COMPARISON OF SOLID-STATE ACTUATORS BASED ON POWER AND ENERGY CRITERIA

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ABSTRACT

A study of published literature and vendor information about solid-state induced-strain actuators has been undertaken to establish their dynamic-operation mechanical and electrical characteristics, and to compare their output power and energy density criteria. A theory was developed from first principles to predict the input and output power and energy amplitudes under dynamic operation including the piezoelectric counter electro-motive force and the bias-voltage effects. The effective electro-mechanical coupling coefficient, $\kappa$, for an assembled active-material stack was calculated from vendor data using a remarkably simple formula. Values between 0.45 and 1.038 were found, somehow in disagreement with the generally accepted value of 0.7 for the basic material. Dynamic energy output at the optimal stiffness match condition was found to have values of up to 0.260 J for some off-the-shelf actuators. Dynamic energy density per unit volume and unit mass varied in the range 0.5-3.7 J/dm$^3$ and 0.058-0.482 J/kg, respectively. Conversion efficiency between input electrical power and output mechanical power was found to vary widely in the range 3.74%-32.39%. This paper presents, with priority, two remarkably simple formulae: one for calculating the effective electro-mechanical coupling coefficient, $\kappa$, using standard vendor data; the other for estimating the peak reactive power for bias-voltage operation of the actuator.

INTRODUCTION

The use of solid-state induced-strain actuators has experienced a great expansion in recent years. Initially developed for high-frequency, low-displacement acoustic applications, these revolutionary concepts are currently expanding in their field of application in many other areas of mechanical and aerospace design. Compact and reliable, induced-strain actuators directly transform the input electrical energy into output mechanical energy. One application area in which solid-state induced-strain devices have a very promising perspective is that of linear actuation. At the moment, the linear actuation market is dominated by hydraulic and pneumatic cylinders, and by electromagnetic solenoids and shakers. Hydraulic and pneumatic cylinders offer reliable performance, with high force and large displacement capabilities. When equipped with servovalves, the hydraulic cylinders can deliver variable stroke output over a relatively large frequency range. Servovalve-controlled hydraulic devices are the actuator of choice for most aerospace, automotive, and robotic applications. However, a major drawback in the use of conventional hydraulic actuators is the need for a separate hydraulic power unit equipped with large electric motors and hydraulic pumps that send the high-pressure hydraulic fluid to the actuators through hydraulic lines. These features can be a major drawback in certain applications; for example, in the actuation of a servo-tab placed at the tip of a rotating blade, the high-g environment and the fact that the blade rotates, prohibit the use of conventional hydraulics. In such situations, an electro-mechanical actuation that directly converts electrical energy into mechanical energy is preferred. Conventional electro-mechanical actuator devices, that are based on electric motors, either deliver only rotary motion or require gearboxes and eccentric mechanisms to achieve linear motion. This route is cumbersome and leads to additional weight being added to the device, thus reducing its design effectiveness. Linear-action electro-mechanical devices, such as solenoids and electrodynamic shakers, exist, but are known for their typical low-force performance. The use of solenoids or electrodynamic shakers to perform the actuator duty-cycle of hydraulic cylinders is not presently conceivable.

Solid-state induced-strain actuators offer a viable alternative. Though their output displacement is relatively small, they can produce remarkably high force. Through the use of well-architected displacement amplification, induced-strain actuators can achieve dynamic output strokes similar to those of conventional hydraulic actuators. Additionally, unlike conventional hydraulic actuators, solid-state induced-strain actuators do not require separate hydraulic power units and long hydraulic lines, and use the much more efficient route of direct electric supply to the actuator site.
The development of solid-state induced-strain actuators has entered the production stage, and actual actuation devices based on these concepts are likely to reach the applications market in the next few years. An increasing number of vendors are producing and marketing solid-state actuation devices based on induced-strain principles. However, the performance of the basic induced-strain actuation materials used in these devices, and the design solutions used in their construction, are found to vary from vendor to vendor. This variability aspect presents a difficulty for the application engineer who simply wants to utilize the solid-state induced-strain actuators as prime movers in a design, and does not desire to investigate the intricacies of active materials technology.

Recognizing this need, the present paper sets out to perform a comparison of commercially-available induced-strain actuators based on a common criterion: the amount of energy that they can deliver, and the density of this energy per unit volume, unit mass, and unit cost. Additionally, this paper also compares the efficiency with which various induced-strain actuators convert the input electrical energy into output mechanical energy for use in the application.

The comparison is done using vendor-supplied information collected in an extensive survey performed over approximately a one-year period.

THEORETICAL BACKGROUND

Basic Equations of Linear Electro-Active Material Behavior

The general constitutive equations of linear electro-active material behavior\(^1\) describe a tensorial relation between mechanical and electrical variables (mechanical strain \(S_{ij}\), mechanical stress \(T_{ij}\), electrical field \(E_i\), and electrical displacement \(D_i\)) of the form:

\[
S_{ij} = s_{ijkl}^{E} T_{kl} + d_{kij} E_k \\
D_j = d_{jkil}^{T} T_{kl} + \varepsilon_{jk}^{T} E_k ,
\]

(1)

where \(s_{ijkl}^{E}\) is the mechanical compliance of the material measured at zero electric field \((E = 0)\), \(\varepsilon_{jk}^{T}\) is the dielectric constant (permittivity) measured at zero mechanical stress \((T = 0)\), and \(d_{kij}\) is the piezoelectric coupling between the electrical and mechanical variables. The second equation reflects the direct piezoelectric effect, while the first equation refers to the converse piezoelectric effect.

