWAS were found to be more responsive to the high frequencies but to give a lower voltage (Figure 12.10).

![Figure 12.10 Vibration signal and spectrum of magnitude (Fourier transform) recorded by (CH 1) strain gauge; (CH 2) 28 $\mu$m thick PVDF-PWAS; (CH 3) 110 $\mu$m thick PVDF-PWAS, (CH 4) 200 $\mu$m thick PZT-PWAS.](image-url)
12.4.1.2 Data Comparison and Analysis

Vishay P3 strain indicator and recorder has an analog output from 0 to 2.5V which equals strain value from -320 με to +320 με. The peak to peak analog output of strain indicator was 1.32 V, which means the peak to peak vibration strain equaled 338 με.

Assuming the oscilloscope’s capacitance is 3 nF, the theoretical peak to peak voltage of PZT-PWAS, 110 μm PVDF-PWAS and 28 μm PVDF-PWAS are 30V, 0.361V and 0.346V respectively. The experimental results can be read from Table 12.2, which were 30.8 V, 0.508 V and 0.332 V respectively. The experimental response voltage (Table 12.3) and natural frequencies (Table 12.4) agreed with the theoretical prediction. The theoretical and experimental frequency values are closer with increasing frequency because the resolution of the built-in Fourier transform of oscilloscope.

Table 12.2 Comparison of the strain gauge, PVDF PWAS, PZT PWAS responses

<table>
<thead>
<tr>
<th>Channel</th>
<th>Sensor</th>
<th>$V_{pp}$ (V)</th>
<th>$f_1$ (Hz)</th>
<th>$A_1$ (mV)</th>
<th>$f_2$ (Hz)</th>
<th>$A_2$ (mV)</th>
<th>$A_2/A_1$ (%)</th>
<th>$f_3$ (Hz)</th>
<th>$A_3$ (mV)</th>
<th>$A_3/A_1$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH 1</td>
<td>Strain Gauge</td>
<td>1.32</td>
<td>29.69</td>
<td>327.4</td>
<td>182.8</td>
<td>12</td>
<td>3.66</td>
<td>509.4</td>
<td>20</td>
<td>6.1</td>
</tr>
<tr>
<td>CH 2</td>
<td>28 μm PVDF-PWAS</td>
<td>0.332</td>
<td>29.69</td>
<td>55.64</td>
<td>181.3</td>
<td>28.75</td>
<td>51.67</td>
<td>501.6</td>
<td>6.16</td>
<td>11.07</td>
</tr>
<tr>
<td>CH 3</td>
<td>110 μm PVDF-PWAS</td>
<td>0.508</td>
<td>29.69</td>
<td>82.5</td>
<td>181.3</td>
<td>45.65</td>
<td>55.33</td>
<td>501.6</td>
<td>11</td>
<td>13.33</td>
</tr>
<tr>
<td>CH 4</td>
<td>200 μm PZT-PWAS</td>
<td>30.8</td>
<td>29.69</td>
<td>8568</td>
<td>181.3</td>
<td>800</td>
<td>9.337</td>
<td>502.5</td>
<td>100</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 12.3 Comparison of PWAS responses

<table>
<thead>
<tr>
<th></th>
<th>PZT-PWAS (V)</th>
<th>110mm PVDF-PWAS (V)</th>
<th>28mm PVDF-PWAS (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical</td>
<td>30</td>
<td>0.361</td>
<td>.346</td>
</tr>
<tr>
<td>Experimental</td>
<td>30.8</td>
<td>0.508</td>
<td>0.332</td>
</tr>
<tr>
<td>Relative error (%)</td>
<td>2.67</td>
<td>40.7</td>
<td>-4.04</td>
</tr>
</tbody>
</table>
### Table 12.4  Comparison of the cantilever beam natural frequencies

<table>
<thead>
<tr>
<th></th>
<th>( f_1 ) (Hz)</th>
<th>( f_2 ) (Hz)</th>
<th>( f_3 ) (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theoretical</td>
<td>28.6</td>
<td>179</td>
<td>501.3</td>
</tr>
<tr>
<td>Experimental</td>
<td>29.69</td>
<td>181.7</td>
<td>503.8</td>
</tr>
<tr>
<td>Relative error (%)</td>
<td>3.81</td>
<td>1.51</td>
<td>0.50</td>
</tr>
</tbody>
</table>

12.4.2 Long Rod Impact Test

12.4.2.1 Experimental Setup

The PZT-PWAS, PVDF-PWAS and strain gauge for dynamic measurement were investigated based on a long rod impact test. The longitudinal waves were generated by impacting the rod. The resulting waves were detected by using electrical resistance strain gages and PVDF-PWAS.

![Rod impact experimental setup](image)

A schematic of the apparatus is shown in Figure 12.12. A long rod is desirable so as to lengthen the time between reflections and make the pulses more distinct. The rod was suspended in three places by a monofilament line. One strain gage and three PVDF-PWAS were used. The BLH semiconductor type strain gauge was mounted 23 inches...
from the left end of the rod. This type of strain gauge was selected for its high sensitivity good dynamic response. 7-mm square PVDF-PWAS were cut from piezo film sheets provided by Measurement Specialties, Sensor Products Div. Three PVDF-PWAS have three different thicknesses, 28 μm, 52 μm and 110 μm, mounted 20 inches (50.8 mm) from the right end of the rod. The impactor was a 0.625 inch (1.58 mm) diameter steel ball supported by monofilament strings. The string was adjustable so that the ball’s center can be placed at the center of the end of the rod.

![Diagram of stress wave propagation in a rod](image)

**Figure 12.12 Stress wave propagation in a rod.**

There are three types of waves that can be formed upon impact of a long rod: longitudinal waves, flexural waves and torsion waves. If the impact is axial, the type of propagated wave is the longitudinal wave. The longitudinal wave speed propagated in a thin rod is \( c = \sqrt{\frac{E}{\rho}} \). where \( E \) and \( \rho \) are the material’s Young’s modulus and density, respectively. When the impactor hits the rod at time \( T_0 \), the compression wave arrives at the PVDF-PWAS at time \( T_1 \), then arrives at the strain gauge at time \( T_1' \). When the compression wave reaches the free end of the rod, it will be reflected as tension and vice versa. Stress reversal is a characteristic of the free end reflection. The tension wave will
reach the strain gauge first at time $T_2$ and reach the PVDF-PWAS at time $T_2$. It will be reflected by the free end again as compression wave.

12.4.2.2 Data Analysis

Two pictures of the history of strain gauge and PVDF-PWAS are shown in Figure 12.13 and Figure 12.14. Figure 12.13 has a time base of 200 $\mu$s per division to allow recording of several transits of the wave pulse. Figure 12.14 is at 20 $\mu$s per division so as to observe the detail of the initial pulse more closely.

From the test, three features are apparent for the long rod. First, each cycle contains a compressive peak for the PVDF-PWAS (positive) and for the strain gauge (negative). Second, each cycle contains a tensile peak from the reflected wave for the PVDF-PWAS (negative) and the strain gauge (positive). Third, there is a dwell while the wave traverses the length of the rod.

The rod is made of 6061-T6 aluminum alloy, which Young’s modulus is 69 Gpa and density is 2700 kg/m$^3$. The rod is 98 inches (2489 mm) in length and 0.25 inch (6.35 mm) diameter. The theoretical velocity is $c=5055$ m/s. The speed of the pulse moving up and down the rod can be estimated from the time between peaks. The distances traveled by the two adjacent positive or negative pulses are $2L$, where $L$ is the length of the rod. The experimental wave speed is 4860 m/s. The relative error is 3.8%.
Figure 12.13  Impact responses for free-free boundary condition recorded by (CH 1) strain gauge; (CH 2) 28 $\mu$m thick PVDF-PWAS; (CH 3) 52 $\mu$m thick PVDF-PWAS, (CH 4) 110 $\mu$m thick PVDF-PWAS.

Figure 12.14  Impact responses recorded by (CH 1) strain gauge; (CH 2) 28 $\mu$m thick PVDF-PWAS; (CH 3) 52 $\mu$m thick PVDF-PWAS, (CH 4) 110 $\mu$m thick PVDF-PWAS.
12.4.3 Comparison of PZT-PWAS and PVDF-PWAS

Two square (dimension: 7 mm x 7 mm x 0.2 mm) PWAS were attached to a 1200mm x 1100mm x 1.6mm thin aluminum plate with a distance of L (Figure 12.15).

Figure 12.15 Two PWAS mounted on an aluminum plate; the left PWAS as a transmitter, the right PWAS as a receiver.

The left PWAS was a transmitter with a smoothed 300 kHz tone-burst excitation with a 10 Hz repetition rate to generate the Lamb wave in the thin plate. It can be directly connected to an HP33120A function generator that can provide 20V peak to peak excitation, or connected to a high voltage amplifier that can provide 300V peak to peak excitation. The right PWAS was a receiver to collect the Lamb wave signal. It can be directly connected to a Tektronix digital oscilloscope or connected to a charge amplifier module first then the oscilloscope. Transmitter PZT-PWAS can generate a Lamb wave of 2 \( \mu e \) strain under 20V peak to peak excitation. After the Lamb wave propagates to the
receiver PZT-PWAS, the receiver PWAS can generate a 50-mV signal that can be calculated from equation.

A complete comparison of PZT-PWAS and PVDF-PWAS in pitch-catch is shown in Table 12.5. PZT-PWAS and PVDF-PWAS were placed at a distance of 65mm. High Voltage was selected to use to amplify the signals.

<table>
<thead>
<tr>
<th>Transmitter HV Amplifier</th>
<th>Input Voltage(V)</th>
<th>Receiver</th>
<th>Output Voltage(mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 PZT N</td>
<td>17</td>
<td>PZT</td>
<td>116</td>
</tr>
<tr>
<td>2 PZT Y</td>
<td>57</td>
<td>PZT</td>
<td>318</td>
</tr>
<tr>
<td>3 PZT N</td>
<td>17</td>
<td>PVDF</td>
<td>9.3</td>
</tr>
<tr>
<td>4 PZT Y</td>
<td>57</td>
<td>PVDF</td>
<td>30.0</td>
</tr>
<tr>
<td>5 PVDF N</td>
<td>18</td>
<td>PZT</td>
<td>0.96</td>
</tr>
<tr>
<td>6 PVDF Y</td>
<td>57</td>
<td>PZT</td>
<td>2.1</td>
</tr>
<tr>
<td>7 PVDF N</td>
<td>18</td>
<td>PVDF</td>
<td>N/A</td>
</tr>
<tr>
<td>8 PVDF Y</td>
<td>250</td>
<td>PVDF</td>
<td>0.85</td>
</tr>
</tbody>
</table>
13 THIN-FILM PWAS

The type and efficiency of the SHM sensors play a crucial role in the SHM system success. Ideally, SHM sensors should be able to actively interrogate the structure and find out its state of health, its remaining life, and the effective margin of safety. Essential in this determination is to find out the presence and extend of structural damage. Currently, structural damage is determined during scheduled inspections with sophisticated nondestructive evaluation (NDE) equipment and extensive labor costs. The challenge of SHM is to develop inexpensive active sensors that can be permanently placed on the monitored structure and assess, continuous or on-demand, the state of structural health.

13.1 INTRODUCTION TO THIN-FILM TECHNOLOGY

Thin-film technology is good candidate for SHM. However, several scientific challenges, specific to SHM implementation on engineering structures, exist and have to be overcome. The following fundamental scientific challenges need to be addressed

1. **Deposition and growth of ferroelectric thin films and structural materials**: The direct write technology developed for microelectronic substrates (silicon, etc.) never considered structural materials (steel, titanium, aluminum, etc.).

2. **Understanding and predictive simulation** of piezoelectric thin films polarization and electroacoustic interaction with the structural substrate to achieve active damage detection
3. **Design and analysis** of the layered active sensor architecture to achieve ultra low voltage performance and wireless interrogation capability

4. **In-situ fabrication of active sensors on structural materials** to achieve the layered architecture need for ultra low voltage functionality

5. **Predictable and controlled durability** of the active sensors on structural materials under environmental attacks and mechanical fatigue loading

Thin film technology has been shown to have better piezoelectric properties that are close to those of single-crystal ferroelectrics. In addition, the thin films require much smaller poling voltage/ power. They have been successfully integrated in micro/nano electromechanical systems, tunable wireless communication elements, and other modern devices. Progress in this area depends upon the ability to selectively and controllably deposit and characterize thin films with specified physical properties.

The chapter first gave a review of deposition and characterization methods for piezoelectric thin-film active sensors. Comparison of the deposition and characterization methods gave us an idea of which method is most suitable for structural health monitoring. In corporations with other universities and research groups, chemical vapor deposition and pulsed laser deposition were used to deposit the films separately and a variety of characterization methods were used to give us the properties of the thin films. The author also got hand-on experiences on how to operate deposition and analysis tools for material characterization and be able to interpreting data and solving materials-related problems.

265
13.2 State of the Art

Thin-film deposition is a technique for depositing a thin film of material onto a substrate or onto previously deposited layers. "Thin" is a relative term, but most deposition techniques allow layer thickness to be controlled within a few tens of nanometers, and some (molecular beam epitaxy) allow one layer of atoms to be deposited at a time.

13.2.1 Thin-film Deposition Methods

It is useful in the manufacture of optics (for reflective or anti-reflective coatings, for instance), electronics (layers of insulators, semiconductors, and conductors form integrated circuits), and packaging (i.e., aluminum-coated PET film).

Deposition techniques fall into two broad categories, based on whether they are understood in terms of chemistry, or of physics.

13.2.1.1 Chemical Deposition

Here, a fluid precursor undergoes a chemical change at a solid surface, leaving a solid layer. An everyday example is the formation of soot on a cool object when it is placed inside a flame. Since the fluid surrounds the solid object, deposition happens on every surface, with little regard to direction; thin films from chemical deposition techniques tend to be conformal, rather than directional.

Chemical deposition is further categorized by the phase of the precursor:

**Plating** relies on liquid precursors, often a solution of water with a salt of the metal to be deposited. Some plating processes are driven entirely by reagents in the solution (usually for noble metals), but by far the most commercially important process is electroplating. It
was not commonly used in semiconductor processing for many years, but has seen resurgence with more widespread use of chemical-mechanical polishing techniques.

**Chemical vapor deposition** (CVD) is a chemical process often used in the semiconductor industry for the deposition of thin films of various materials. In a typical CVD process the substrate is exposed to one or more volatile precursors, which react and/or decompose on the substrate surface to produce the desired deposit. Frequently, volatile byproducts are also produced, which are removed by gas flow through the reaction chamber.

In a typical CVD process, reactant gases (often diluted in a carrier gas) at room temperature enter the reaction chamber. The gas mixture is heated as it approaches the deposition surface, heated radiatively or placed upon a heated substrate. Depending on the process and operating conditions, the reactant gases may undergo homogeneous chemical reactions in the vapor phase before striking the surface. Near the surface thermal, momentum, and chemical concentration boundary layers form as the gas stream heats, slows down due to viscous drag, and the chemical composition changes. Heterogeneous reactions of the source gases or reactive intermediate species (formed from homogeneous pyrolysis) occur at the deposition surface forming the deposited material. Gaseous reaction by-products are then transported out of the reaction chamber. Chemical vapor deposition (CVD) is a widely used method for depositing thin films of a large variety of materials. Applications of CVD range from the fabrication of microelectronic devices to the deposition of protective coatings.
CVD has different types:

1) Metal-Organic CVD (MOCVD)
2) Plasma Enhanced CVD (PECVD)
3) Rapid Thermal CVD (RTCVD)
4) Atmospheric Pressure CVD (APCVD)
5) Low Pressure CVD (LPCVD)
6) Ultra-High Vacuum CVD (UHVCVD)
7) Atomic Layer CVD (ALCVD).

CVD Shortcoming: Manufacturing issues include control of the amount and uniformity of the deposition's resistivity and thickness, the cleanliness and purity of the surface and the chamber atmosphere, the prevention of the typically much more highly doped substrate wafer's diffusion of dopant to the new layers, imperfections of the growth process, and protecting the surfaces during the manufacture and handling.