In a linear electro-active material stack, mechanical stress and electric field act only in the 3-direction (the stack axis), and hence their transverse components are zero. The transverse mechanical strain and electric displacement are not necessarily zero, since the compliance and piezoelectric matrices are usually fully populated. The focus of this analysis is directed towards the axial, "33", behavior, and the transverse strains and electric displacements are ignored. The one-dimensional equivalent of the general Equation (1), is simply:

\[
S = s \cdot T + d \cdot E \\
D = d \cdot T + \varepsilon \cdot E ,
\]

(2)

where the subscripts "3", "33", "333", and "3333" are implied, as appropriate. The compliance \(s\) is assumed to be measured at zero electric field, while the permittivity \(\varepsilon\) is assumed to be measured at zero mechanical stress.

Electro-mechanical Description of an Induced-Strain Actuator Operating a Mechanical Load

Figure 1 gives a schematic representation\(^9\) of a solid-state induced-strain actuator (a PZT stack) operating a mechanical load of parameters \(k_e\), \(m_e\), \(c_e\). The PZT stack is energized by a voltage source, \(v(t)\), which sends a current, \(i(t)\), that builds up the internal charge. As the charge is built up, the voltage and the electric field increase. Under the action of the electric field, the material expands and produces an output displacement, \(u(t)\), which generates a reaction force from the mechanical system, \(F(t)\). The reaction force, \(F(t)\), acting on the PZT stack induces loss of output displacement through the stack compressibility and through the counter electric motive force (emf) due to the piezo-electric effect. An actuator under load
always has a lower output displacement than a load-free actuator energized by the same voltage. A detailed analysis of the configuration shown in Figure 1 starting from the basic electro-mechanical constitutive equations of active material behavior and using wave propagation techniques was given in a previous publication. The equivalent input admittance was found as:

\[ Y(\omega) = i\omega C \left( 1 - \frac{d^2}{\tilde{s} \tilde{\varepsilon}} \frac{\tilde{\tau}(\omega)}{1 + \tilde{\tau}(\omega)} \right), \]  

(3)

where \( C \) is the zero-load capacitance of the stack, \( d \) is the zero-load induced-strain coefficient, \( s \) is the open-circuit (zero-field) complex mechanical compliance of the stack, and \( \tilde{\varepsilon} \) is the zero-load complex electrical permittivity (dielectric constant) of the active material, i.e.,

\[ \tilde{s} = s(1 - i\eta), \]  

(4)

\[ \tilde{\varepsilon} = \varepsilon(1 - i\delta), \]  

(5)

where \( \eta \) is the hysteresis internal damping coefficient, and \( \delta \) is the dielectric loss coefficient. The coefficient \( \tilde{\tau}(\omega) \) is the complex stiffness

\[ \tilde{\tau}(\omega) = \frac{\tilde{k}_e(\omega)}{\tilde{k}_i}, \]  

(6)

where the complex external stiffness, \( \tilde{k}_e(\omega) \), is given by Equation (11), while the complex stiffness expressions are:

\[ \tilde{k}_i = \frac{A}{s} \text{ (complex internal stiffness)}, \quad \tilde{k}_e(\omega) = \left[ (k_e - \omega^2 m_e) + i\omega \varepsilon \right] \text{ (complex external stiffness)}. \]  

(7)

Equation (3) can be simplified using the electro-mechanical coupling coefficient, \( \kappa \), defined as:

\[ \kappa^2 = \frac{d^2}{s \varepsilon}, \]  

(8)

and, hence:

\[ Y(\omega) = i\omega C \left( 1 - \kappa^2 \frac{\tilde{\tau}(\omega)}{1 + \tilde{\tau}(\omega)} \right). \]  

(9)

**Calculation of the Electromechanical Coupling Coefficient for an Existing Induced-Strain Actuator**

According to the IEEE Standard on Piezoelectricity\(^1\), the electromechanical coupling coefficient relevant to the operation of an induced-strain actuator is:

\[ \left( k_{33}^l \right)^2 = \frac{d_{33}^2}{s_{33} \varepsilon_{33}^T}. \]  

(10)

For simplicity, the subscripts and superscripts in Equation (10) can be considered implied. To avoid confusion with the previously introduced notation, \( k \), for the mechanical stiffness, the symbol \( \kappa \) will be used for the coupling coefficient instead of the letter \( k \). Hence,

\[ \kappa^2 = \frac{d^2}{s \varepsilon}. \]  

(11)

The constants, \( d \), \( s \), and \( \varepsilon \), can be calculated from the manufacturers' data sheet information about the stack, i.e.,

\[ \hat{u}_{ISA} = \frac{lV}{l}, \quad d = \frac{\hat{u}_{ISA}}{l} \]  

(12)
\[ C = \frac{\epsilon A}{t} \quad N_{\text{layers}} = \frac{\epsilon A}{t} / t, \quad \varepsilon = \frac{C}{A}, \quad l^2 = \frac{s}{lk_i} \]  

Hence, the effective electromechanical coupling coefficient for an electro-active material stack is:

\[ k_i^2 = \frac{\ddot{u}_{ISA}^2}{\frac{V^2}{l^2} = \frac{k_i \ddot{u}_{ISA}}{CV^2}} \]

which can be easily recognized as the ratio between the mechanical energy amplitude, \( \dot{E}_{\text{mechanical}} = \frac{1}{2} k_i \ddot{u}_{ISA} \), and the electrical energy amplitude, \( \dot{E}_{\text{electrical}} = \frac{1}{2} CV^2 \).

**Electrical Power with Bias Voltage**

Many solid-state induced-strain actuators do not have a symmetrical behavior when the polarity of the applied voltage is reversed. They have a preferred polarity which generates the maximum expansion. Under reversed polarity, some limited contraction may be achieved. Hence, dynamic operation must take place about a mid-range position, which is achieved by superposing a bias component onto the dynamic component. Thus, the applied voltage has the general expression:

\[ v(t) = V_0 + V \sin \alpha t, \]

where \( V_0 \) is the bias voltage and \( V \) is the amplitude of the dynamic component. The corresponding induced-strain displacement has the expression:

\[ u_{ISA}(t) = u_0 + \ddot{u}_{ISA} \sin \alpha t, \]

where, \( u_0 \) is the bias position, and \( \ddot{u}_{ISA} \) is the dynamic displacement amplitude. The values \( V_0, V, u_0 \) and \( \ddot{u}_{ISA} \) are easily calculated from manufacturers' specifications. For example (Figure 2), the solid-state induced-strain actuator P247.70 produced by Polytec PI, Inc., obtains its maximum free expansion of 120 \( \mu \text{m} \) under a voltage of -1000V, and has a maximum contraction of -30 \( \mu \text{m} \) under a voltage of 250V. For this actuator, the applied voltage for most productive dynamic operation will be \( v(t) = (-375) + (-625) \sin \alpha t \), with \( V_0 = [(1000) - (-250)]/2 = -375 \text{V} \) and \( V = [(1000) + (-250)]/2 = -625 \text{V} \). The resulting dynamic displacement is \( u_{ISA}(t) = 45 + 75 \sin \alpha t \mu \text{m} \), with \( u_0 = [120 - 30]/2 = 45 \mu \text{m} \) and \( \ddot{u}_{ISA} = (120 + 30)/2 = 75 \mu \text{m} \). Other solid-state induced-strain actuators have no reversed polarity operation, like, for example, the actuator E300P-4 produced by EDO Corporation which attains its maximum expansion at 800 V, but accepts no voltage reversal. For E300P-4, the excitation voltage for dynamic operation will be \( v(t) = 400 + 400 \sin \alpha t \), with \( V_0 = V = 800/2 = 400 \text{V} \), whereas the resulting displacement is \( u_{ISA}(t) = 30 + 30 \sin \alpha t \mu \text{m} \) with and \( u_0 = \ddot{u}_{ISA} = 30 \mu \text{m} \).
The power requirements of an induced-strain actuator operating with bias voltage are calculated as follows. Assume the equivalent circuit has the complex admittance given by:

$$\hat{Y} = \left[ R + i\left(\omega L - \frac{1}{\omega C}\right) \right]^{-1},$$

and write the current as:

$$i(t) = I \sin(\omega t - \phi), \quad \phi = \tan^{-1}\left[\left(\frac{\omega L}{\omega C}\right) / R\right],$$

where the current amplitude is:

$$I = Y(\omega)V.$$  \hspace{1cm} (20)

Use the fundamental definition, power = voltage x current, and write:

$$P(t) = v(t) \cdot i(t) = (V_0 + V \sin \omega t) \cdot I \sin(\omega t - \phi).$$  \hspace{1cm} (21)

Expansion of Equation (21) gives:

$$P(t) = \frac{1}{2} VI \cos \phi - \frac{1}{2} VI \cos(2\omega t - \phi) + V_0 I \sin(\omega t - \phi)$$

$$= P_{\text{active}} + P_{\text{reactive}}(t).$$  \hspace{1cm} (22)

Note that the active component of power is not influenced by the bias voltage and has the expression $P_{\text{active}} = \frac{1}{2} VI \cos \phi$, while the reactive component of power has the modified expression:

$$P_{\text{reactive}}(t) = -\frac{1}{2} VI \cos(2\omega t - \phi) + V_0 I \sin(\omega t - \phi)$$  \hspace{1cm} (23)

The influence of the bias voltage is to increase significantly the reactive power component. The complex power amplitude, $|\hat{P}| = \frac{1}{2} VI$, can be factored out of Equation (22), and hence,

$$P_{\text{reactive}}(t) = |\hat{P}|(-\cos(2\omega t - \phi) + 2v_0 \sin(\omega t - \phi)),$$  \hspace{1cm} (24)

where,

$$v_0 = V_0 / V,$$  \hspace{1cm} (25)

is the normalized bias voltage coefficient. Solid-state induced-strain actuators are either mainly inductive or capacitive, and their phase angle is close to $\pm90^\circ$. For values of the phase angle $\phi = -90^\circ$ (capacitive induced-strain actuator), Equation (24) becomes:

$$P_{\text{reactive}}(t) = |\hat{P}|(\sin(2\omega t) + 2v_0 \cos(\omega t)),$$  \hspace{1cm} (26)