13.2.1.2 Physical Deposition

Physical deposition uses mechanical or thermodynamic means to produce a thin film of solid. An everyday example is the formation of frost. Since most engineering materials are held together by relatively high energies, and chemical reactions are not used to store these energies, commercial physical deposition systems tend to require a low-pressure vapor environment to function properly; most can be classified as physical vapor deposition.

The material to be deposited is placed in an energetic, entropic environment, so that particles of material escape its surface. Facing this source is a cooler surface which draws energy from these particles as they arrive, allowing them to form a solid layer. The whole system is kept in a vacuum deposition chamber, to allow the particles to travel as freely.
as possible. Since particles tend to follow a straight path, films deposited by physical means are commonly directional, rather than conformal.

Some examples of physical deposition are given below:

A **thermal evaporator** uses an electric resistance heater to melt the material and raise its vapor pressure to a useful range. This is done in a high vacuum, both to allow the vapor to reach the substrate without reacting with or scattering against other gas-phase atoms in the chamber, and reduce the incorporation of impurities from the residual gas in the vacuum chamber. Obviously, only materials with a much higher vapor pressure than the heating element can be deposited without contamination of the film. Molecular beam epitaxy is a particular sophisticated form of thermal evaporation.

An **electron beam evaporator** fires a high-energy beam from an electron gun to boil a small spot of material; since the heating is not uniform, lower vapor pressure materials can be deposited. The beam is usually bent through an angle of 270° in order to ensure that the gun filament is not directly exposed to the evaporator flux. Typical deposition rates for electron beam evaporation range from 1 to 10 nanometers per second.

**Sputtering** is a physical process whereby atoms in a solid target material are ejected into the gas phase due to bombardment of the material by energetic ions. It is commonly used for thin-film deposition, as well as analytical techniques. Sputtering Phenomenon first described 150 years ago Grove (1852) and plücker (1858) first reported vaporization and film formation of metal films by sputtering. “Sputtering” is a vacuum process used to deposit very thin films on substrates for a wide variety of commercial and scientific purposes. It is performed by applying a high voltage across a low-pressure gas (usually argon at about 5 millitorr) to create a “plasma,” which consists of electrons and gas ions
in a high-energy state. During sputtering, energized plasma ions strike a “target,” composed of the desired coating material, and cause atoms from that target to be ejected with enough energy to travel to, and bond with, the substrate. (Figure 13.1) Sputtering is a physical process whereby atoms in a solid target material are ejected into the gas phase due to bombardment of the material by energetic ions. One important advantage of sputtering as a deposition technique is that the deposited films have the same concentration as the target material.

![Diagram of sputtering](image)

**Figure 13.1 Diagram of sputtering.**

**Pulsed laser deposition (PLD)** is a thin film deposition technique where a high power pulsed laser beam is focused inside a vacuum chamber to strike a target of the desired composition. Material is then vaporized from the target and deposited as a thin film on a substrate, such as a silicon wafer facing the target. This process can occur in ultra high vacuum or in the presence of a background gas, such as oxygen which is commonly used when depositing oxides to fully oxygenate the deposited films.

While the basic-setup is simple relative to many other deposition techniques, the physical pheomena of laser-target interaction and film growth are quite complex. When the laser
pulsed is absorbed by the target, energy is first converted to electronic excitation and then into thermal, chemical and mechanical energy resulting in evaporation, ablation, plasma formation and even exfoliation. The ejected species expand into the surrounding vacuum in the form of a plume containing many energetic species including atoms, molecules, electrons, ions, clusters, particulates and molten globules, before depositing on the typically hot substrate.

The detailed mechanisms of PLD are very complex including the ablation process of the target material by the laser irradiation, the development of a plasma plume with high energetic ions, electrons as well as neutrals and the crystalline growth of the film itself on the heated substrate. The process of PLD can generally be divided into four stages:

1. Laser ablation of the target material and creation of a plasma
2. Dynamic of the plasma
3. Deposition of the ablation material on the substrate
4. Nucleation and growth of the film on the substrate surface

Figure 13.2 One possible configuration of a PLD deposition chamber.
13.2.1.3 Comparison of Deposition Methods

The comparison of three methods used in this dissertation was shown here.

CVD is a chemical deposition method.

Advantage:

Versatile – any element or compound

High Purity – typically 99.99-99.999%

High Density – nearly 100%

Material Formation well

Coatings deposited by CVD are conformal and near net shape

Economical in production

Shortcoming:

Low deposition rate

PLD is a physical deposition method.

Advantage:

Simple – the same composition as the target.

Versatile – many materials can be deposited in a wide variety of gases over a broad range of gas pressures.

Cost-effective – one laser can serve many vacuum systems

Fast – high quality samples can be grown reliably in 10 or 15 minutes.

Scalable – as complex oxides move toward volume production
Shortcoming:

Complex: 4-stage process.

Annealing method is difficult.

Sample size: The film area is determined by the dimensions of the central part of the plume and it is typically 1 cm². The film thickness is typically 100-200 nm.

Sputtering is also a physical deposition method.

Advantage:

Same composition as the source material.

Speed.

Lower impurity.

Shortcoming:

An alloy or mixture component sputter faster than the other components, leading to an enrichment of that component depends on the atomic weight.

Plastic substrates cannot be used.

### 13.2.2 Thin-film Characterization Methods

#### 13.2.2.1 X-Ray Diffraction

X-Ray diffraction (XRD) phenomenon is that the atomic planes of a crystal cause an incident beam of X-rays (if wavelength is approximately the magnitude of the interatomic distance) to interfere with one another as they leave the crystal. Diffraction can occur whenever Bragg's law is satisfied. With monochromatic radiation, an arbitrary setting of
a single crystal in an x-ray beam will not generally produce any diffracted beams. There would therefore be very little information in a single crystal diffraction pattern from using monochromatic radiation. X-ray diffraction was utilized to assess film crystallinity and to determine whether the films possessed a solid solution or multiphase structure.

13.2.2.2 Scanning Electron Microscope

**Scanning electron microscope (SEM)** is a type of electron microscope capable of producing high-resolution images of a sample surface. Due to the manner in which the image is created, SEM images have a characteristic three-dimensional appearance and are useful for judging the surface structure of the sample. Scanning Electron Microscopes use a beam of highly energetic electrons to examine objects on a very fine scale.

This examination can yield the following information:

1) **Topography**: The surface features of an object or "how it looks", its texture; direct relation between these features and materials properties (hardness, reflectivity...etc.).

2) **Morphology**: The shape and size of the particles making up the object; direct relation between these structures and materials properties (ductility, strength, reactivity...etc.).

3) **Composition**: The elements and compounds that the object is composed of and the relative amounts of them. Direct relationship between composition and materials properties (melting point, reactivity, hardness...etc.)

4) **Crystallographic Information**: How the atoms are arranged in the object, direct relation between these arrangements and materials properties (conductivity, electrical properties, strength...etc.)
13.2.2.3 Energy-Dispersive X-ray Spectroscopy

Energy dispersive X-ray spectroscopy (EDX or EDS) is a method used to determine the energy spectrum of X-ray radiation. It is mainly used in chemical analysis, in an X-ray fluorescence spectrometer (especially portable devices), or in an electron microprobe (e.g. inside an scanning electron microscope). Energy Dispersive Spectroscopy (EDS) is a standard procedure for identifying and quantifying elemental composition of sample areas as small as a few cubic micrometers.

The detector is a semiconductor detector, usually a Silicon Drift Detector or a silicon crystal doped with lithium (Si(Li) detector). The semiconductor is polarized with a high voltage; when an X-ray photon hits the detector, it creates electron-hole pairs that drift due to the high voltage. The electric charge is collected, it is like charging a capacitor; the increment of voltage of the condensator is proportional to the energy of the photon, it is thus possible to determine the energy spectrum. The condensator voltage is reset regularly to avoid saturation.

13.2.2.4 Auger Electron Spectroscopy

Auger electron spectroscopy (AES) is an analytical technique in surface chemistry and materials science. Auger electron spectroscopy probes the chemistry of a surface by measuring the energy of electrons emitted from that surface when it is irradiated with electron of energy in the range 2–50 keV. Some of the electrons emitted from the surface have energies characteristic of the element from which they were emitted, and in some cases, the bonding state of those atoms. The physical process by which these electrons are made is called the Auger effect. An Auger spectrum plots a function of electron signal intensity versus electron energy. The Auger energies fall between secondary electron
energies on the low end and backscattered electron energies on the high end. Auger electrons fail to emerge with their characteristic energies if they start from deeper than about 1 to 5 nm into the surface. Thus, Auger analysis is surface specific.

13.2.2.5 Rutherford Backscattering Spectrometry

Rutherford Backscattering (or RBS, for Rutherford Backscattering Spectrometry) is an analytical technique in materials science. It is named for Ernest Rutherford who in 1911 first explained Geiger and Marsden's experimental results for alpha particle scattering from a very thin gold foil in a backward direction by using the Coulomb electrostatic force between the positively charged nucleus and the positively charged alpha particle. Rutherford first correctly described the atom as a tiny positive nucleus surrounded by negatively charged electrons (essentially the Bohr atom) based on this experiment.

A high energy beam (2 - 4 MeV ) of low mass ions ( e.g. He ++ ) is directed at a sample. A detector is placed such that particles which scatter from the sample at close to a 180 degree angle will be collected. The energy of these ions will depend on their incident energy and on the mass of the sample atom which they hit, because the amount of energy transferred to the sample atom in the collision depends on the ratio of masses between the ion and the sample atom. Thus, measuring the energy of scattered ions indicates the chemical composition of the sample.

13.2.2.6 Transmission Electron Microscope

Transmission electron microscope (TEM) operates on the same basic principles as the light microscope but uses electrons instead of light. TEMs use electrons as “light source”
and their much lower wavelength make it possible to get a resolution a thousand times better than with a light microscope. You can see objects to the order of a few angstroms ($10^{-10}$ m). For example, you can study small details in the cell or different materials down to near atomic levels. The possibility for high magnifications has made the TEM a valuable tool in both medical, biological and materials research.

In material science/metallurgy the specimens tend to be naturally resistant to vacuum, but must be prepared as a thin foil, or etched so some portion of the specimen is thin enough for the beam to penetrate. Preparation techniques to obtain an electron transparent region include ion beam milling and wedge polishing. The focused ion beam (FIB) is a relatively new technique to prepare thin samples for TEM examination from larger specimens. Because the FIB can be used to micro-machine samples very precisely, it is possible to mill very thin membranes from a specific area of a sample, such as a semiconductor or metal. Furthermore, a less time consuming and far more common technique is the deposition of a dilute sample containing the specimen on copper or nickel grids just to name a few. These grids consist of thin films of the respective elements, which rest atop sturdier grid bars and act as relatively transparent supports for specimens. Samples are normally deposited as a suspension in a volatile solvent such as ethanol.

There are a number of drawbacks to the TEM technique. Many materials require extensive sample preparation to produce a sample thin enough to be electron transparent, which makes TEM analysis a relatively time consuming process with a low throughput of samples. The structure of the sample may also be changed during the preparation process. Also the field of view is relatively small, raising the possibility that the region analyzed
may not be characteristic of the whole sample. There is potential that the sample may be damaged by the electron beam, particularly in the case of biological materials.

13.2.2.7 Atomic Force Microscope

**Atomic force microscope (AFM)** is another scanning technique used to characterize minute objects. The advantage of AFM is that it creates a three-dimensional, topographic map of a surface, as opposed to SEM and traditional imaging processes, which create only a two-dimensional image. AFM is a very high-resolution type of scanning probe microscope, with demonstrated resolution of fractions of an Angstrom, more than a 1000 times better than the optical diffraction limit. The AFM was invented by Binnig, Quate and Gerber in 1986, and is one of the foremost tools for imaging, measuring and manipulating matter at the nanoscale.

The AFM consists of a micro scale cantilever with a sharp tip (probe) at its end that is used to scan the specimen surface. The cantilever is typically silicon or silicon nitride with a tip radius of curvature on the order of nanometers. When the tip is brought into proximity of a sample surface, forces between the tip and the sample lead to a deflection of the cantilever according to Hooke's law. Depending on the situation, forces that are measured in AFM include mechanical contact force, Van der Waals forces, capillary forces, chemical bonding, electrostatic forces, magnetic forces (see Magnetic force microscope (MFM)), Casimir forces, salvation forces etc. Typically, the deflection is measured using a laser spot reflected from the top of the cantilever into an array of photodiodes.

The AFM has several advantages over the scanning electron microscope (SEM). Unlike the electron microscope which provides a two-dimensional projection or a two-
dimensional image of a sample, the AFM provides a true three-dimensional surface profile. Additionally, samples viewed by AFM do not require any special treatments (such as metal/carbon coatings) that would irreversibly change or damage the sample. While an electron microscope needs an expensive vacuum environment for proper operation, most AFM modes can work perfectly well in ambient air or even a liquid environment. This makes it possible to study biological macromolecules and even living organisms. A disadvantage of AFM compared with the scanning electron microscope (SEM) is the image size.

13.2.3 Thin-film Sensors

In recent years, considerable progress has been achieved in developing NDE methods that actively interrogate the structure using guided Lamb and Rayleigh waves. However, conventional NDE transducers are relatively large and expensive; for SHM applications, smaller and inexpensive active sensors are needed. Recent SHM work has shown that piezoelectric wafers adhesively bonded to the structure may successfully emulate the NDE methodology (pitch-catch, pulse-echo, phased array) while being sufficiently small and inexpensive to allow permanent attachment to the monitored structure.

![Figure 13.3](image)

Figure 13.3 (a) pitch-catch method; (b) pulse-echo method; (c) bonded interface between PWAS and structure.
However, the current methods for fabrication and installation of these sensors on engineering structures are rather “primitive”: pre-manufactured piezoelectric wafers are adhesively bonded to the structural surface. The bonding layer is susceptible to environmental ingression that may lead to loss of contact with the structural substrate. The bonding layer may also induce acoustic impedance mismatch with detrimental effects on damage detection. Not surprisingly, several unresolved fundamental issues impede its development towards industrial acceptance and implementation:

1. Unacceptable durability and survivability of adhesively-mounted piezoceramic wafers
2. Large power and voltage requirements due to poor efficiency of piezoceramic wafers
3. Excessive variability and uncertainty in functional properties due to manual installation methods, which are labor intensive and subjected to extensive human error
4. Inability to utilize advanced sensor architectures due to the use of bulk piezoceramics
5. Unsuitable for efficient wireless interrogation due to the inherent limitations of the basic approach

13.3 PZT THIN FILM USING SPUTTER DEPOSITION

13.3.1 Sample Preparation

Sputter deposition techniques was used to deposit PZT thin film. The basic sputter techniques includes planar diode sputtering (DC sputtering), radio frequency (RF)
sputtering and magnetron sputtering, et al. DC sputtering uses target as a cathode. It must be a conductive material (metal). RF sputtering uses radio frequency waves instead of the traditional DC current. DC sputtering can only be used to deposit conductive materials, whereas RF sputtering can work on nonconductive materials as well. This is essential to piezoelectric sensor applications, as PZT, among other materials that demonstrate piezoelectric properties, are not conductive and therefore cannot be deposited using DC sputtering.

Our substrate sample was sputtered onto a glass microscope slide, on top of a layer of aluminum (Al), and under a layer of silver (Ag). The schematic of the sample is shown in Figure 2.3. Procedure for producing a sputtering sample involves starting off with the DC sputtering technique to deposit the aluminum layer, then employing the RF sputtering technique to deposit the PZT, and finally, using DC sputtering once again for the silver layer.

The samples preparation was done on a standard glass microscope slide with the dimensions of 7.5 x 2.5 cm (Figure 13.4). The first layer, aluminum, was sputtered on using DC sputtering over the entire glass surface. The second layer, PZT, was then sputtered on top of the Aluminum layer using RF sputtering, this time over an area of 4.5 x 2.5 cm centered over the Al layer and leaving sections of the Al exposed at the edges. The third and final layer, Ag, was then sputtered on using DC sputtering, over an area of 3 x 1.7 cm, again centered, leaving sections of Al and PZT exposed as well.
13.3.2 Characterization of PZT Thin Film

Observation of sputtered samples under a light microscope showed nothing of significance, due to the fact that magnification under a light microscope is not anywhere near the magnification required to see variations at the scale of the layer thickness (1 micron meter.).