Figure 3 presents the time-variation of the normalized reactive power given by Equation (26) for various values of the bias voltage coefficient, $v_0$. Note that, for $-1 < v_0 < 1$, the normalized reactive power curve presents a pair of local maxima, of which only one is also global maximum, and represents the peak value of the reactive power per cycle. For $v_0 = 1$ (not shown in Figure 3), the curve has a horizontal-tangent inflection point at $3\pi/2$. For $v_0 > 1$ and for $v_0 < -1$, the normalized reactive
power curve presents only one maximum since its behavior is dominated by the bias-voltage component. For \( \phi \neq \pm 90^\circ \), but close to \( 90^\circ \), curves similar to those given in Figure 7 are obtained.

To calculate the peak value per cycle of the reactive power, we differentiate Equation (26) with respect to \( \alpha \) and set the derivative to zero. The resulting quadratic equation has a pair of solutions of which only one corresponds to the peak reactive power. Thus, the peak reactive power takes place at the following critical angular values:

\[
(\alpha_{cr}) = \sin^{-1}\left(-\frac{v_0}{4} + \frac{1}{4}\sqrt{\frac{v_0^2}{2} + 8}\right), \text{ for } v_0 > 0,
\]

\[
(\alpha_{cr}) = \sin^{-1}\left(-\frac{v_0}{4} - \frac{1}{4}\sqrt{\frac{v_0^2}{2} + 8}\right), \text{ for } v_0 < 0.
\] (27) \hspace{1cm} (28)

The variation of the critical angular values, \( \alpha_{cr} \), with the bias voltage coefficient is shown in Figure 4. The position of the per cycle, \( \alpha_{cr} \), decreases as the bias voltage coefficient, \( v_0 \), increases (Figure 4a). The corresponding value peak reactive power, normalized with respect to the value at \( v_0 = 0 \), varies with \( v_0 \) as shown in Figure 4b.

![Figure 4 Influence of the bias voltage coefficient on the reactive power: (a) variation of the critical angular values, \( \alpha_{cr} \); (b) reactive power correction coefficient.](image)

![Diagram](image)

The normalized value of peak reactive power can be viewed as a reactive power correction coefficient, \( \chi(v_0) \), to account for the modifications in peak reactive power due to the presence of the bias voltage. Thus,

\[
P_{\text{peak reactive}} = \chi(v_0)|\overline{P}|.
\] (29)

Note that the variation shown in Figure 4b is almost linear, though the corresponding algebraic expressions (not given here for brevity) are quite elaborate. A reasonable approximation for the reactive power correction coefficient in the range \(-1.5 < v_0 < 1.5\) is given by:

\[
\chi(v_0) \approx 1 + 1.62|v_0|.
\] (30)

**Electrical Power Input to an Induced-Strain Actuator**

Assume operation under constant amplitude sinusoidal voltage, \( V \sin \alpha \), and bias voltage, \( V_0 \). The voltage amplitude and bias voltage are chosen so as to obtain maximum dynamic displacement from the actuator. Using the reactive power peak value per cycle expression, given by Equation (29), in terms of the bias voltage coefficient, \( v_0 = V_0 / V \), and the correction factor \( \chi(v_0) \), we write:

\[
P_{\text{peak reactive}} = \chi(v_0)|\overline{P}_{\text{electr}}|,
\] (31)
where \( \overline{P}_{\text{elect}} = \frac{1}{2} \overline{Y} V^2 \) is the complex-notations expression of the electrical power input to the system. Upon substitution, the peak value per cycle of the electrical power input to the system becomes:

\[
P_{\text{in}} = \frac{1}{2} \chi(v_0) \left| i \alpha C \left( 1 - \kappa^2 \frac{\overline{F}(\omega)}{1 + \overline{F}(\omega)} \right) \right| V^2.
\] (32)

The power input varies with the complex stiffness, \( \overline{F}(\omega) \), which depends strongly on frequency. For low-damping mechanical system driven at low frequencies, well below the mechanical resonance frequency, the complex stiffness ratio is predominantly real and almost equal to the static stiffness ratio, \( \kappa \).

Thus, the peak value per cycle of the electrical power input takes the simpler form:

\[
P_{\text{in}} = \omega \cdot \chi(v_0) \left( 1 - \kappa^2 \frac{r}{1 + r} \right) \left( \frac{1}{2} CV^2 \right).
\] (33)

As expected, the input power increases linearly with frequency. The last factor in Equation (33) represents the product between the angular frequency, \( \omega \), and the reference electrical energy amplitude, \( \frac{1}{2} CV^2 \), that can be easily calculated, for a given active-material stack, from manufacturers' specifications. The second and third factors in Equation (33) represent modifiers that are applied to account for the bias voltage effect and for the external loading conditions, respectively. At a given frequency, the peak value per cycle of the electrical power input varies strongly with the stiffness ratio, \( \kappa \). The normalized plot of its variation, given in Figure 5, indicates that, from the very low stiffness ratio case (free operation) to the very high stiffness ratio case (fully constrained operation), the peak value per cycle of the electrical power input can decrease as much as 50%. The exact percentage value of this decrease is given by the square of electromechanical coupling coefficient, i.e., \( \kappa^2 \).