Under further magnification, with the use of an SEM (scanning electron microscope), we were able to see discontinuities on the PZT sputtered surface. However, since the RF sputtering process occurs at room temperatures (as opposed to high temperatures), the resulting thin film layer is already considerably more homogenous than one deposited using more traditional, high temperature methods. The patterns of discontinuity might be significant because the sputtering process produces supposedly continuous, homogenous thin film layers. The PZT layer as seen in Figure 13.5 shows blemishes, most likely attributed to dust and other particles attached to either the surface of the PZT and not as a result of actual discontinuity in the surface.
Figure 13.5 (a) PZT layer observed through scanning electron microscope (SEM). Note the defects in the surface. (b) AFM surface roughness analysis.

The thickness of the PZT layer is of the order of a few microns, whereas the surface variations are in mere nanometers. This means that for all reasons of comparison, the surface is rather smooth and homogenous. This is important because the smoother the surface of the PZT, the more accurate and consistent the sensor will be when used in such
an application. A long-term goal of thin-film deposition technology has always been, and still is, to construct the layer with as little variance range on the z-axis as possible, therefore resulting in less variation in measurements. One of the main advantages of sputtering versus more traditional ways of depositing thin-films is that it can achieve a smoother and generally more homogenous layer on top of a substrate.

Since the AFM unit essentially does a 3-D scan of the surface, we can obtain visuals of the surface in just about any direction, along with a variable amount of area observed, much like a zoom function on a traditional microscope. The roughness analysis shown in Figure 13.5 covers an area of 10 x 10 microns, whereas in Figure 13.6 our area is 5x5 and 1 x 1 microns. The 3-D scan allows us a better representation of how the surface would look if it were on a larger scale.

From the data collected, we can conclude that through the use of RF sputtering, PZT thin film layers can be deposited on a substrate surface. The fact that homogeneity seems consistent to a nanoscale is determined by atomic force microscopy. Further characterization of the sample was done through the process of X-Ray diffraction. The X-ray diffraction process uses X-ray waves to determine the presence of different substances. In our case, X-ray diffraction was used to determine the presence of aluminum, silver, and PZT. Our deposited PZT thin film layer is amorphous as opposed to the required crystalline structure for visibility under an XRD survey; we were not able to see the PZT under the XRD. The reason for amorphous structure of the sputtered film may due to the lack of oxygen during the sputtering process. We need to provide oxygen during sputtering process to get the crystalline structure. The piezoelectric characteristics of the PZT are not discovered as opposed to the physical characteristics so far.
13.4 BSTO Thin-Films Using Metal-Organic Solution Deposition

Piezoelectric thin films $\text{Ba}_{0.8}\text{Sr}_{0.2}\text{TiO}_3$ (BST) were directly integrated with titanium utilizing a metal-organic chemical solution approach in collaboration with US Army Research Laboratory (ARL). This research addresses the need of directly integrating the piezoelectric thin films with the structural materials (alpha titanium for this research) by
developing and utilizing a metal-organic solution deposition (MOSD) approach to optimize the integration strategy.

13.4.1 Thin-film MOSD Experimental Setup

The Ba$_x$Sr$_{1-x}$TiO$_3$ (BST) piezoelectric thin films with Ba/Sr compositions of 80/20 were fabricated on structural alpha titanium stubs via the metal-organic solution deposition (MOSD) film fabrication techniques. Pre-deposition preparation of the substrates involved a standard multi-step cleaning process. The substrates were dipped in acetone and methanol; the bare Ti stubs were also exposed to a weak HF solution to remove native oxide, and then rinsed in de-ionized (DI) water. A subset of titanium stubs (1 cm x 1 cm x 0.25 cm) was thermally oxidized at 500 °C to grow a TiO$_2$ oxide buffer layer on the surface in an attempt to minimize the reaction between the BST thin-film and the titanium during the annealing step. A trace layer of platinum was deposited on all stubs to monitor the rate and extent of reaction between the BST thin-film and titanium. For the MOSD BST film fabrication, barium acetate, strontium acetate, and titanium isopropoxide were used as precursors to form BST. Acetic acid and 2-methoxyethanol were used as solvents. The precursor films were spin coated onto the substrates. Particulates were removed from the solution by filtering through 0.2 mm syringe filters. Subsequent to each coat, the films were pyrolyzed at 350 °C for 10 min in order to evaporate solvents and organic addenda and form an inorganic amorphous film. The spin coat pyrolyzation process was repeated until a nominal film thickness of 300 nm was achieved. Crystallinity was achieved via post-deposition annealing in an oxygen ambience.
A series of films (16 samples total) were deposited at ARL using metal organic solution deposition on titanium stubs polished by USC. An appropriate nanometer grain size piezoelectric BST 80/20 composition powder was used for the composite thin films using MOSD method. The films were annealed at varying temperatures after deposition. The details of each combination on samples are shown in Table 13.1.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Oxide Layer *</th>
<th>Trace Layer **</th>
<th>Anneal Method</th>
<th>Anneal Temp</th>
<th>BST Film</th>
<th>Roughness</th>
<th>Grain Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>USC 11</td>
<td>N</td>
<td>N</td>
<td>none</td>
<td>none</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>USC 12</td>
<td>Y</td>
<td>N</td>
<td>none</td>
<td>none</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>USC 13</td>
<td>N</td>
<td>Y</td>
<td>none</td>
<td>none</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>USC 14</td>
<td>Y</td>
<td>Y</td>
<td>none</td>
<td>none</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>USC 15</td>
<td>Y</td>
<td>Y</td>
<td>standard ***</td>
<td>650 C</td>
<td>80/20</td>
<td>2.316</td>
<td>30.303</td>
</tr>
<tr>
<td>USC 16</td>
<td>Y</td>
<td>Y</td>
<td>standard</td>
<td>700 C</td>
<td>80/20</td>
<td>4.355</td>
<td>38.461</td>
</tr>
<tr>
<td>USC 17</td>
<td>Y</td>
<td>Y</td>
<td>standard</td>
<td>750 C</td>
<td>80/20</td>
<td>6.388</td>
<td>unavailable</td>
</tr>
<tr>
<td>USC 18</td>
<td>Y</td>
<td>Y</td>
<td>standard</td>
<td>800 C</td>
<td>80/20</td>
<td>10.85</td>
<td>52.631</td>
</tr>
<tr>
<td>USC 19</td>
<td>N</td>
<td>Y</td>
<td>standard</td>
<td>650 C</td>
<td>80/20</td>
<td>3.292</td>
<td>45.454</td>
</tr>
<tr>
<td>USC 20</td>
<td>N</td>
<td>Y</td>
<td>standard</td>
<td>700 C</td>
<td>80/20</td>
<td>3.526</td>
<td>55.555</td>
</tr>
<tr>
<td>USC 21</td>
<td>N</td>
<td>Y</td>
<td>standard</td>
<td>750 C</td>
<td>80/20</td>
<td>4.487</td>
<td>unavailable</td>
</tr>
<tr>
<td>USC 22</td>
<td>N</td>
<td>Y</td>
<td>standard</td>
<td>800 C</td>
<td>80/20</td>
<td>10.759</td>
<td>76.923</td>
</tr>
<tr>
<td>USC 23</td>
<td>N</td>
<td>Y</td>
<td>RTA ****</td>
<td>650 C</td>
<td>80/20</td>
<td>3.392</td>
<td>amorphous</td>
</tr>
<tr>
<td>USC 24</td>
<td>N</td>
<td>Y</td>
<td>RTA</td>
<td>700 C</td>
<td>80/20</td>
<td>7.14</td>
<td>25.641</td>
</tr>
<tr>
<td>USC 25</td>
<td>N</td>
<td>Y</td>
<td>RTA</td>
<td>750 C</td>
<td>80/20</td>
<td>3.379</td>
<td>35.714</td>
</tr>
<tr>
<td>USC 26</td>
<td>N</td>
<td>N</td>
<td>RTA</td>
<td>800C</td>
<td>80/20</td>
<td>3.389</td>
<td>50</td>
</tr>
</tbody>
</table>

* Oxide Layer: 500 C under flowing O2 to grow a thermal TiO2 layer several hundred nm thick
** Trace Layer: Pt layer a few nm thick to monitor the diffusion of Ti
*** Standard: at anneal temp for 60 minutes under flowing O2
**** RTA (Rapid Thermal Anneal.) at anneal temp for 5 minutes under flowing O2

The samples with the thermal oxide and a reference set of BST/Ti stubs were annealed under standard conditions at 650, 700, 750, 800 °C for 60 minutes under flowing oxygen.
A final set of BST/Ti stubs without a thermal oxide layer was exposed to a rapid thermal annealing at 650, 700, 750, 800 °C for 5 minutes under flowing oxygen. There are four groups. The first group (USC11 - USC14) was used to deposit oxide layer and trace layer and no piezoelectric thin film. The second group (USC15 - USC18) uses both the oxide layer and trace layer and was deposit the BST 80/20 film with a standard anneal method at varying temperatures. The third group (USC19 – USC22) uses only trace layer and was deposit the BST 80/20 film with a standard anneal method at varying temperatures. The fourth group (USC23 – USC26) uses only trace layer and was deposit the BST 80/20 film with a RTA anneal method at varying temperatures.

13.4.2 Thin-film Experimental Results

The active thin films were characterized for structural, microstructural, compositional, and surface morphological properties. A Hitachi S4500 field emission scanning electron microscopy (FESEM) was employed to detail the films’ plan-view microstructure, compositional uniformity, phase formation, and surface morphology. Specifically, the surface roughness and grain size of the constituent films and the composite heterostructure was assessed by a Digital Instrument’s Dimension 3000 atomic force microscope (AFM) employing tapping mode. Rutherford backscattering spectrometry (RBS) was utilized to assess the film elemental composition, a real thickness, interdiffusion, and film-substrate interface quality. The RBS measurements were obtained using 1.2 MeV He\(^+\) ion beams from an NEC 5SDH-2 tandem positive ion accelerator. All spectra were fit and interpreted using the program RUMP. Glancing angle x-ray diffraction (GAXRD) with a Rigaku diffractometer and CuK\(\alpha\) radiation at 40°.
289 kV was employed to assess film crystallinity, phase formation, and reaction rate and extent.

13.4.2.1 Atomic Force Microscope (AFM)

The SEM and AFM image of sample USC15 are shown in Figure 13.7. The sample USC15 with both oxide and trace layer and a standard anneal at 650 gave a low surface roughness and small grain size.

![Figure 13.7 USC 15 sample (a) FESEM image (b) AFM image.](image)
AFM was employed to monitor the surface evolution during heat treatment. It is well known that annealing thin film oxide materials has both advantageous and disadvantageous results. Higher heat treatment temperatures will substantially increase the degree of film crystallinity, which increases the piezoelectric properties, but it will also lead to grain growth. Though larger grains also improve the piezoelectric properties, they are also responsible for the substantial roughening of the surface, making it very difficult to deposit further layer, such as electrodes, on top of the BST film. The effects of annealing and thermal oxide layer are evident in the data shown in Figure 13.8.

Figure 13.8 The atomic force microscopy (AFM) results detailing the roughness and grain size as a function of the annealing temperature for samples that underwent a standard annealing treatment, had a thermal oxide layer deposited before undergoing a standard annealing treatment, and underwent a Rapid Thermal Annealing (RTA) treatment.

The surface roughness as measured by AFM indicates that the growth of a thermal oxide substantially roughens the surface prior to any thin-film deposition. Comparing the surface roughness with different groups, RTA group gave a relative constant roughness. The roughness of oxide layer group and standard anneal group increased with the anneal
temperature increased. The final thin-film surface roughness increases substantially with increasing annealing temperature for the standard annealing conditions.

The grain size, however, increased with increasing annealing temperature for all treatment methods. Most mechanical properties were improved as the size of the grains decreased. Comparing the average grain size with different group, the average grain size in all groups got increased with the anneal temperature increased. The RTA anneal method gave a relative low grain size. The largest grain size was observed in the BST films grown on the titanium without a thermal oxide buffer layer. However, the rate of increase in grain size between the BST samples grown on a thermal oxide and those without is comparable, indicating that the observed increase in grain size under the standard annealing conditions without a thermal oxide buffer layer is largely due to the increase in grain size within the titanium substrate.

13.4.2.2 Rutherford Backscattering Spectrometry (RBS)

Rutherford backscattering spectrometry (RBS) was utilized to examine the atomic level interactions of the BST layer with the underlying substrate for all treatment conditions, and the results are shown in Figure 13.9. From the RBS it is possible to directly measure the BST thicknesses at around 500 nm in all cases. The “preoxidation” of the Ti created a TiO₂ layer around 280 nm +/- 100 nm thick, indicating an oxide of somewhat non-uniform thickness. The 2 nm Pt marker layer was detectable after deposition, but it was undetectable after every anneal, standard or RTA, indicating that it must have diffused easily; it may have even reacted with the Ti during the pyrolysis step of the MOSD process.
Figure 13.9. Rutherford backscattering spectrometry results for the titanium stubs with (a) BST, trace Pt layer, no thermal oxide, and standard anneal; (b) BST, trace Pt layer, no thermal oxide layer, and RTA; (c) BST, trace Pt layer, thermal oxide, and standard anneal; and, (d) unannealed Ti stub and unannealed Ti stub with trace Pt layer.

For standard anneals, the BST reacted with the underlying Ti or TiO$_2$ in every case. The Ti samples that underwent the standard annealing treatment show a monotonically increasing reaction rate, as measured by examining the slope of the low energy side of the BST peak in Figure 13.9. It appears that the thermally grown TiO$_2$ slightly delayed the reaction onset (from 700°C in the standard anneal case to 750°C in the case of the samples with a thermal oxide). Somewhere between 700 and 750°C the BST started to
significantly react with the underlying TiO$_2$. There was significantly less BST/substrate reaction in the case of RTA; however, there was still evidence of interaction, which seems to be independent of the RTA temperature. For all three sets of samples, except for the 800°C standard anneals, all of the samples remained still substantially “BST” on top of an interaction layer (i.e., they weren’t fully consumed by the substrate).

13.4.2.3 Glancing Angle X-ray Diffraction (GA-XRD)

Glancing angle x-ray diffraction (GA-XRD) was utilized to measure the crystallinity of the BST layer, the presence of a piezoelectric phase, and to verify the measurements made by RBS regarding the interlocution of the oxide and BST layers. Figure 13.10A-C shows the results of the GA-XRD, where the peaks represented by a normal script (100) are for the BST, while peaks with italic script (100) are for the rutile phase of titania. The most striking item, when first examining the GA-XRD spectrum, is that for all annealing temperatures, even as low as 650°C, there are substantial BST peaks apparent. Previous research on semiconductor substrates had indicated that an annealing temperature of at least 700°C was necessary to generate reasonable crystallinity and 750°C was preferred.

Examining Figure 13.10A, there are several items of interest for the samples that underwent a standard annealing treatment with no thermally grown buffer layer. First, the titania and the BST peaks become narrower with increasing annealing temperature indicating improved crystallinity. Secondly, the area beneath the titania peaks grows at the expense of the BST peaks, verifying the RBS results that indicated that the BST was reacting with the underlying titanium. Finally, the BST peaks show noticeable peak broadening and splitting, which is indicative of the piezoelectric, tetragonal phase of BST. Figure 13.10D shows a field emission scanning electron microscope (FE-SEM) of
the surface and grain structure sample that underwent the standard annealing process at 650°C without a thermally grown oxide buffer layer. The grains are uniform and well developed and do not demonstrate any voiding or pin holes. This BST thin film on the structural titanium stub is representative of the BST films.

When the samples with a thermally grown oxide buffer layer that underwent a standard annealing process are examined, in Figure 13.10B, many similarities are seen to the samples that did not have a thermal buffer layer but also underwent the standard annealing process. Both materials become more crystalline with increasing annealing temperature. The titania layer grows at the expense of the BST layer, but it appears to do so at a slower rate than the samples without a thermal buffer layer. The peak splitting associated with the piezoelectric tetragonal phase is especially apparent in the (200) and (211) peaks. When the samples that underwent a RTA treatment without a thermally grown oxide were examined, many of the same features are observed. The materials become more crystalline with increasing annealing temperature; the oxide material increases with increased annealing temperature, and the presence of the piezoelectric tetragonal phase is observed for all annealing temperatures. Additionally, there are several interesting aspects seen in the RTA samples. The * in Figure 13.10C indicates the presence of additional phases. When the peaks centered around 39° are examined, it is apparent that there are three distinct peaks, where Figure 13.10A and Figure 13.10B had only a single dominant peak. In Figure 13.9C the peak at 38° and 40° are for barium titanate and strontium titanate respectively, while the peak centered at 39° is for the atomistic mixture BST.
Figure 13.10  Glancing Angle X-Ray Diffraction patterns for Ti stubs with BST for (a) standard annealing conditions and no thermal oxide, (b) standard annealing conditions with a thermal oxide, and (c) RTA with no thermal oxide. The diffraction peaks for BST are the lower set of numbers, while the diffraction peaks for rutile are shown in italics as the higher set of numbers. (d) Shows a representative planar FE-SEM image of the grain structure of the BST thin films on the Ti stubs.

Similar segregations can be seen for the 650°C anneals observed in Figure 13.10A and Figure 13.10B. When the annealing temperature is increased the BST peak becomes more dominant and the barium titanate and strontium titanate gradually disappear. However in the RTA samples, even at 800°C there is a residual strontium titanate peak. This indicates that the atomistic mixing typically observed for MOSD samples, even at the pyrolization step, is inhibited. While crystallinity appears to occur at substantially lower annealing temperatures for the MOSD samples on structural titanium substrates, a much higher activation energy for atomistic mobility on the surface is required. This is indicative a
substrate surface is substantially rougher than the atomically smooth semiconductor substrates.

13.4.3 Summary of BST MOSD Deposition

The BST thin-film deposition treatments were all successfully applied to the polished structural titanium stubs. Glancing angle X-ray diffraction illustrated that all of the BST films were crystalline, possessed BST and exhibited peak splitting of the (110) and (200) peaks, which is indicative of the piezoelectric phase, even at the lowest annealing temperature of 650°C. However, maximum crystallinity was achieved at 800 °C annealing for all samples. For the RTA samples in addition to BST, there were also barium titanate and strontium titanate phases present. The surface, which was rougher than semiconductor substrates, substantially lowered the activation energy necessary for nucleation and growth of the crystalline phases; however, the same roughness also raised the activation energy for surface hopping of the various atoms, impeding atomic level mixing.

The AFM results indicated that grain size increased with annealing temperature, indicative of increasing piezoelectric properties. The difference between grain size of the oxide buffer layer samples and the standard anneal samples indicates that the larger grain size observed in the standard anneal samples is largely due to a growth of the substrates grains. The roughness substantially increased for the standard and the oxide buffer layer samples, but remained nearly constant for the RTA samples as the annealing temperature increased. RBS indicated that only minimal reaction occurred for the RTA samples at all annealing temperatures, but all of the standard and the oxide buffer layer samples experience some intermixing. The oxide buffer layer, though, delayed the intermixing as
compared to the standard annealing process. The presence of a reaction or interaction layer is not necessarily adverse from a performance perspective. The interaction layer will substantially increase the mechanical and the piezoelectric coupling between the sensor and the substrate. The higher heat treatment temperatures responsible for the large interaction zone also caused a substantially improved piezoelectric and crystallographic material to be formed. It may even be beneficial to intentionally sacrifice a portion of the thin film to create a bonding/coupling layer, so the interaction may not be a bad thing per se. Therefore, the optimum treatment was a mechanical polishing treatment followed by deposition of the BST thin films and an RTA annealing at 750-800°C. The use of an oxide buffer layer would substantially roughen the surface, but would also inhibit the intermixing between the BST thin-film and the substrate. For the RTA annealing there is only minimal intermixing, so the use of an oxide buffer layer would not be necessary. All of these processes are also the most readily adaptable to a Depot.

### 13.5 BTO Thin-Film Using Pulsed Laser Deposition

In this section, ferroelectric thin films directly on the structural substrate were produced using pulsed laser deposition (PLD). The methods for in-situ fabrication of piezoelectric wafer active sensors arrays using the nano technology approach was developed.

#### 13.5.1 BaTiO3 Thin Films on Ni Tape

Thin-film active nano-PWAS research consists of two parts, thin-film fabrication and nano-PWAS construction. The first part is how to fabricate the piezoelectric thin-film on structures. Recently, Chen at UTSA has developed a unique technique for the first time to achieve in-situ fabrication of BTO thin films on the typical structural material Ni and Ti.
using PLD technique. Ferroelectric BaTiO3 (BTO) thin films were successfully deposited on Ni tape and Ti plate by pulsed laser deposition under the optimal synthesis conditions. The interfaces between the BTO films and substrates were found to be with/without a buffer layer NiO that was usually formed during the deposition of metal on oxygen or oxide materials at high temperature. BaTiO3 thin films were deposited on Ni and Ti substrates in a PLD system using a KrF excimer laser. The BTO thin films were fabricated with various conditions, details can be found next. X-ray diffraction (XRD) was employed to understand the crystal phases and the transmission electron microscopy (TEM), plan-view and cross-section, were employed to study the microstructure of the as-grown films and interfacial layers. The dielectric properties were characterized by using a Radiant RT6000 for understanding the physical properties of the as-grown films.

13.5.1.1 BaTiO3 Thin Films on Ni Tape with NiO Buffer Layer

A KrF excimer PLD system with a wavelength of 248 nm was used to perform the fabrication of ferroelectric BTO thin films on Ni substrates. A layer of NiO with its thickness of 80 nm was synthesized via in situ oxidation treatment of the Ni tape in oxygen atmosphere of 300 Torr at 800°C for 3 min. The optimal growth conditions for ferroelectric BTO thin film growth on Ni tape were found to be an energy density of about 2.0 J/cm² with a laser repetition rate of 5Hz in an oxygen pressure of 300 mTorr at 800°C. Stoichiometric BTO target was used for the BTO growth. The typical growth rate and the film thickness were about 6 nm/min and 400 nm, respectively. The experimental setup is shown in Figure 13.11.
Figure 13.11  Pulsed laser deposition system experimental setup.

Figure 13.12 shows the as-deposited BTO thin film on Ni with NiO buffer layer. A layer of NiO with its thickness of 80 nm was synthesized via in situ oxidation treatment of the Ni tape in oxygen atmosphere of 300 Torr at 800°C for 3 min. NiO has rock salt structure with good electrical conductivity, which can be used as conductive electrodes and avoiding the formation of a dead layer formed between the ferroelectric film and metal substrate. It should be pointed out that the rock salt structural NiO has good crystallographic compatibility to the perovskite microstructure of BTO growth.

Figure 13.12  TEM image of the BTO films on Ni with NiO interlayer (a) Plane-view (b) Cross-section. Inset is SAED pattern.
13.5.1.2 TEM

Jiang from UTA conducted a thorough and systematic microstructure and interface analysis of the structurally integrated thin film sensors. The effect of the processing conditions on the quality of the sensors and its interface with respect to the substrate was investigate, which can be used for optimizing the processing condition to achieve the high quality film sensors and strong sensor/substrate bonding. Microstructure studies suggest that the films have polycrystalline nanopillar structures with an average size of approximately 90 nm as reveal by plan-view and cross-section transmission electron microscopy (TEM). The size of the grains in Figure 13.12(a) varies from 35 nm to 160 nm in diameter, while the majority grains have a size of about 90 nm in diameter. The inset SAED pattern of the film shows sharp diffraction rings, indicating that all the grains are in plan randomly oriented. The structure of the as-grown BTO film was identified as a tetragonal structure with a space group of p4mm and lattice parameters of a=3.992 Å and c=4.036 Å. For example, the inner 6 diffraction rings 1, 2, 3, 4, 5 and 6 have a lattice spacing of 4.0 Å, 2.8 Å, 2.3 Å, 2.0 Å, 1.8 Å, 1.64 Å and 1.4 Å, respectively, which can be identified as the (001), (101), (111), (002), (102) and (112) reflection of the tetragonal BTO. Figure 13.12 (b) is a cross-sectional TEM image showing the interface structures of the BTO films on NiO buffered Ni tapes and the inset is the low magnification image of the films. The BTO film has a thickness of about 500 nm and consists of nanopillar structures. Most of the nanopillars extend from the film/substrate interface to the film surface with a length of about 500 nm and show a lateral dimensions from 30 nm to 100 nm (inset), which is close to the value obtained from the plan-view TEM. An intermediate layer that can be identified as NiO was observed between the BTO film and
Ni substrate (Figure 13.12 b) indicating that a NiO oxidized layer was successfully produced prior to the deposition of BTO film. The BTO film is found to be very well bound the NiO layer with a sharp interface in between. The NiO layer has a thickness of about 80 nm and a clear interface with respect to the Ni substrate.

### 13.5.1.3 BaTiO3 Thin Films on Ni Tape Without NiO Buffer Layer

The ferroelectric BTO films can be directly grown on the nano Ni surface without the NiO interlayer by nano fabrication technique. As seen in Figure 13.13 (a), a cross-sectional TEM image shows the interface structure of the BTO films on Ni tapes. The BTO film has a thickness of about 200 nm and consists of nanopillar structures with lateral dimensions of about 100 nm, which is close to the value obtained from the plan-view TEM. The intermediate (IM) layer between the BTO film and Ni substrate shows nanocrystalline structure. Figure 13.13(b) is a plane-view TEM image and the SAED pattern (inset) of the IM layer. The grain size of the nanostructures varies from 30 to 100 nm in diameter and is smaller than that of the BTO grains. The nanostructures in the IM layer were found to be pure Ni (\(a = 3.52 \text{ Å}\)) as identified by the electron diffraction analysis. For example, diffraction rings 1, 2, 3 and 4 have a lattice spacing of 2.03 Å, 1.76 Å, 1.25 Å and 1.06 Å, respectively, which can be identified as the (111), (200), (220) and (311) reflection of Ni. More than 60% of the nanopillars in the films possess the same orientation with the \(a\)-axis lying in the film plane and the [011] axis perpendicular to the film plane. This achievement suggests that the BTO films directly on Ni tape with no NiO layer has paved the way to develop supercapacitance devices for the energy harvest applications.
13.5.1.4 X-ray Diffraction

X-ray diffraction $\theta - 2\theta$ scan (XRD) were employed to characterize the microstructures and crystallinity of the as-grown BTO films. As seen in Figure 13.14 (a), the XRD pattern from the as-deposited BTO thin film on Ni shows all the peaks from the polycrystalline BTO phases and Ni substrate. These peak positions suggest that the Ni substrate is cubic phase and the BTO film belongs to the tetragonal phase. This indicates that the BTO film has randomly oriented grains. Figure 13.14 $\theta - 2\theta$ The impact of the microstructure difference on the physical properties can be clearly seen from the PRM image of the films (Figure 13.14 (c) and (d)). The films with orientation preferred structures possess nearly the same polarization direction over 60% areas. The dielectric and ferroelectric property studies demonstrated that such BaTiO$_3$ films on Ni substrate exhibit a very high resistivity value of $10^{10}$ $\Omega$cm.

13.5.1.5 Ferroelectric Property

The ferroelectric property measurements were also performed at room temperature, as seen Figure 13.15. It is surprisingly found that the as-grown BTO films on Ni metal tapes
with a NiO buffered layer exhibit very high resistivity value of $10^{10} \ \Omega cm$. The ferroelectricity of the BTO films was evidenced from the hysteresis loop. The room temperature spontaneous polarization, remnant polarization, and coercive field from the as-deposited BTO layer can be obtained to be about 2.0 $\mu C/cm^2$ and 1.0 $\mu C/cm^2$, respectively, with a coercive field of 25 kV/cm. It is known that the lattice dipole along the c-axis for a tetragonal perovskite structure is the origin of the ferroelectric properties associated with BTO.

Figure 13.14  (a) X-ray diffraction $\theta$-2$\theta$ scans of randomly oriented BTO/NiO/Ni thin film; (b) X-ray diffraction $\theta$-2$\theta$ scans of oriented BTO/Ni thin film; (c) PRM images of the films with randomly oriented BTO/NiO/Ni; (d) orientation preferred BTO/Ni nanostructures.
In other words, the ferroelectric dipole originates from ionic displacement in the c-axis direction, only c-axis oriented BTO thin films exhibit ferroelectricity. The a-axis oriented BTO film cannot show ferroelectric hysteresis due to the randomly oriented polarization, the large spontaneous polarization obtained in the as-deposited film is consistent with the result of the microstructure measurement that the film has highly c-axis oriented texture structure. The piezoelectric response of the as-deposited BTO film was surprisingly found to be 130 (x 10^{-12} C/N) which is about 30% larger than the values (90 – 100 x 10^{-12} C/N) of BTO single crystalline and polycrystalline bulk materials. The large piezoelectric response might result from the uniform nanodomain structures. The use of Ni substrate and alloys with Ni buffer layer offer better crystallinity of thin films according our preliminary results. However, despite the widespread application of PLD, the epitaxial quality and the microstructures of the as-grown oxide thin films are highly dependent upon the synthesis conditions and the selected materials. Stoichiometric films can only be synthesized under certain processing conditions (temperature, pressure, incident beam energy density, etc.).
13.5.1.6 Scanning Electron Acoustic Microscopy

Bhalla and Guo at UTSA have developed by a novel characterization technique, scanning electron acoustic microscopy (SEAM). It has the resolution of an electron microscope but penetration depth one to two orders of magnitude larger. (The penetration depth is adjustable using acoustic signal modulation.) SEAM is based on the detection of electron acoustic (EA) signals that are generated by heating with a pulsed electron beam modulated by a driving electric signal applied on a beam blanker in a scanning electron microscope (SEM) column. The propagated EA signal is sensed by a piezoelectric transducer in contact with the specimen. Through computer controlled scan and amplification, the SEAM images are formed for a particular depth of penetration, as a function of modulation frequency. SEAM has been used successfully in the study of ferroelectric ceramics, semiconductors, magnetic materials, and many others. It has the advantage of permitting a microstructural view into the bulk of a small sample. This is in contrast to X-ray diffraction and SEM examinations that give only information on the surface regions. This newly developed technique seems ideally suited to provide both the needed resolution and the in-depth imaging capability in the real state of the sensor deposited on the substrate.

13.5.2 BaTiO₃ thin films on Ti

BaTiO₃ films on the Ti substrates fabricated using PLD by modifying the growth conditions have also been studied. BaTiO₃ films are composed of crystalline nanopillars with good interface structures with respect to the Ti substrates (Figure 13.16). The grain size of the BaTiO₃ nanopillars and their interface structures between the film and Ti were found to strongly depend on the fabrication conditions. BaTiO₃ thin films composed of
randomly oriented and orientation preferred nanopillars with a grain size varying from 50 nm to 200 nm were successfully fabricated. The BaTiO$_3$ films have a good interface and strong adhesion with respect to the Ti substrate through a grain sized gradient Rutile TiO$_2$ intermediate interfacial region.

### 13.5.2.1 BTO on Ti Characterization

Figure 13.17 is a $\theta - 2\theta$ XRD pattern of the as-deposited BTO thin film on Ti exhibiting peaks from BTO, Rutile TiO2 and Ti. The BTO films on Ti were found to have the tetragonal structure with lattice constant $a = 4.00\,\text{Å}$ and $c = 4.03\,\text{Å}$. The rutile TiO2 has a lattice constant $a = 4.61\,\text{Å}$ and $c = 2.97\,\text{Å}$. The Ti substrate has a structure of $\alpha$-Ti and the lattice constant was found to be $a = 2.97\,\text{Å}$ and $c = 4.78\,\text{Å}$, which is about 1% larger than the lattice constant of pure Ti ($a=2.95\,\text{Å}$ and $c=4.68\,\text{Å}$). Such a lattice expansion might be understood as the induction of the O atoms into the Ti lattice.