**Mechanical Power Output from an Induced-Strain Actuator**

The mechanical power output is calculated using the output displacement:

\[
\tilde{u} = \frac{1}{1 + \overline{F}(\omega)} \hat{u}_{ISA},
\] (34)

where \( u_{ISA} \) is the induced-strain displacement amplitude,

\[
\hat{u}_{ISA} = l \hat{E} d = l \frac{V}{t},
\] (35)

where \( t \) is the thickness of the active material layers. Hence, the mechanical power output is:

\[
\bar{P}_{\text{mechanical}} = \frac{i \omega}{2} k_e u^2 = \frac{i \omega}{2} \overline{F}(\omega) \hat{u}_{ISA} \hat{u}_{ISA}^* \overline{F}(\omega) \left( \frac{1}{1 + \overline{F}(\omega)} \right)^2
\]

\[
= \frac{i \omega}{2} \overline{k}_e \overline{u}_{ISA} \overline{F}(\omega) \left( \frac{1}{1 + \overline{F}(\omega)} \right)^2 = \frac{i \omega}{2} \overline{k}_e \left( \frac{V d}{t} \right)^2 \overline{F}(\omega) \left( \frac{1}{1 + \overline{F}(\omega)} \right)^2 = \frac{i \omega}{2} \left( \frac{A d^2 l^2}{s l^2} \right) \overline{F}(\omega) \left( \frac{1}{1 + \overline{F}(\omega)} \right)^2 V^2.
\]
Further rearrangement yields:

$$
\bar{P}_{\text{mechanical}} = \frac{i \omega \frac{A l}{2} d^2}{\left(1 + \bar{r}(\omega)\right)^2} \bar{r}(\omega) V^2
= \frac{i \omega \left(\frac{A l}{2} \right) d^2}{s \varepsilon \left(1 + \bar{r}(\omega)\right)^2} \bar{r}(\omega) V^2
= \frac{i \omega \left(\frac{C d^2}{s \varepsilon} \right)}{\left(1 + \bar{r}(\omega)\right)^2} \bar{r}(\omega) V^2
= \frac{i \omega \left(\frac{C d^2}{s \varepsilon} \bar{r}(\omega)\right)}{\left(1 + \bar{r}(\omega)\right)^2} V^2.
$$

Since the mechanical power is the power output, we write:

$$
\bar{P}_{\text{out}} = i \omega \frac{\bar{r}(\omega)}{\left(1 + \bar{r}(\omega)\right)^2} \left[k^2 \left(\frac{1}{2} CV^2\right)\right].
$$

(36)

The power output varies with the complex stiffness, \(\bar{r}(\omega)\) which depends strongly on frequency. For low-damping mechanical systems driven at low frequencies, well below the mechanical resonance frequency, the complex stiffness ratio is predominantly real and almost equal to the static stiffness ratio, \(r\). Thus, the power output amplitude takes the simpler form:

$$
P_{\text{out}} = \omega \frac{r}{\left(1 + r\right)^2} \left[k^2 \left(\frac{1}{2} CV^2\right)\right].
$$

(37)

The output power can also be written in terms of the free dynamic amplitude of the actuator, \(\hat{u}_{ISA}\), and the internal stiffness, \(k_i\), i.e.:

$$
P_{\text{out}} = \omega \frac{r}{\left(1 + r\right)^2} \left(\frac{1}{2} k_i \hat{u}_{ISA}^2\right).
$$

(38)

It can be easily proved that Equations (37) and (38) are equivalent. As expected, the output power increases linearly with frequency. The last factors in Equations (37) and (38) represent reference values that can be easily calculated, for a given active-material stack, from manufacturers' specifications. For example,

$$
\bar{P}_{\text{mechanical}}^{\text{ref}} = \frac{1}{2} k_i \hat{u}_{ISA}^2.
$$

(39)

is the reference mechanical energy of an active-material stack operating under dynamic conditions. The second factor represents a modifier that accounts for the external loading conditions. An expression similar to Equation (39) was derived by Giurgiutiu et al.\(^{10}\) for an actuator operating under static conditions, but attention should be given to the fact that the static induced strain displacement, \(u_{ISA}\), and the dynamic induced strain displacement amplitude, \(\hat{u}_{ISA}\), have different values. At a given frequency, as the stiffness ratio varies, the output power increases and then decreases (Figure 6). The maximum output power is obtained at \(r = 1\), and its value is:

$$
P_{\text{out}}^{\text{max}} = \omega \frac{1}{4} \left(\frac{1}{2} k_i \hat{u}_{ISA}^2\right).
$$

(40)

It can be seen that the maximum output power is the product

![Figure 6 Variation of output power with stiffness ratio, \(r\)](image)

![Figure 7 Variation of electromechanical power conversion efficiency with \(r\) and \(k\), (no bias voltage, \(v_0 = 0\)).](image)
between the angular frequency, \( \omega \), and \( \frac{\nu}{4} \) of the reference mechanical energy output. Hence, the maximum amplitude of the reactive energy output is given by:

\[
\hat{E}_{\text{out}}^{\text{max}} = \frac{1}{4} \left( \frac{1}{2} k_i \hat{u}_{\text{ISA}}^2 \right).
\]  

The quantity \( \hat{E}_{\text{out}}^{\text{max}} \) is an effective metric for comparing the dynamic performance of various active-material stacks.

**Electromechanical Power Conversion Efficiency of an Induced-Strain Actuator**

The ratio between the output power and the input power is the conversion efficiency of the system, i.e.:

\[
\eta = \frac{|\hat{p}_{\text{out}}|}{|\hat{p}_{\text{in}}|} = \frac{|\omega \frac{\hat{r}(\omega)}{(1 + \hat{r}(\omega))^2} \left( l - \kappa^2 \frac{\hat{r}(\omega)}{l + \hat{r}(\omega)} \right) |}{\omega |\chi(v_0)\left( 1 - \kappa^2 \frac{\hat{r}(\omega)}{l + \hat{r}(\omega)} \right) |} = \frac{\hat{r}(\omega)}{(1 + \hat{r}(\omega))^2} \frac{k_i \hat{u}_{\text{ISA}}^2}{\chi(v_0) \left( 1 - \kappa^2 \frac{\hat{r}(\omega)}{l + \hat{r}(\omega)} \right) \frac{1}{2} CV^2}.
\]

Upon simplification, the conversion efficiency becomes:

\[
\eta = \frac{k^2 r(\omega)}{\chi(v_0) \left[ 1 + \left( 1 - \kappa^2 \right) r(\omega) \right] (1 + r)}.
\]  

The conversion efficiency varies with the complex stiffness, \( r(\omega) \), which depends on frequency. For low-damping mechanical systems driven at low frequencies, well below the mechanical resonance frequency, the complex stiffness ratio is predominantly real and almost equal to the static stiffness ratio, \( r \). Thus, the conversion efficiency takes the simpler form:

\[
\eta = \frac{k^2 r}{\chi(v_0) \left[ 1 + \left( 1 - \kappa^2 \right) r \right] (1 + r)}.
\]  

The conversion coefficient increases with the stiffness ratio and then decreases, as shown in Figure 7. It has a peak at stiffness ratio values that are close to \( r = 1 \), but not exactly equal to 1. The optimal stiffness value that maximizes the conversion coefficient can be found by setting the derivative of the conversion efficiency with respect to \( r \) equal to zero. This yields:

\[
r_\eta(\kappa) = \frac{1}{\sqrt{1 - \kappa^2}}.
\]

Note that the optimal stiffness ratio varies with the electromechanical coupling coefficient. For \( \kappa = 0 \), the optimal stiffness ratio is \( r_\eta = 1 \), i.e., it coincides with the optimal stiffness ratio for maximum power output. However, for \( \kappa = 0 \), the conversion efficiency would also be zero. Practical values of the electromechanical coupling coefficient vary between 0.6 and 0.7. Figure 8a shows the variation of the optimal stiffness ratio and of the corresponding maximum value of the conversion coefficient with \( \kappa \). It can be seen that, practical values of the optimal stiffness ratio vary between 1.25 and 1.4. The corresponding values of the best conversion coefficient can be calculated with the formula:
\[ \eta_{\text{max}}(\kappa, v_0) = \frac{1}{\chi(v_0)} \frac{\kappa^2}{2\sqrt{1 - \kappa^2} + 2 - \kappa^2}. \]  

(45)

Plots of Equation (45), given in Figure 8b, show that the values of the best conversion coefficient vary between 11% and 17% for no bias voltage operation \( (v_0 = 0) \) and decrease progressively as more bias voltage is applied. For the wide-spread case of the bias voltage equal to the dynamic voltage amplitude \( (v_0 = 1) \), the best conversion coefficient values may vary between 3% and 5%.

**DATA COLLECTION AND PROCESSING**

A large variety of induced-strain actuators are presently available in the commercial market. In our study, we collected as much data as possible by directly contacting the vendors and manufacturers of these products. Details about the data collection process were given in ref. 10. An initial processing of the data, done for static operation of the induced-strain actuators, was also presented. In this paper, a further reduction of data was performed to predict the dynamic operation of the induced-strain actuators. The process and results of this data reduction are presented in Table 1. The free expansion and free contraction, \( u^+_\text{ISA} \) and \( u^-\text{ISA} \), were used to calculate the bias position, \( u_0 \), and the load-free dynamic displacement amplitude, \( \hat{u}_\text{ISA} \), i.e.,

\[ u_0 = \frac{1}{2} \left( u^+\text{ISA} + u^-\text{ISA} \right), \quad \hat{u}_\text{ISA} = \frac{1}{2} \left( u^+\text{ISA} - u^-\text{ISA} \right). \]  

(46)

The expansion and contraction voltages, \( V^+ \) and \( V^- \), were used to calculate the bias voltage, \( V_0 \), and the dynamic voltage amplitude, \( V \), i.e.,

\[ V_0 = \frac{1}{2} \left( V^+ + V^- \right), \quad V = \frac{1}{2} \left( V^+ - V^- \right). \]  

(47)

Hence, the bias voltage coefficient, \( v_0 = V_0 / V \), and the reactive power correction factor, \( \chi(v_0) = 1 + 1.62v_0 \) were computed.