![Figure 13.16](image.png)

**Figure 13.16** BTO on Ti (a) Cross-section and (b) plan-view TEM image of the BTO films on Ti. Inset is SAED pattern of (b).
A revolutionary new approach to SHM active sensors architecture is described herein based on the following fundamental objectives:

1. Seamless atomic bond between the active sensor and the structure to ensure a durable and reliable connection between the active sensor and the structure

2. Enhanced piezoelectric properties (coherent crystalline structure, well-oriented electric domains, new piezoelectric formulations) to improve the sensor’s inherent response

3. In-situ fabrication of the active sensors directly on the structure, using direct-write technology

Figure 13.17 $\theta - 2\theta$ XRD pattern of the BTO films on Ti substrate.
4. Ultra-low voltage layered architecture to permit direct wireless interrogation and autonomous operation using environmental energy harvesting.

It is estimated that implementation of this approach will permit orders of magnitude improvements in active SHM sensors performance and durability. The direct-write achievements in the microelectronics industry give in principle credibility to the approach described herein.

**13.6.1 Nano-PWAS Approach**

The novel approach illustrated here consists in using layered architecture in concern with guided wave SHM understanding and modeling to move from current single-layer PWAS to future multilayer nano-PWAS, as shown in Figure 13.18.

The thin film technologies enable the development of several highly-efficient active sensors. Our goal is to achieve wireless battery-less capability similar to the RFID tags ubiquitous in the commercial world. In our vision, we envisage a battery-less active sensor to be powered only by an interrogating microwave beam aimed at it from a standoff distance.

![Figure 13.18 Sequential development of the multi-layer battery-less nano-PWAS phased array.](image)

308
Our new sensors (dubbed “nano-PWAS” due to the thin-films nano size) concept will be developed in incremental steps as follows:

1. Single-layer thin-film nano-PWAS

2. Multi-layer thin-film nano-PWAS

3. Micro phased array of nano-PWAS transducers

4. Wireless battery-less operation via tag antenna

13.6.1.1 Single-layer Thin-film nano-PWAS

Figure 13.19  (a) PZT plate with SAW type electrode; (b) Impedance resonance of SAW type sensor.

The single-layer thin-film nano-PWAS consists of thin piezoelectric film directly deposited on a structural substrate with an electrode pattern deposited on top. We envisage the use of an interdigitated (IDT) electrode pattern that will permit tuning in selected MHz frequencies, as appropriate to the size and type of structural damage that needs to be detected. Previous work has proven that the deposition of ferroelectric thin film on structural materials is feasible. We have also tested the feasibility of constructing IDT electrodes on piezoelectric substrates and using them to energize surface acoustic
waves (SAW). The IDT electrode approach was selected because it permits a better coupling with SAW and Lamb waves used in the damage detection process. Figure 13.19a shows an IDT electrode pattern fabricated on Dupont Pyralux copper-clad flexible laminate by photolithography technique. Each SAW type sensor in array consists of five pairs of electrodes. The electrode finger is 200 \( \mu \text{m} \) width with 200 \( \mu \text{m} \) space. The aperture of the device was 4 mm. The resonance of this SAW-type devise was measured with an HP4194 impedance analyzer; the resonance frequency is \( \sim 5 \text{MHz} \) (Figure 13.19b) is close to the design value. This method will be used to test single-layer thin-film nano-PWAS to be supplied by the UTSA team members.

13.6.1.2 Multi-layer nano-PWAS

The realization of a multi-layer thin-film construction is essential for the success of the nano-PWAS concept. This aspect can be easily illustrated through the following 1-D analysis of a piezo wafer with top and bottom electrodes: recall the linear piezoelectric equations for 31 coupling between the \( E_3 \) electric field and \( S_1 \) strain and \( T_1 \) stress, i.e.,

\[
S_1 = s_{11} T_1 + d_{31} E_3 
\]

Equation (6.1) expresses the strain in terms of two variables, the mechanical stress \( T_1 \) and the electric field \( E_3 \). The part of the strain due to the piezoelectric effect, \( S_1^{\text{piezo}} \), is obtained by making \( T_1 = 0 \), i.e.,

\[
S_1^{\text{piezo}} = d_{31} E_3 
\]

For practical PZT properties, Equation (6.2) shows that an inplane strain \( S_1^{\text{piezo}} = 87.5 \mu \text{e} \) could be obtained with an electric field \( E_3 = 0.5 \text{kV/mm} \). For a typical PZT wafer of
thickness $h = 200 \, \mu m$, this means a quite sizable applied voltage $V = hE_z = 100 \, V$. The associate power can be approximated as

$$P = \frac{1}{2} \omega CV^2$$

(6.3)

where $\omega$ is the operating frequency and $C = N\varepsilon_{33}A/h$ is the capacitance, with $N$ the number of layer, $A = bl$ the electrode area, $\varepsilon_{33}$ the electric permittivity of the piezo material. At, say, 100 kHz operation, Equation yields $P_{200\mu m}^{1-layer} = \sim 12 \, W$. This situation, 100 V and $\sim 12 \, W$, reflects the state-of-the-art piezoceramic wafer technology (Figure 13.20a).

Replacing the 200-\(\mu\)m piezoceramic wafer with a 3-\(\mu\)m ferroelectric thick-film will decrease the voltage needed to achieve $S_1^{\text{piezo}} = 87.5 \, \mu e$ induced strain from 100 V to only 1.5 V (Figure 13.20b). The required power decreases from $\sim 12 \, W$ to a mere $\sim 0.180 \, W$. However, the voltage value of 1.5 V is still too large for microwave-powered battery-less operation.

![Figure 13.20](image_url)

Figure 13.20 Proposed thin-film layered nano-PWAS would require orders of magnitude lower voltage (0.015 V vs. 100 V) to achieve the same inplane strain ($S_1 = 87.5 \, \mu e$).

We can further decrease the voltage requirements and bring them within the microwave energized capabilities by adopting a multi-layer construction (Figure 13.20c); for $N = 100$
layers of 30 nm each, the above calculations yield a voltage of mere 0.015 V. This indicates that the thin-film nano-PWAS will decrease the required voltage by orders of magnitude less than current piezoceramic wafers (0.015 V vs. 100 V) and enable a microwave-powered battery-less wireless operation.

13.6.1.3 Micro phased array of nano-PWAS transducers

The next step in complexity is to use the multi-layer nano-PWAS to create micro phased arrays. We have priority claims and extensive experience with the use of PWAS phased arrays for efficient large-area damage detection from a single location. However, an inherent shortcoming and limitation of the phased array approach is the existence of a blind area in the array’s near field. The blind area radius $R$ is commensurable with the array aperture $D$, where $D = M \lambda/2$, where $M$ is the number of elements in the array and $\lambda$ is the wavelength of the particular Lamb wave mode excited in the structure ($\lambda = c/f$). Our previous studies on Lamb wave tuning with PWAS transducers have indicated that maximum excitation of a certain Lamb wave mode happens when the PWAS size $a$ is in a certain relationship with the half wavelength $\lambda/2$. (More details of the PWAS Lamb-wave tuning principles are given in ref.) From these considerations, it is apparent that in order to reduce the blind area one has to construct PWAS transducers of smaller size $a$. E.g., a ten fold reduction in PWAS size $a$ will ensure a ten fold reduction in the blind area $D$ as well as a ten fold increase in the damage detection resolution $1/\lambda$. For these reasons, we are currently pursuing the development of nano-PWAS with small surface footprint (10–100 μm) that will ensure a small phased array aperture and hence a small blind areas, as well very good damage detection resolution.
A schematic of the proposed nano-PWAS phased array concept using annular IDT electrodes is presented in Figure 13.21. The annular IDT electrodes design was selected to ensure axisymmetric wave propagation from each transducer. (The electrodes planform was selected to be quasi-square in order to ensure an optimum coverage of the ferroelectric thin-film area; the corner effects in the wave field quickly even out, as revealed by numerical simulation). The IDT electrodes, which exploit the half-wavelength tuning with the selected Lamb-wave mode, will ensure that a quasi-radial electric field that is created and that this field alternates in sign between one electrode pair and the next. Through the half-wavelength relationship, this excitation pattern will ensure that the elastic ultrasonic Lamb waves emanate outward from the nano-PWAS center, and each nano-PWAS can be approximated with a point source in the idealized phased-array scheme. The IDT electrodes are reproduced identically at each after each ferroelectric thin-film layer.

13.6.1.4 Wireless battery-less operation via tag antennas

A major challenge in the SHM usability of the proposed nano-PWAS device is to give it a non-battery wireless operational capability. Our concept is to provide each nano-PWAS...
with a miniature tag antenna that can be remotely interrogated with microwave wireless power. The microwave beam pulse will energize the nano-PWAS into sending an interrogating Lamb wave/SAW pulse into the surrounding structure. The reflected/diffracted acoustic waves received back by the nano-PWAS will be transduced back into microwave field and emitted through the tag antenna. From a electromagnetic standpoint, this operation is similar to the well proven RFID technology. What is not yet fully understood is how to use the transduction between electromagnetic and acoustic domains to detect the incipient damage presence. Perceived research challenges include:

Separation of damage diffractions from those due to structural features and boundaries

Capability to detect weak acoustic wave signals and transduce them into electromagnetic waves of sufficient power to ensure adequate reception and interpretation

Development of new phased-array principles amenable to remote wireless interrogation

These fundamental challenges will be addressed in the proposed work using the previous work on damage detection with embedded PWAS arrays. Challenge (a), which is pervasive in the SHM research, will be addressed by developing a novel statistics-based differential imaging approach. Challenge (b) will be addressed by building on our previous work on efficient power and energy transfer through optimal interaction structural acoustic waves and PWAS transducers. Challenge (c) will be addressed by developing new principles of indexed-addressing of phased-array elements through wave modulation/encoding.
13.6.2 Experimental Results and Discussion

We have developed analytical models for the electro-mechanical-acoustical analysis of the interaction between piezoelectric wafer active sensors (PWAS) and the guided-Lamb waves in the substrate structure. Analytical models of the bonding layer shear transfer and space-domain wavenumber Fourier analysis of the guided Lamb waves predicted the possibility of preferential tuning of various Lamb-wave modes. These predictions have been experimentally confirmed. Tuned Lamb waves generated with PWAS arrays have been used to create guided-wave scanning beams to interrogate large structural areas. However, these arrays were much less effective when applied to small and compact structural parts of complicated geometry. Upon investigation, it was found that the tuned wavelength was insufficiently small to permit small area/incipient damage detection and that the blind area was relatively large. The thin-film approach of this project will provide a much smaller array for effective damage detection.

Predictive modeling and experimental test is to be used to achieve the nano-PWAS developmental steps outlined on page 309, from single-layer thin-film nano-PWAS up to the micro-phased array and the non-battery wireless operation. Careful analysis using advanced simulation and precise measurements is to be conducted. In brief, the main activities should consist of:

1. Modeling and experimental analysis of the piezoelectric interactions in the ferroelectric thin-film sensor applied to the structural substrate shows the use a multi-physics FEM code to analyze the poling process in a simple configuration explored in preliminary work. To achieve full understanding, one should use this approach to analyze the multitude of piezoelectric interactions and optimize the multi-layer
configuration, ferroelectric thin-film/electrode thin-film ratio, antenna configuration, etc. In addition, one should develop reduced-order analytical methods to perform wider parameter search and optimization.

2. Modeling and experimental analysis of the tuning between ferroelectric thin-film active sensor array and multi-mode dispersive Lamb waves in the structure. One should build on PWAS tuning expertise to address the complicated problems appearing in multi-layer nano-PWAS working at MHz frequencies in micro phased arrays.

3. Damage detection simulation and experimental validation with multi-layer nano-PWAS.

4. Performance and durability testing of multi-layer nano-PWAS and development of guidelines and recommendations for quality improvement in collaboration with the other team members.

13.6.3 Nano-PWAS Advantages and Potential Industrial Applications

This section described herein introduces the term nano-PWAS to designate a new type of sensor that uses for PWAS-based structural health monitoring in nano scale. Further this invention encompasses the following:

1. The concept of using single-layer interdigitated electrode pattern for PWAS fabrication.

2. The concept of multi-layer nano-PWAS for structural health monitoring.

3. The concept of micro phased array of nano-PWAS transducer for structural health monitoring.
4. The concept of wireless battery-less operation of a nano-PWAS via tag antenna

The invention is better and more advantageous than present state-of-the-art technology because:

1. It provides a durable and reliable way to integrate active sensors onto the structure subjected to structural health monitoring (SHM).

2. It provides a very efficient way to reduce the power and voltage requirement for SHM.

3. It provides an accurate way to detect small cracks and incipient damage in a monitored structure based on active sensing technology.

4. It provides wireless battery-less operation of active sensors for SHM.

The major potential industrial application of this invention is in PWAS SHM. With the durable nano-PWAS fabrication and wireless battery-less signal collection, the nano-PWAS provides a simple solution for SHM that is likely to be adopted by many industrial users.

The organizations likely to use the method and the device that make the object of the present invention include federal and industrial laboratories, original equipment manufactures, and operators of large critical infrastructure projects (bridges and buildings), aerospace, energy generation, nuclear oil, automotive and related industrials that are required to assure the safety of their product by structure healthy monitoring and nondestructive evaluation.
14 CONCLUSIONS, MAJOR FINDINGS, AND FUTURE WORK

The application of PWAS allows the user to obtain the information on structural health by interrogating the corresponding sensor network installed on structure. This dissertation addresses three major aspects associated with PWAS design, and PWAS are studied in both theoretical and practical aspects.

14.1 CONCLUSIONS AND MAJOR FINDINGS

The research focuses on three areas: (I) modeling of PWAS power and energy transduction; (II) PWAS installation and durability study; (III) novel miniaturized PWAS approaches for SHM.

In part I, a systematic investigation of power and energy transduction in PWAS attached to a structure was conducted using the wave propagation and normal mode expansion methods. With 1-D wave propagation, ideal bonding and ideal excitation source assumption, frequency response function and power and energy transduction of PWAS transmitter and receiver were developed.

For PWAS transmitter, the active power, reactive power, power rating of electrical requirement were determined under harmonic voltage excitation. It indicates that the reactive power is dominant and gives the power requirement for PWAS actuation application. The electrical and mechanical power analysis at the PWAS structure interface indicates that the active electrical power provides the mechanical power at the
interface. This provides the power and energy for the axial and flexural wave’s power and energy travelling in the structure. Under 1-D wave propagation assumption, the axial and flexural waves propagate in both direction and the forward and backward wave power and energy are equal. The sum of forward and backward wave power equals the mechanical power PWAS applied to the structure. The parametric study of PWAS transmitter size shows the proper size and excitation frequency selection based on the tuning effects.

For PWAS receiver under harmonic strain excitation, the structure interface acoustic and electrical energy transduction was also developed. The parametric study of receiver size, receiver impedance and external electrical load gives the PWAS design direction for PWAS sensing and power harvesting application.

The power and energy flow of a complete pitch-catch setup was also considered. The electro-acoustic transduction of both PWAS transmitter and receiver were examined thoroughly. Both PWAS transmitter and receive pin-forces were considered for frequency transfer function and power and energy modeling. The power flow at each step in pitch-catch was monitored. The numerical simulation and graphical chart show the trends in the power and energy flow behavior with remarkable peaks and valleys that can be exploited for optimum design.

The power and energy under non-harmonic excitation was considered with frequency response function using the Fourier transform. The power and energy under non-harmonic excitation was also simulated.

CF-FEM performs the numerical simulation of complicated structures using small-element discretization. The FEM mesh size for wave propagation was discussed and the
proper time step and mesh size for transient analysis was determined. PWAS impedance, PWAS mode shape, electric field, poling process, and pitch-catch were modeled. Closed-form solution, coupled-field finite element simulation and experimental data were compared.

In part II, the PWAS fabrication, characterization and installation were important aspects for achieving consistent and reliable results of experimental testing. The statistical design of experiments concept was used to investigate PWAS installation procedure. The effects of four factors (adhesive type, PWAS polarization, surface preparation and curing method) were investigated by comparing the thickness of the adhesive layer, capacitance, and pitch-catch signal amplitude of the PWAS installed under different conditions. The PWAS polarization does not affect the pitch-catch property under the test conditions. The adhesive type affects the thickness of the adhesive layer and the Lamb wave generating amplitude. Alkyl cyanoacrylate compound based M-200 is easy to apply but gives a thinner bond layer and weaker wave generating property compared to Epoxy based AE-15. AE-15 requires specific curing conditions and it provides a better pitch signal than M-200. The receiving ability does not show statistically significant differences with different adhesive types. The interaction among adhesive, curing and surface preparation affects the wave generating ability, but does not show statistically significant effects on receiving. Overall, these full factorial designs provide us some basic understanding and guidance for the improvement of the PWAS installation procedure.