Using Equation (15), we calculated the effective electro-mechanical coupling coefficient, \( \kappa = k_i u^2\text{ISA} / CV^2 \). The amplitude of the reactive energy output in the most favorable conditions (stiffness match, \( r = 1 \)) was calculated with Equation (41), i.e.,

\[ \hat{E}_{\text{out}}^\text{max} = \frac{1}{2} \left( \frac{1}{2} k_i \hat{u}^2\text{ISA} \right). \]

By dividing amplitude of the reactive energy output, \( \hat{E}_{\text{out}}^\text{max} \), by the active-material volume and mass, we obtained the volume and mass energy densities of the actuators for dynamic operation. Note that these dynamic-operation energy densities are substantially lower than the static-operation energy densities presented in ref. 10. The main difference stems from the fact that, due to the asymmetric operation of the induced-strain actuator, the dynamic displacement amplitude is lower (in many cases \( \frac{1}{2} \)) than the static displacement.

The best energy conversion efficiency was calculated with Equation (45). (In one case, EDO E400P-3, when the apparent electro-mechanical coupling coefficient returned a value greater than 1, Equation (45) could not be applied and the best energy conversion efficiency was not calculated.) We also calculated the value of the stiffness ratio at which the best conversion efficiency is obtained. This value was called "best stiffness ratio" and denoted by, \( r_\eta \), in Table 1.

**DISCUSSION OF RESULTS**

Following the data reduction procedure outlined in the previous section, the charts contained in Figure 9 were obtained. Figure 9a presents the effective electro-mechanical coupling coefficient for the electro-active material stacks discussed here. Values of \( \kappa \) from 0.45 to 1.038 are observed. This range is somehow contradictory, since, as mentioned in ref. 1, the electro-mechanical coupling coefficients of the most electro-active materials vary around 0.7. This aspect has to be further discussed with the manufacturers for finding the necessary data corrections that will lead to reconciliation. Figure 9b presents a comparison of the maximum dynamic energy output that can be extracted from the commercially-available induced-strain actuators.
Table 1 Vendor data and calculated results for 14 commercially-available induced-strain actuators for dynamic applications.

<table>
<thead>
<tr>
<th>Identification</th>
<th>Type</th>
<th>Price</th>
<th>Stiffness (K/Nmm)</th>
<th>Electrical capacitance (µF)</th>
<th>Max. free expansion (µm)</th>
<th>Max. free contraction (µm)</th>
<th>Expansion voltage (V)</th>
<th>Contraction voltage (V)</th>
<th>Bias position (µm)</th>
<th>Dynamic displacement (µm)</th>
<th>Bias voltage (V)</th>
<th>Dynamic voltage (V)</th>
<th>Correction factor</th>
<th>Electro-mechanical coupling coefficient</th>
<th>Maximum dynamic energy output (J)</th>
<th>Best energy conversion efficiency (%)</th>
<th>Best stiffness ratio for energy conversion</th>
<th>Active material volume (mm³)</th>
<th>Active material mass (g)</th>
<th>Output energy per active material volume (J/mm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polytec PI</td>
<td>HVPZT</td>
<td>2250</td>
<td>8.0</td>
<td>0.45</td>
<td>120</td>
<td>30.0</td>
<td>-1000</td>
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<td>375</td>
<td>375</td>
<td>375</td>
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<td>0.0056</td>
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<td>1.16</td>
<td>7829</td>
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<td>375</td>
<td>375</td>
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<td>69183</td>
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<td>P-844.60</td>
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<td>0.0010</td>
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<td>160.0</td>
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<td>60</td>
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<td>0.0012</td>
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<td>395</td>
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<td>17.5</td>
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<td>0</td>
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<td>0.018</td>
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<td>0.0012</td>
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Figure 9. Charts of predicted behavior under dynamic operation calculated using vendor data for 14 commercially-available induced-strain actuators.

It seems that, at present, only one company, Polytec PI, Inc., has off-the-shelf products with large energy output (0.260 J peak for P-247-70). When discussing this aspect, the other vendors commented that they can also manufacture products with similar output energy, but on special order. Figure 9c compares the energy density per unit volume. The energy density per unit volume values vary from 0.5 to 3.7 J/dm³. For many induced-strain actuators, a mid-range value of around 2 J/dm³ seems to be common. The high end values, around 3.7 J/dm³, are reached by the Polytec PI and AVX products. Figure 9d compares...
the energy density per unit mass. The energy density per unit mass varies from 0.058 to 0.482 J/kg. For many induced-strain actuators, a mid-range value of around 0.2 J/kg seems to be common. The high-end values, around 0.482 J/kg, are reached by the Polytec PI and AVX products. Figure 9e gives a cost-based energy density comparison in terms of the energy per unit cost in mJ/$1000. Examination of these charts indicates that some companies are capable of marketing products with remarkably lower cost of the specific energy than others. This observation does not seem to be influenced by the processing method, since it affects adhesively-bonded and co-fired products equally.

Figure 9f presents a comparison of the best energy conversion efficiency that can be expected from these induced strain actuators under most favorable stiffness ratio conditions. As described by Equation (43), this coefficient represents the ratio between the output mechanical energy and the input electrical energy. These energies were assumed predominantly reactive (capacitance in the electrical energy, and spring in the mechanical energy), and the efficiency is the ratio of the reactive energy peaks per cycle. Note that the peak of the reactive electrical energy is strongly influenced by the presence of the bias voltage through the reactive power correction factor, \( \chi(v_0) \), as described by Equation (29). It is also influenced by the electromechanical coupling coefficient, \( \kappa \), as shown in Equation (43). The results, shown in Figure 9f, indicate that the energy conversion coefficient seems to vary from 2.90% to 32.39%, with the high-end products also having high values of conversion efficiency. However, before giving full confidence to these results, reconciliation of the unusual values of the electromechanical coupling coefficient, \( \kappa \), mentioned above, must be attained.

CONCLUSIONS

In continuation to some earlier work on static energy\(^{10}\), we considered here the dynamic power and energy capability of commercially-available induced-strain actuators operating against external load of adjustable stiffness ratio, \( r \). In the beginning of the study, we developed some additional theoretical concepts needed to clarify the meaning of active and reactive power and energy in the dynamic operation with bias-voltage specific to induced-strain actuators. It was found that the reactive power and energy dominate the dynamic operation of active-material-based induced-strain actuators, which is fundamentally different from the customary analysis of electric motors and other conventional electro-mechanical devices.

A number of specific concepts were introduced and/or clarified in the present study: Equation (15) defined the effective electro-mechanical coupling coefficient, \( \kappa \), and Equation (29) defined the reactive power correction coefficient, \( \chi(v_0) \), as a function of the bias-voltage coefficient, \( v_0 \). A reasonable linear approximation to the coefficient \( \chi(v_0) \), that can be successfully used for pre-design studies in the range -1.5 < \( v_0 < 1.5 \), was given in Equation (30). The electrical input power for a capacitive (piezoelectric or quasi-piezoelectric) induced-strain actuator was expressed by Equation (33) in a novel form that clearly establishes its dependence on the bias voltage coefficient, \( v_0 \), the reactive power correction coefficient, \( \chi(v_0) \), the effective electro-mechanical coupling coefficient, \( \kappa \), and the stiffness ratio, \( r \). The mechanical output power expressed in terms of the electrical characteristics of the induced-strain actuator was given in Equation (37), while Equation (38) expressed it in terms of the mechanical characteristics. The two formulations were shown to be equivalent through the use of the effective electro-mechanical coupling coefficient, \( \kappa \). The maximum possible value of the overall electro-mechanical conversion coefficient, \( \eta \), was given in Equation (45) in terms of a simple expression depending on \( v_0 \), \( \chi(v_0) \), and \( \kappa \).

The resulting numerical data, presented in a table and several charts, shows that maximum values of the dynamic output energy of up to 0.260 J can be achieved with off-the-shelf commercially-available actuators. The corresponding energy density values per unit volume and unit mass were found in the range 0.5-3.7 J/dm\(^3\), and 0.058-0.482 J/kg, respectively. Not surprisingly, these values are considerably lower than those for static operation, since, due to the asymmetric behavior of the induced-strain actuators, their dynamic displacement amplitude, \( \hat{u}_{ISA} \), is much lower (typically \( \frac{1}{2} \)) than their static stroke, \( u_{ISA} \). Electro-mechanical energy conversion efficiency was found to be in the range 0.39% to 26.8%. However, these conversion efficiency values are not, at present, entirely trustworthy, since they depend strongly on the effective electromechanical coupling coefficient, \( \kappa \), which gave, for some actuators, values that seem to violate the variation range accepted for the basic material. Further consultation with the vendors research teams and specialized experimental studies are needed to
build full confidence in the data that affects the subtle aspects of electro-mechanical conversion inside an induced-strain actuator operating in a variable dynamic-load environment.

The study presented here goes beyond simply comparing the properties of various active materials exhibiting the induced-strain actuating effect. In this study, assembled induced-strain actuators, directly available on the commercial market, were considered and their operation was compared in an operation environment containing mechanical loads of variable equivalent stiffness. The advantage of our approach is that it offers data that can be directly incorporated in the design of mechanical and hydraulic devices utilizing off-the-shelf induced-strain actuators. The present study follows an earlier study\textsuperscript{10} that was mainly concerned with the static operation, which is thus complemented and augmented. These two studies, taken together, form a valuable guide for the practicing engineers that wish to design actuation applications using off-the-shelf induced-strain actuators and do not require profound insight in the specific knowledge of active material behavior. It is remarkable that all the important parameters (input power, output power, and conversion efficiency) could be expressed in terms of standard vendor information without requiring insight into the intimate behavior of the active material used in the construction of the induced-actuator. Therefore, the analysis is general and can be readily applied to other similar products. It can also lead directly to formulation of industry standards that will greatly facilitate the development, use, and marketability of this novel class of actuators.

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