The durability and survivability of the PWAS transducers under various exposures (cryogenic and high temperature, temperature cycling, freeze-thaw, outdoor environment, operational fluids, large strains, fatigue load cycling). In most cases, the PWAS survived
the tests successfully. The cases when the PWAS did not survive the tests were closely examined and possible causes of failure were discussed. The test results indicat that PZT PWAS can be successfully used in cryogenic environment; however, it does not seem to be a good candidate for high temperature. Repeated differential thermal expansion and extended environmental attacks can lead to PWAS failure. This emphasizes the importance of achieving the proper design of the adhesive bond between the PWAS and the structure, and of using protective coating to minimize the ingress of adverse agents. The high strain tests indicated that the PWAS remained operational up to at least 3000 micro-strain and failed beyond 6000 micro-strain. The fatigue cyclic loading, conducted up to 12 millions of cycles, showed that the PWAS transducers sustained at least as many fatigue cycles as the structural coupon specimens on which they were installed. These results gave us confidence in this new technology and opened the path toward installation on realistic aerospace specimens.

In part III, novel PWAS architectures (composite-PWAS, PVDF-PWAS and nano-PWAS) were studied to overcome these shortcomings of conventional PWAS. The study of novel PWAS configurations includes: composite-PWAS, PVDF-PWAS, and nano-PWAS. Methods for in-situ fabrications of composite and PVDF-PWAS on curved and/or complicated structural surfaces were explored. In comparison with PZT-PWAS, PVDF-PWAS have been studied with the following experiments: cantilever beam free vibration and long rod impaction test. The experimental results of the PVDF-PWAS and strain gauge have been compared with those of PZT-PWAS. The theoretical and experimental results of PZT-PWAS and PVDF-PWAS performance in this study gave the
demonstration of the piezoelectricity of PWAS. Both PZT-PWAS and PVDF-PWAS are capable of use as a transmitter and receiver in pitch-catch method.

In collaboration with University of Texas at San Antonio, University of Texas Arlington, and Army Research Laboratory, several deposition methods, materials, and substrates were investigated for thin-film nano-PWAS fabrication. As a result, nano-scale BTO thin films with enhanced piezoelectric properties were developed on Ni and Ti substrates. The BST thin-film deposition treatments were all successfully applied to the polished structural titanium stubs. Characterization tests illustrated that all of the BST films were crystalline, possessed BST with piezoelectric phase. However, maximum crystallinity was achieved at 800 °C annealing for all samples. For the RTA samples in addition to BST, there were also barium titanate and strontium titanate phases present. The ferroelectric thin film was able to deposit directly on the structural substrate Ni and Ti using PLD technology. The achievements show that BaTiO$_3$ films consisting of randomly oriented or orientation preferred crystalline nanopillars having excellent properties can be deposited directly on Ni and Ti substrates, which are expected to have significant importance for the development of new piezoelectric thin-film sensors to be structurally integrated to the structural materials.

14.2 RECOMMENDED FUTURE WORK

The work presented in this dissertation focuses on the power and energy transduction of PWAS for SHM. But there are still questions to be solved to fully discover the potential of PWAS for SHM. The following tasks are recommended to be undertaken for further development of PWAS.
The power and energy analysis in the dissertation is based on 1-D model. 2-D model considering the energy spread out can be done to show the power and energy transduction more realistically. The current approach only considered the axial force and bending moment, the shear-lag model can be applied to remove the pin-force assumption. The axial and flexural wave considered in the beam is close to the Lamb wave in plate at the low frequency. At high frequency, multi-mode Lamb waves should be considered. In combination with shear-lag solution, the power and energy analysis can be used for multi-mode Lamb wave propagation.

The modeling of the temperature effect of PWAS impedance could be done by combining the complete analytical model of free and bonded PWAS on circular plates as function of material properties of PWAS and structures. The dielectric and piezoelectric properties of PZT ceramics can be considered for actuator applications at cryogenic temperatures and high temperatures.

Statistical method was used for PWAS installation and durability tests. Statistical method used for thin-film fabrication will be useful to understand the nature of thin-film growth. More thin-film candidates are needed for variety of SHM application.

Traditional model for PWAS may or may not fit in the nano scale. The material properties in nano-scale need to be discovered and used in the model. The model of nano-fabrication of PWAS is needed. Thin-films fabrication and characterization were demonstrated in this dissertation, but actual sensor and sensor network have not completed yet. A step by step approach of nano-PWAS and future work has been proposed.
REFERENCES


APPENDIX A  MATERIAL PROPERTIES INPUT IN ANSYS SOFTWARE

The manufacturer supplied material properties and the format required by ANSYS may be different. This section want to clarify how to convert of material properties of piezoelectric ceramics (such as PZT) to use in ANSYS.

The basic constitutive relationship of piezoelectric materials have been outlined in (2.1). On the other hand, ANSYS requires data in the following form

\[ \{T\} = \left[ e^E \right] \{S\} - [e] \{E\} \]
\[ \{D\} = [e]^T \{S\} + \left[ e^s \right]^T \{E\} \]  \hspace{1cm} (A.1)

where:

\{S\} = mechanical strain vector (six components x, y, z, yz, zx, xy)

\{T\} = mechanical stress vector (six components x, y, z, yz, zx, xy)

\{E\} = electrical field vector (three components x, y, z)

\{D\} = electrical displacement vector (three components x, y, z)

\left[ e^E \right] = mechanical stiffness matrix evaluated at zero electric field, i.e. short circuit

\[ e \] = piezoelectric matrix relating stress / electric field

\[ e^T \] = transposed \[ e \]
\[ \varepsilon^S \] = dielectric matrix evaluated at zero mechanical strain, i.e. mechanically clamped

The relationship between equation (2.1) and (A.1) can be obtained by the following relations.

\[ \left[ e^E \right] = \left[ s^E \right]^{-1} \] \hspace{1cm} (A.2)

\[ \varepsilon^S = \left[ e^E \right] - [d] \left[ s^E \right]^{-1} [d] \] \hspace{1cm} (A.3)

\[ [e] = \left[ s^E \right]^{-1} [d] = [d]^T \left[ s^E \right]^{-1} \] \hspace{1cm} (A.4)

### A.1 Stiffness/Compliance Matrix

For most published piezoelectric materials, the order used for the piezoelectric matrix is x, y, z, yz, xz, xy, based on IEEE standards (see ANSI/IEEE Standard 176–1987), while the ANSYS input order is x, y, z, xy, yz, xz. This means that the matrix of the ANSYS input order need to be modified by switching row data for the shear terms as shown below:

IEEE constants [e61, e62, e63] would be input as the ANSYS xy row

IEEE constants [e41, e42, e43] would be input as the ANSYS yz row

IEEE constants [e51, e52, e53] would be input as the ANSYS xz row

Based on the previous order, there are two ways to input anisotropic elastic matrix in ANSYS. Assuming polarization in the z-axis of a PZT material, we can generate an anisotropic compliance matrix:
\[
\begin{bmatrix}
S^{E}_{11} & S^{E}_{12} & S^{E}_{13} & 0 & 0 & 0 \\
S^{E}_{12} & S^{E}_{12} & S^{E}_{13} & 0 & 0 & 0 \\
S^{E}_{13} & S^{E}_{13} & S^{E}_{13} & 0 & 0 & 0 \\
0 & 0 & 0 & S^{E}_{66} & 0 & 0 \\
0 & 0 & 0 & 0 & S^{E}_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & S^{E}_{44}
\end{bmatrix} =
\begin{bmatrix}
1/E_x & -\nu_{xy}/E_y & -\nu_{xz}/E_z & 0 & 0 & 0 \\
-\nu_{yx}/E_x & 1/E_y & -\nu_{yz}/E_z & 0 & 0 & 0 \\
-\nu_{xz}/E_x & -\nu_{yz}/E_y & 1/E_z & 0 & 0 & 0 \\
0 & 0 & 0 & 1/G_{xy} & 0 & 0 \\
0 & 0 & 0 & 0 & 1/G_{yz} & 0 \\
0 & 0 & 0 & 0 & 0 & 1/G_{xz}
\end{bmatrix}
\] . (A.5)

If \( S^{E}_{66} \) is not available, it can be determined from \( S^{E}_{66} = 2(S^{E}_{11} - S^{E}_{12}) \).

An alternative method uses manufacturer data. For z-axis of a PZT material, according to ANSYS theory manual,

Using the above relationship, the stiffness can be calculated from manufacturer data,

\[ E_x = E_y = \frac{1}{S^{E}_{11}} \] (A.7)

\[ E_z = \frac{1}{S^{E}_{33}} \] (A.8)

\[ G_{xy} = \frac{1}{S^{E}_{66}} \] (A.9)

\[ G_{yz} = G_{xz} = \frac{1}{S^{E}_{44}} \] (A.10)
\[ V_{xy} = -\frac{s_{12}^E}{s_{11}^E} \]  \hspace{1cm} (A.11)

\[ V_{yz} = V_{zx} = \frac{s_{13}^E}{s_{33}^E} \]  \hspace{1cm} (A.12)

### A.2 Permittivity Matrix

The permittivity matrix evaluated at constant strain is input into ANSYS. Oftentimes, manufacturers’ data has permittivity evaluated at constant stress. The equation (A.3) can be used for conversion. The permittivity matrix has only diagonal terms:

\[
\begin{bmatrix}
\varepsilon_{11}^S & 0 & 0 \\
0 & \varepsilon_{11}^S & 0 \\
0 & 0 & \varepsilon_{33}^S
\end{bmatrix}
= \varepsilon_0
\begin{bmatrix}
K_{11}^S & 0 & 0 \\
0 & K_{11}^S & 0 \\
0 & 0 & K_{33}^S
\end{bmatrix}
\]  \hspace{1cm} (A.13)

where \( K_{11}^S = \frac{\varepsilon_{11}^S}{\varepsilon_0} \) is relative permittivity.

### A.3 Piezoelectric Constant Matrix

Usually, manufacturers’ data has \([d]\), which relates mechanical strain to electric field. However, ANSYS requires \([e]\), relating mechanical stress to electric field. The conversion can be done using equation (A.4).

When polarization is in the z-direction, we consider the symmetry in the unpolarized directions \(d_{32} = d_{31} \) and \(d_{24} = d_{15}\) and recall the ANSYS order, the piezoelectric constant can be written as:
\[
[d] = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & d_{15} \\
0 & 0 & 0 & 0 & d_{15} & 0 \\
d_{31} & d_{31} & d_{33} & 0 & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (A.14)

\[
[e] = \begin{bmatrix}
0 & 0 & e_{31} \\
0 & 0 & e_{31} \\
0 & 0 & e_{43} \\
0 & 0 & 0 \\
0 & e_{15} & 0 \\
e_{15} & 0 & 0
\end{bmatrix}
\]  \hspace{1cm} (A.15)
APPENDIX B   TRANSFORMATION OF MATERIAL PROPERTIES

The general vector transformation law is

\[ v'_j = a_{ij} v_j \rightarrow i, j = x, y, z \]  \hspace{1cm} (B.1)

where the coefficients \( a_{ij} \) define a transformation matrix \([a]\).

W. L. Bond developed a very efficient matrix technique to convert material properties. The Bond stress transformation matrix \([M]\) define a 6 x 6 transformation matrix.

\[
[M] = \begin{bmatrix}
    a_{xx}^2 & a_{xy}^2 & a_{xz}^2 & 2a_{xy}a_{xz} & 2a_{xz}a_{xx} & 2a_{xx}a_{xy} \\
    a_{yx}^2 & a_{yy}^2 & a_{yz}^2 & 2a_{yx}a_{yz} & 2a_{yz}a_{yy} & 2a_{yy}a_{yx} \\
    a_{zx}^2 & a_{zy}^2 & a_{zz}^2 & 2a_{zx}a_{zz} & 2a_{zz}a_{zx} & 2a_{zz}a_{zy} \\
    a_{xy}a_{xz} & a_{yx}a_{yz} & a_{zx}a_{zz} & a_{yz}a_{zz} + a_{yz}a_{zx} & a_{yx}a_{xz} + a_{xy}a_{zz} & a_{yx}a_{yz} + a_{xy}a_{zx} \\
    a_{xz}a_{xx} & a_{zx}a_{xy} & a_{zz}a_{zy} & a_{zx}a_{yy} + a_{zx}a_{zy} & a_{xz}a_{xx} + a_{xz}a_{yy} & a_{xz}a_{yx} + a_{xz}a_{yx} \\
    a_{xx}a_{yy} & a_{yx}a_{yy} & a_{zx}a_{zy} & a_{yz}a_{xx} + a_{yz}a_{xx} & a_{yx}a_{yy} + a_{yx}a_{yy} & a_{yx}a_{yy} + a_{yx}a_{yy}
\end{bmatrix}
\hspace{1cm} (B.2)

The Bond strain matrix \([N]\) is the same as \([M]\), except for a shift of the factor 2 from the upper right-hand corner to the lower left-hand corner.
The transformed stress $[T']$ can be written with Bond stress matrix $[M]$ and stress $[T']$

$$[T'] = [M][T']$$ \hspace{1cm} (B.4)

Similar, the transformed strain has the following relation

$$[S'] = [N][S]$$ \hspace{1cm} (B.5)

Based on Hooke’s Law

$$[T] = [c][S]$$ \hspace{1cm} (B.6)

Application of the Bond stress transformation matrix (B.4) to (B.6) leads to

$$[T'] = [M][c][S]$$ \hspace{1cm} (B.7)

The inverse of (B.5) is

$$[S] = [N]^{-1}[S']$$ \hspace{1cm} (B.8)

Substitution for (B.7) gives

$$[T'] = [M][c][N]^{-1}[S']$$ \hspace{1cm} (B.9)

This shows that the transformed stiffness matrix is simply
The other transformed material properties have a similar relation.

\[ [c'] = [M][c][N]^{-1} \]  
(B.10)

\[ [e'] = [a][e][N]^{-1} \]  
(B.11)

\[ [\varepsilon'] = [a][\varepsilon][a]^{-1} \]  
(B.12)

If using the another set of parameters, the relation is

\[ [s'] = [N][s][M]^{-1} \]  
(B.13)

\[ [d'] = [a][d][M]^{-1} \]  
(B.14)

For a clockwise rotation of the coordinate axes through an angle $\alpha$ about the z axis. The coordinate transformation matrix is therefore

\[
[a] = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  
(B.15)

Bond stress transformation matrix is

\[
[M] = \begin{bmatrix}
\cos^2 \alpha & \sin^2 \alpha & 0 & 0 & 0 & \sin 2\alpha \\
\sin^2 \alpha & \cos^2 \alpha & 0 & 0 & 0 & -\sin 2\alpha \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \alpha & -\sin \alpha & 0 \\
0 & 0 & 0 & \sin \alpha & \cos \alpha & 0 \\
-\frac{\sin 2\alpha}{2} & \frac{\sin 2\alpha}{2} & 0 & 0 & 0 & \cos 2\alpha
\end{bmatrix}
\]  
(B.16)

Bond strain transformation matrix is
\[
\begin{bmatrix}
\cos^2 \alpha & \sin^2 \alpha & 0 & 0 & 0 & \frac{\sin 2\alpha}{2} \\
\sin^2 \alpha & \cos^2 \alpha & 0 & 0 & 0 & -\frac{\sin 2\alpha}{2} \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \alpha & -\sin \alpha & 0 \\
0 & 0 & 0 & \sin \alpha & \cos \alpha & 0 \\
-\sin 2\alpha & \sin 2\alpha & 0 & 0 & 0 & \cos 2\alpha \\
\end{bmatrix}
\] (B.17)

All the transformed material properties can be calculated with the matrix calculation.
APPENDIX C  INTEGRAL OF ANALYTICAL FLEXURAL MODES

C.1  INTEGRAL OF EXACT FORMULA

To prove the integral for the flexural mode in a free-free beam has the following relation

\[ I = \int_0^L W^2(x) dx = L \]  \hspace{1cm} (C.1)

Given the following equations

\[ W(x) = \cosh \gamma x + \cos \gamma x - \beta (\sinh \gamma x + \sin \gamma x) \]  \hspace{1cm} (C.2)

\[ \beta = \frac{\cosh \gamma L - \cos \gamma L}{\sinh \gamma L - \sin \gamma L} = \frac{\sinh \gamma L + \sin \gamma L}{\cosh \gamma L - \cos \gamma L} \]  \hspace{1cm} (C.3)

\[ \cosh \gamma L \cos \gamma L - 1 = 0 \]  \hspace{1cm} (C.4)

First, \( W^2(x) \) can be expanded as

\[ W^2(x) = \cosh^2 \gamma x + 2 \cosh \gamma x \cos \gamma x + \cos^2 \gamma x \]
\[ -2 \beta (\sinh \gamma x + \sin \gamma x)(\cos \gamma x + \cos \gamma x) \]  \hspace{1cm} (C.5)
\[ + \beta^2 (\sinh^2 \gamma x + 2 \sinh \gamma x \sin \gamma x + \sin^2 \gamma x) \]

Next, the integral is calculated separately from three parts:

(1) The first part \( I_1 = \int_0^L (\cosh^2 \gamma x + 2 \cosh \gamma x \cos \gamma x + \cos^2 \gamma x) dx \) does not contain \( \beta \),

\[ \int_0^L \cosh^2 \gamma x dx = \frac{\sinh 2\gamma L}{4\gamma} + \frac{L}{2} \]  \hspace{1cm} (C.6)
\[
\int_0^L \cos^2 \gamma \, x \, dx = \frac{L}{2} + \frac{\sin 2\gamma L}{4\gamma} \quad (C.7)
\]

\[
2\int_0^L \cosh \gamma x \cos \gamma x \, dx = \frac{\sin \gamma L \cosh \gamma L + \cos \gamma L \sinh \gamma L}{\gamma} \quad (C.8)
\]

Hence, the integral of the first part becomes

\[
I_1 = \frac{\sinh 2\gamma L}{4\gamma} + \frac{L}{2} + \frac{\sin 2\gamma L}{4\gamma} + \frac{\sin \gamma L \cosh \gamma L + \cos \gamma L \sinh \gamma L}{\gamma} \\
= L + \frac{\sinh 2\gamma L}{4\gamma} + \frac{\sin 2\gamma L}{4\gamma} + \frac{\sin \gamma L \cosh \gamma L + \cos \gamma L \sinh \gamma L}{\gamma} \quad (C.9)
\]

(2) The second part

\[
-2\beta \left( \sinh \gamma x + \sin \gamma x \right) \left( \cosh \gamma x + \cos \gamma x \right) \, dx \quad \text{contains } \beta
\]

\[
-2\beta \int_0^L \sinh \gamma x \cosh \gamma x \, dx = -\frac{\beta \sinh^2 \gamma L}{\gamma} \quad (C.10)
\]

\[
-2\beta \int_0^L \left( \sinh \gamma x \cos \gamma x + \cosh \gamma x \sin \gamma x \right) \, dx = -\frac{2\beta \sin \gamma L \sinh \gamma L}{\gamma} \quad (C.11)
\]

\[
-2\beta \int_0^L \sin \gamma x \cos \gamma x \, dx = -\frac{\beta \sin^2 \gamma L}{\gamma} \quad (C.12)
\]

Hence, the integral of the second part becomes

\[
I_2 = -\frac{\beta \sinh^2 \gamma L}{\gamma} - \frac{2\beta \sin \gamma L \sinh \gamma L}{\gamma} - \frac{\beta \sin^2 \gamma L}{\gamma} \\
= -\frac{\beta}{\gamma} \left( \sinh \gamma L \sin \gamma L \right)^2 \quad (C.13)
\]

(3) The third part

\[
I_3 = \int_0^L \beta^2 \left( \sinh^2 \gamma x + 2 \sinh \gamma x \sin \gamma x + \sin^2 \gamma x \right) \, dx \quad \text{contains } \beta^2, \text{ i.e.}
\]

\[
\beta^2 \int_0^L \sinh^2 \gamma x \, dx = \beta^2 \left( \frac{\sinh 2\gamma L}{4\gamma} - \frac{L}{2} \right) \quad (C.14)
\]
\begin{align*}
2\beta^2 \int_0^L \sinh \gamma x \sin \gamma x \, dx &= \beta^2 \frac{\sin \gamma L \cosh \gamma L - \cos \gamma L \sinh \gamma L}{\gamma} \tag{C.15} \\
\beta^2 \int_0^L \sin^2 \gamma x \, dx &= \beta^2 \left( \frac{L}{2} - \frac{\sin 2\gamma L}{4\gamma} \right) \tag{C.16}
\end{align*}

Hence, the integral of the third part becomes

\begin{align*}
I_3 &= \beta^2 \left( \frac{\sin 2\gamma L}{4\gamma} - \frac{L}{2} \right) + \beta^2 \frac{\sin \gamma L \cosh \gamma L - \cos \gamma L \sinh \gamma L}{\gamma} \\
&+ \beta^2 \left( \frac{L}{2} - \frac{\sin 2\gamma L}{4\gamma} \right) \tag{C.17} \\
&= \frac{\beta^2}{\gamma} \left( \frac{\sin 2\gamma L}{4} + \sin \gamma L \cosh \gamma L - \cos \gamma L \sinh \gamma L - \frac{\sin 2\gamma L}{4} \right)
\end{align*}

Combining the first and third parts and multiply by \( \frac{\gamma}{\beta} \), we got

\begin{align*}
(I_1 - L + I_3) \frac{\gamma}{\beta} &= \frac{1}{\beta} \left( \frac{\sin 2\gamma L}{4} + \frac{\sin 2\gamma L}{4} + \sin \gamma L \cosh \gamma L + \cos \gamma L \sinh \gamma L \right) \\
&+ \beta \left( \frac{\sin 2\gamma L}{4} + \sin \gamma L \cosh \gamma L - \cos \gamma L \sinh \gamma L - \frac{\sin 2\gamma L}{4} \right) \\
&= \frac{\sin 2\gamma L}{4} \left( \frac{1}{\beta} + \beta \right) + \frac{\sin 2\gamma L}{4} \left( \frac{1}{\beta} - \beta \right) \\
&+ \sin \gamma L \cosh \gamma L \left( \frac{1}{\beta} + \beta \right) + \cos \gamma L \sinh \gamma L \left( \frac{1}{\beta} - \beta \right) \tag{C.18} \\
&= \left( \frac{\sin 2\gamma L}{4} + \sin \gamma L \cosh \gamma L \right) \left( \frac{1}{\beta} + \beta \right) \\
&+ \left( \frac{\sin 2\gamma L}{4} + \cos \gamma L \sinh \gamma L \right) \left( \frac{1}{\beta} - \beta \right)
\end{align*}

Recall equation (C.3)

\begin{align*}
\beta &= \frac{\cosh \gamma L - \cos \gamma L}{\sinh \gamma L - \sin \gamma L} = \frac{\sinh \gamma L + \sin \gamma L}{\cosh \gamma L - \cos \gamma L} \tag{C.19}
\end{align*}
We have the following relation

\[
\frac{1}{\beta} + \beta = \frac{\sinh \gamma L - \sin \gamma L}{\cosh \gamma L - \cos \gamma L} + \frac{\sinh \gamma L + \sin \gamma L}{\cosh \gamma L - \cos \gamma L} = \frac{2 \sinh \gamma L}{\cosh \gamma L - \cos \gamma L} \tag{C.20}
\]

\[
\frac{1}{\beta} - \beta = \frac{\sinh \gamma L - \sin \gamma L}{\cosh \gamma L - \cos \gamma L} - \frac{\sinh \gamma L + \sin \gamma L}{\cosh \gamma L - \cos \gamma L} = -\frac{2 \sin \gamma L}{\cosh \gamma L - \cos \gamma L} \tag{C.21}
\]

Substitution of (C.20) and (C.21) into Equation (C.18)

\[
(I, -L + I_1) \frac{\gamma}{\beta} = \left( \frac{\sinh 2\gamma L}{4} + \sin \gamma L \cosh \gamma L \right) \left( \frac{1}{\beta} + \beta \right) + \left( \frac{\sin 2\gamma L}{4} + \cos \gamma L \sinh \gamma L \right) \left( \frac{1}{\beta} - \beta \right)
\]

\[
= \left( \frac{\sinh \gamma L \cosh \gamma L}{2} + \sin \gamma L \cosh \gamma L \right) \frac{2 \sinh \gamma L}{\cosh \gamma L - \cos \gamma L}
\]

\[
- \left( \frac{\sin \gamma L \cos \gamma L}{2} + \cos \gamma L \sinh \gamma L \right) \frac{2 \sin \gamma L}{\cosh \gamma L - \cos \gamma L}
\]

\[
= \frac{1}{\cosh \gamma L - \cos \gamma L} \left( \sinh^2 \gamma L \cosh \gamma L + 2 \sin \gamma L \sinh \gamma L \cosh \gamma L \right. - \sin^2 \gamma L \cos \gamma L - 2 \sin \gamma L \cos \gamma L \sinh \gamma L \biggr)
\]

\[
= \frac{1}{\cosh \gamma L - \cos \gamma L} \left( \sinh^2 \gamma L \cosh \gamma L - \sin^2 \gamma L \cos \gamma L \right) + 2 \sin \gamma L \sinh \gamma L
\]

Recall Equation (C.4)

\[
\cos \gamma L = \frac{1}{\cosh \gamma L} \quad \text{and} \quad \cosh \gamma L = \frac{1}{\cos \gamma L} \tag{C.23}
\]

Substitution into (C.22)
\[
(I_1 - L + L_3) \frac{\gamma}{\beta} = 0
\]

\[
= \frac{1}{\cosh \gamma L - \cos \gamma L} (\sinh^2 \gamma L \cosh \gamma L - \sin^2 \gamma L \cos \gamma L) + 2 \sin \gamma L \sinh \gamma L
\]

\[
= \frac{1}{\cosh^2 \gamma L - 1} (\sinh^2 \gamma L \cosh \gamma L - \sin^2 \gamma L) + 2 \sin \gamma L \sinh \gamma L
\]

\[
= \cosh^2 \gamma L - \sin^2 \gamma L \frac{1}{\cosh^2 \gamma L - 1} + 2 \sin \gamma L \sinh \gamma L
\]

\[
= \cosh^2 \gamma L - \sin^2 \gamma L \frac{\cos^2 \gamma L}{1 - \cos^2 \gamma L} + 2 \sin \gamma L \sinh \gamma L
\]

\[
= \cosh^2 \gamma L - \cos^2 \gamma L + 2 \sin \gamma L \sinh \gamma L
\]

\[
= \sinh^2 \gamma L + 1 - (1 - \sin^2 \gamma L) + 2 \sin \gamma L \sinh \gamma L
\]

\[
= \sinh^2 \gamma L + 2 \sin \gamma L \sinh \gamma L + \sin^2 \gamma L
\]

\[
= (\sinh \gamma L + \sin \gamma L)^2
\]

Hence

\[
(I_1 - L + L_3) \frac{\gamma}{\beta} + I_2 \frac{\gamma}{\beta} = 0
\]  
(C.25)

Rearrangement and assumption \( \frac{\gamma}{\beta} \neq 0 \) yields

\[
I = I_1 + I_2 + I_3 = L
\]  
(C.26)

This is the exact solution for all flexural wave mode that satisfy the free-free beam Equation (C.2) - (C.4)

### C.2 Integral of Approximate Formula

For simulation, after 10th mode, \( \gamma L \gg 1 \), recall Equation (C.4)

\[
cosh \gamma L \cos \gamma L - 1 = 0
\]  
(C.27)

The numerical result of \( \gamma L \) simulated by the computer is very close to
\[ \gamma L = (2j + 1) \frac{\pi}{2} \quad j = 11, 12 \ldots \]  
(C.28)

The difference between the approximate form (C.28) and the exact solution of (C.27) cannot be shown in the computer, we have to use the approximate form (C.28).

Hence, the coefficient \( \beta \) is

\[
\beta = \frac{\cosh \gamma L - \cos \gamma L}{\sinh \gamma L - \sin \gamma L} \\
= \frac{1 - \cos \gamma L / \cosh \gamma L}{\tanh \gamma L - \sin \gamma L / \cosh \gamma L} \xrightarrow{\gamma L \to \infty} 1
\]
(C.29)

Substitution of \( \beta = 1 \) into Equation (C.2) yields

\[ W(x) = \cos \gamma x - \sin \gamma x + e^{-\gamma x} \]  
(C.30)

However, Equation (C.30) is not symmetric with beam center. Symmetry must be restored by taking the first half, \( x \in \left[ 0, \frac{L}{2} \right] \), and flipping it over to create the second half,

\[ x \in \left[ \frac{L}{2}, L \right] \]. If we do not do this, the integral of (C.1) gets distorted, i.e.

\[
\int_{0}^{L} (\cos \gamma x - \sin \gamma x + e^{-\gamma x})^2 \, dx = L + \frac{1}{2\gamma} \left( 4e^{-\gamma L} \sin \gamma L - e^{-2\gamma L} + \cos 2\gamma L \right) 
\]
(C.31)

Here \( \gamma \gg 0 \), the exponential part decreases to zero very quickly,

\[ \gamma L = (2j + 1) \frac{\pi}{2} \quad j = 11, 12 \ldots \]  
(C.32)

\[ \cos 2\gamma L = \pm 1 \]  
(C.33)

\[ I \equiv L + \frac{\cos 2\gamma L}{2\gamma} = L \pm \frac{1}{2\gamma} \]  
(C.34)
However, if we use the first part only and to make the second half by symmetry, then the integral is well behaved and consistent with the general result, \( I = L \). The proof is follows.

Substitution of (C.30) into (C.1), and consider the symmetry of the mode shape, the integral can be calculate from the first half of the beam

\[
I = 2\int_0^L \left[ \cos \gamma x \sin \gamma x + e^{-\gamma x} \right]^2 \, dx
\]

\[
= 2\int_0^L \left[ (\cos \gamma x - \sin \gamma x)^2 + 2(\cos \gamma x - \sin \gamma x)e^{-\gamma x} + e^{-2\gamma x} \right] \, dx \quad \text{(C.35)}
\]

\[
= 2\int_0^L \left[ (1 - 2\cos \gamma x \sin \gamma x) + 2(\cos \gamma x - \sin \gamma x)e^{-\gamma x} + e^{-2\gamma x} \right] \, dx
\]

Integral of each part, we got

\[
2\int_0^L e^{-\gamma x} \, dx = L
\]

\[
2\int_0^L -2\cos \gamma x \sin \gamma x \, dx = -2\frac{\sin^2 \frac{\gamma L}{2}}{\gamma} \quad \text{(C.37)}
\]

\[
2\int_0^L \left[ 2(\cos \gamma x - \sin \gamma x)e^{-\gamma x} \right] \, dx
\]

\[
= 4\int_0^L \left( e^{-\gamma x} \cos \gamma x \right) \, dx - 4\int_0^L \left( e^{-\gamma x} \sin \gamma x \right) \, dx
\]

\[
= \left\{ 4\frac{e^{-\gamma x}}{2\gamma^2} (-\gamma \cos \gamma x + \gamma \sin \gamma x) - 4\frac{e^{-\gamma x}}{2\gamma^2} (-\gamma \sin \gamma x - \gamma \cos \gamma x) \right\} \bigg|_0^L \quad \text{(C.38)}
\]

\[
= \left\{ 4\frac{e^{-\gamma x}}{\gamma} \sin \gamma x \right\} \bigg|_0^L = -4\frac{e^{-\gamma L}}{\gamma} \sin \frac{\gamma L}{2}
\]

\[
2\int_0^L e^{-2\gamma x} \, dx = 2\left. \frac{1}{-2\gamma} e^{-2\gamma x} \right|_0^L = -\frac{1}{\gamma} (e^{-\gamma L} - 1) \quad \text{(C.39)}
\]
\[ I = L - 2 \frac{\sin^2 \frac{\gamma L}{2}}{\gamma} + \frac{4}{\gamma} e^{-\frac{\gamma L}{2}} \sin \frac{\gamma L}{2} - \frac{1}{\gamma} (e^{-\gamma L} - 1) \]

\[ = L + \frac{1}{\gamma} \left( 4e^{-\frac{\gamma L}{2}} \sin \frac{\gamma L}{2} + e^{-\gamma L} + 1 - 2 \sin^2 \frac{\gamma L}{2} \right) \quad (C.40) \]

\[ = L + \frac{1}{\gamma} \left( 4e^{-\frac{\gamma L}{2}} \sin \frac{\gamma L}{2} - e^{-\gamma L} + \cos \gamma L \right) \]

Here \( \gamma \gg 0 \), the exponential part goes to zero quickly when \( \gamma L \) becomes large.

\[ \gamma L = (2j + 1) \frac{\pi}{2} \quad j=1,12... \quad (C.41) \]

\[ \cos \gamma L = 0 \quad (C.42) \]

\[ I \equiv L + \frac{\cos(\gamma L)}{\gamma} = L \quad (C.43) \]
APPENDIX D  PWAS CHARGE AMPLIFIER

The charge amplifier principle was patented by W.P. Kistler in 1950 and gained practical significance in the 1960s. The charge amplifiers have high input impedance; they integrate weak charge pulses and convert them into voltage pulses for amplification then provide a low-impedance output. Piezoelectric wafer active sensors (PWAS) are charge mode active sensors and charge amplifier concept can be used to amplify the signal. The use of a PWAS charge sensitive amplifier can also reduce the adverse effects of low frequency noise.

D.1 CHARGE AMPLIFIER PRINCIPLE

Basically a charge amplifier consists of a high-gain inverting voltage amplifier at its input to achieve high insulation resistance. PWAS measuring systems are active electrical systems. That is, PWAS produce an electrical output signals only when they experience a change in load. PWAS can be considered as a charge mode device with a capacitor $C_{PWAS}$. A charge amplifier schematic for PWAS application is shown in Figure D.1.
Figure D.1 The charge amplifier consists of an op-amp. In this simplified schematic, \( V_o \) = output voltage; \( C_{PWAS} \) = PWAS capacitance; \( C_f \) = feedback capacitor; \( R_f \) = time constant resistor; \( Q \) = charge generated by PWAS.

PWAS charge amplifier voltage output for a static load,

\[
V_o = \frac{Q_{PWAS}}{C_f}
\]  

(D.1)

For a harmonic load,

\[
I_{PWAS} = V_{PWAS} Y_{PWAS}
\]  

(D.2)

\[
I = -V_o Y_f
\]  

(D.3)

where

\[
Y_{PWAS} = j\omega C_{PWAS}
\]  

(D.4)

\[
Y_f = j\omega C_f + \frac{1}{R_f}
\]  

(D.5)

The ideal relation between charge amplifier output voltage \( V_o \) and PWAS output voltage \( V_{PWAS} \) is

351
Ideally a charge amplifier is a high pass filter. It reduces the adverse effects of low frequency noise. The turnover frequency (-3 dB from that of DC) is

\[ f_H = \frac{1}{2\pi R_f C_f} \quad (D.7) \]

The op-amp gain calculated at low frequency does not apply at higher frequencies. To a first approximation, the gain of a typical op-amp is inversely proportional to frequency. This means that an op-amp is characterized by its gain-bandwidth product. This low-pass characteristic is introduced deliberately, because it tends to stabilize the circuit by introducing a dominant pole. Typical low cost, general purpose op-amps exhibit a gain bandwidth product of a few megahertz.

The PWAS charge amplifier amplification ratio can be considered in two frequency range. At low frequencies less than 100 kHz, it follows the equation. The capacitance of a PWAS bonded on an aluminum plate was 3.25 nF. Using the feedback components shown in Figure D.1, the maximum voltage gain is around 70. The lower cutoff frequency of charge amplifier is at 60 kHz. At high frequencies, the gain is limited by the bandwidth of the op-amp. The frequency limitation of the operational amplifier OPA627 used in the experiments starts to dominate the amplification after the frequency 100 kHz. The OPA627 amplification ratio experimental results fitted well with Equation(D.8). The upper cutoff frequency is at 500 kHz.

The PWAS charge amplifier becomes a band-pass filter with lower cutoff frequency at 60 kHz and the upper cutoff frequency at 500 kHz.
\[ \text{Ratio} = \begin{cases} \frac{Y_{P\text{WAS}}}{Y_f} & \rightarrow 0 \leq f \leq 100\text{kHz} \\ 320 - 50 \times \log_{10}(f) & \rightarrow 100\text{kHz} \leq f \leq 1000\text{kHz} \end{cases} \] (D.8)

### D.2 Operational Amplifier Open-loop Gain

Because the magnitude of the open-loop gain is typically very large and not well controlled by the manufacturing process, op-amps are not usually used without negative feedback. Unless the differential input voltage is extremely small, open-loop operation results in op-amp saturation. The open-loop gain of an operational amplifier falls very rapidly with increasing frequency. Along with slew rate, this is one of the reasons why operational amplifiers have limited bandwidth.

![Measurement Circuit](attachment:image.png)

(a)

![Results](attachment:image.png)

(b)

**Figure D.2** The open loop gain vs frequency (a) measurement circuit (b) results.
Burr-Brown precision high-speed difet® operational amplifier OPA627 was selected for its advantage of very low noise and fast settling time. The OPA627 op-amp’s open loop gain and frequency relation was measured with the circuit shown in Figure D.2(a). An AC input signal as indicated was applied. The amplifier output voltage and the voltage at the junction of the two 10 kΩ resistors (V_x) were measured. The amplifier gain is 101 times V_out/V_x. The measurement result is in Figure D.2 (b). It is almost the same as datasheet provided by manufacture.

**D.3 PWAS Charge Amplifier**

Two square (dimension: 7mm x 7 mm x 0.2 mm) PWAS made of PZT 850 from APC international Ltd was used in the experimental study. The static free PWAS capacitance was measured at 3.45 nF. Two PWAS were attached to a 1200mm x 1100mm aluminum plate with a distance of 250mm (Figure D.3). After bonding the plate, the capacitance of the PWAS reduced to 3.25 nF. Using the pitch-catch method, the left PWAS as a transmitter was connected to an HP33120A function generator with tone-burst excitation to generate the lamb wave in the thin plate. The right PWAS as a receiver was connected to a Tektronix digital oscilloscope to collect the Lamb wave signal.

Figure D.4 shows the signal with and without the charge amplifier. Ch 1 was 18V peak to peak 300 kHz tone-burst excitation signal. The Lamb wave was generated and propagated in the plate. Without the charge amplifier, the right PWAS received the Lamb wave S0 mode signal with amplitude of 20 mV (R1). With the charge amplifier, the charge amplifier output signal was around 800 mV.
The signals in different frequency ranges were measured to determine the efficiency of the charge amplifier. At a low frequency range from 15 kHz to 150 kHz, the Lamb wave A0 mode was measured because A0 mode was the dominant signal in this range. At a high frequency range from 150 kHz to 800 kHz, the Lamb wave S0 mode was measured because S0 was the dominant signal in this range. The voltage gain was calculated and compared with the theory value, shown in Figure D.5.

In conclusion, PWAS are charge mode active sensors and we can use charge amplifier to amplify the signal. The use of a charge sensitive amplifier can reduce the adverse effects of low frequency noise. From the comparison of the theoretical and experimental results, we can find the charge amplifier was suitable for the PZT-PWAS working frequency range. The voltage gain was adjustable and the amplification ratio decreased when the frequency became high due to the connection wire impedance.
Figure D.4 Charge amplifier experimental results.

Figure D.5 Comparison of the theory and experiment voltage gain of the specific charge amplifier.
APPENDIX E CAUCHY'S RESIDUE THEOREM FOR COMPLEX INTEGRALS EVALUATION

The residue theorem, sometimes called Cauchy's Residue Theorem, in complex analysis is a powerful tool to evaluate line integrals of analytic functions over closed curves and can often be used to compute real integrals as well. It generalizes the Cauchy integral theorem and Cauchy's integral formula.

E.1 CAUCHY’S INTEGRAL THEOREM

If $C$ is a curve in the complex plane joining the points $z_0$ and $z_1$, the line integral of a function $f(z) = u + iv$ along $C$ is defined by the equation

$$\int_C f(z) \, dz = \int_C (u + iv)(dx + idy) = \int_C (udx - vdy) + i \int_C (vdx + udy) \quad (E.1)$$

The real and imaginary parts of Equation (E.1) are ordinary real line integrals in the plane. The integral $\int_C f(z) \, dz$ is independent of the path $C$ joining the end points $z_0$ and $z_1$ if $C$ can be enclosed in a simply connected region inside which $f(z)$ is analytic.

In such a case the curve need not be prescribed and we may indicate the integral by the notation $\int_{z_0}^{z_1} f(z) \, dz$. It follows also that here $f(z) \, dz$ is the exact differential of a function $F(z)$,
\[ f(z)dz = dF(z) \] (E.2)

And that the integral can be evaluated in the form

\[ \int_{z_0}^{z_1} f(z)dz = dF(z_1) - dF(z_0) \] (E.3)

where \( F(z) \) is a function whose derivative is \( f(z) \).

By considering the case where the end points \( z_0 \) and \( z_1 \) coincide, we conclude that if \( C \) is any closed curve lying in a simply connected region where \( f(z) \) is analytic, the line integral of \( f(z) \) around \( C \) will vanish. We deduce that if \( f(z) \) is analytic inside and on a closed curve \( C \), then

\[ \oint_C f(z)dz = 0 \] (E.4)

This is known as Cauchy’s integral theorem.

For the single-valued function

\[ f(z) = \frac{1}{z} \] (E.5)

The derivative \( f'(z) = -\frac{1}{z^2} \) exists at all points except \( z = 0 \). Hence \( f(z) \) is analytic in any region not including the origin in the complex plane. Any closed curve \( C \) not surrounding the origin satisfies the conditions of Cauchy’s Theorem, and hence for any such curve there follows

\[ \oint_C f(z)dz = 0 \] (E.6)
However, if $C$ encloses the origin, the line integral need not vanish. If $C$ is taken as the unit circle $|z|=1$, with center at the origin, then on $C$ we may write

$$z = e^{i\theta}, \quad dz = ie^{i\theta} \, d\theta \quad (E.7)$$

And hence for this closed curve we obtain

$$\oint_{C_i} \frac{dz}{z} = \int_0^{2\pi} e^{-i\theta} ie^{i\theta} \, d\theta = \int_0^{2\pi} i \, d\theta = 2\pi i \quad (E.8)$$

where $\oint_{C_i}$ indicates integration around the positive direction of the unit circle.

In this case we may notice that $\frac{dz}{z}$ is the exact differential of the multi-valued function $\log z$. The value given by (E.8) is merely the increase experienced by the imaginary part $i\theta$ of $\log z$ as $z$ describes a positive closed circuit about the origin. Now consider any other closed curve $C$ which surrounds the origin. If we make a “cross-cut” from $1_C$ to $C$, and so determine the simply connected region $\Re$ shown in E.1, we see that since $f(z) = \frac{1}{z}$ is analytic in $\Re$, the integral around the complete boundary of $\Re$ must vanish, by Cauchy’s theorem. As the width of the cut is decreased towards zero the integrals along the edges of the cut are taken in opposite directions and hence cancel. Since the part of the complete integration which is carried out along $C_i$ is taken in the negative direction, there follows in the limit

$$\oint_C \frac{dz}{z} - \oint_{C_i} \frac{dz}{z} = 0 \quad (E.9)$$
And hence, for any closed curve $C$ enclosing the origin once in the positive direction, we have

$$\oint_C \frac{dz}{z} = 2\pi i \quad (E.10)$$

More generally, if $f(z) = z^n$, where $n$ is an integer, it can be concluded that

$$\oint_C z^n dz = 0 \quad (n \neq -1) \quad (E.11)$$

for any closed curve not passing through the origin. If $n = -1$, the integral is zero unless $C$ includes the origin, in which case the value is $2\pi i$.

If we replace $z$ by $z-a$, where $a$ is any complex number, we deduce from these results the additional results

$$\oint_C (z-a)^n dz = 0 \quad (n \neq -1)$$

$$\oint_C \frac{dz}{z-a} = 2\pi i \quad (E.12)$$
where $C$ is a closed curve enclosing the point $z = a$ in the positive direction and $n$ is an integer.

### E.2 Cauchy’s Residues Theorem

Suppose that the analytic function $f(z)$ has a pole of order $m$ at the point $z = a$. Then $(z - a)^m f(z)$ is analytic and hence can be expanded in the Taylor series

$$
(z - a)^m f(z) = A_0 + A_1(z - a) + ... + A_m (z - a)^m + ...
$$

(E.13)

where

$$
A_k = \frac{1}{k!}[\frac{d^k}{dz^k}(z-a)^m f(z)]_{z=a}
$$

(E.14)

The Taylor series converges within any circle about $z = a$ which does not include another singularity. If $z \neq a$, Equation (E.13) can be rewritten as in the form

$$
f(z) = \frac{A_0}{(z-a)^m} + \frac{A_1}{(z-a)^{m-1}} + ... + \frac{A_{m-1}}{(z-a)} + A_m + A_{m+1}(z-a) + ...
$$

(E.15)

Now let $C_a$ be any closed contour surrounding $z = a$ which lies inside the circle of convergence of (E.13) and $f(z)$ is analytic inside and on $C_a$, except at $z = a$. If we integrate around this contour and review Equation (E.12), we obtain

$$
\oint_{C_a} f(z)dz = 2\pi i A_{m-1}
$$

(E.16)

The only term contributing to the integration being $\frac{A_{m-1}}{z-a}$.
We call the coefficient $A_{m-1}$ the residue of $f(z)$ at $z=a$ and denote its value by $\text{Res}(a)$. Thus, if $f(z)$ has a pole of order $m$ at $z=a$, then

$$\oint_{C_a} f(z)dz = 2\pi i \text{Res}(a) \quad (E.17)$$

with

$$\text{Res}(a) = \frac{1}{(m-1)!} \left[ \frac{d^{m-1}}{dz^{m-1}} \left((z-a)^m f(z)\right) \right]_{z=a} \quad (E.18)$$

where $C_a$ is a closed contour enclosing $z=a$ but excluding all other singularities of $f(z)$.

Suppose now that $C$ is the boundary of a finite region inside which $f(z)$ is single-valued and has only isolated singularities, at a finite number of points $z=a_1, a_2, \ldots, a_n$. We enclose these points by small nonintersecting closed curves $C_1, C_2, \ldots, C_n$, each of which lies inside $C$ and encloses only one singularity. Then, by introducing a crosscut from each curve $C_k$ to $C$, a simply connected region $\mathcal{R}$ is obtained inside which $f(z)$ is analytic (E.2).

Thus the line integral of $f(z)$ around the complete boundary of this region vanishes. Noticing that the integrals along the crosscuts cancel, and that the integrations taken around the small contours are in the negative sense, there follows

$$\oint_C f(z)dz - \left[ \oint_{C_1} f(z)dz + \oint_{C_2} f(z)dz + \ldots + \oint_{C_n} f(z)dz \right] = 0 \quad (E.19)$$
Figure E.2 A finite number of interior isolated singularities in contour $C$.

Since the integral taken in the positive sense around $C_k$ is $2\pi i$ times the residue of $f(z)$ at $z = a_k$, Equation (E.19) leads to the result

$$\oint_{C_k} f(z)\,dz = 2\pi i \sum_{k=1}^{n} \text{Res}(a_k)$$  \hspace{1cm} (E.20)

Thus, if $f(z)$ is analytic inside and on a closed curve $C$, except at a finite num of interior isolated singularities, then $\oint_{C} f(z)\,dz$ is given by $2\pi i$ times the sum of the residues of $f(z)$ at those points. This result is known as Cauchy’s residue theorem.