Predictive Methodologies for the Design of Lamb-Wave Piezoelectric Wafer Active Sensors for Structural Health Monitoring, Damage Detection, and Failure Prevention

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Abstract: The piezoelectric wafer active sensors (PWAS) are low-power non-resonant devices with broadband capabilities. They interrogate the structure through guided Lamb waves. PWAS capabilities for Lamb-wave damage detection have been validated experimentally, but the modeling of the in-situ PWAS behavior is still incomplete.

This paper addresses fundamental aspects of the Lamb-wave interaction between the PWAS transducers and the host structure. The goal of our research is to develop predictive methodologies to be used in the optimum design of these active sensors. The objective is to develop an understanding of the interaction between the PWAS and the Lamb-wave modes, and how this affects the PWAS effectiveness. A set of predictive methodologies used in the design of active SHM systems have been resulted.

1. INTRODUCTION

Current ultrasonic inspection of thin-wall structures (e.g., aircraft shells, storage tanks, large pipes, etc.) requires meticulous through-the-thickness C-scans over large areas. One method to increase the efficiency of thin-wall structures ultrasonic inspection is to utilize guided waves (e.g. Lamb waves) instead of the conventional pressure waves. Today’s advanced ultrasonic techniques rely on the generation, propagation, and detection of guided waves in various structures. Guided waves have opened many new opportunities for the cost-effective detection of damage in pipelines and aircraft structures [1][2].

In guided wave nondestructive evaluation, the most commonly used transducers are wedge transducers comb transducers, and electromagnetic acoustic transducers (EMATs). While these types of transducers function well for maintenance inspection when the structure is offline for service, they are not compact enough to be permanently installed on the structure during its operation as required for in-situ SHM. This is particularly true in aircraft structures, when the mass and space penalties associated with the additional transducers on the structure should be minimal [3]. For in-situ SHM, a different type of guided wave transducer that is smaller, lighter, and cheaper is required.

One convenient and often used transducers for in-situ guided wave generation during SHM studies are the piezoelectric wafer active sensors (PWAS). These inexpensive transducers are available in the form of thin piezoceramic wafers, that can be unobtrusively attached to existing structures. The use of PWAS for damage detection with Lamb wave propagation was pioneered by Chang and his coworkers [4]–[7] and was also studied by many other researchers [8]–[11]. The purpose of this paper is to present several results on the development of predictive methodologies for the use of PWAS transducers for structural health monitoring of metallic and composite structures. The results were obtained by the team of authors working in the Laboratory for Active Materials and Smart Structures at the University of South Carolina.

2. PWAS LAMB WAVES IN METALLIC THIN-WALL STRUCTURES

2.1 PWAS Ultrasonic Transducers: PWAS operate on the piezoelectric principles that couple the mechanical and electrical properties of the material. PWAS generate an electric field when they are subjected to a mechanical stress (direct effect), or, conversely, generate a mechanical strain in response to an applied electric field. Hence they can be used as both actuators and sensors. The coupling between the electrical and the mechanical variables (the charge per unit stress and the strain per unit electric field) is signified by the coefficients $d_{ij}$ ($i=1,\ldots,6; j=1,2,3$), also known as the polarization coefficient.

In practical applications, many of the piezoelectric coefficients $d_{ij}$ have negligible values as the piezoelectric materials respond preferentially along certain directions depending on their intrinsic (spontaneous) polarization. For PWAS depicted in Figure 1a, assume that the applied electric field $E_3$ is parallel to the spontaneous polarization $P_s$, with $P_s$
aligned with the $x_3$ axis. $E_3$ can be created through the application of a voltage $V$ between the top and bottom electrode of the wafer represented by the shading. The application of $E_3/p_{33}$ results in a vertical (thickness wise) expansion $\varepsilon_3 = d_{31}E_3$ and a lateral (in plane) contractions $\varepsilon_1 = d_{31}E_3$ and $\varepsilon_2 = d_{32}E_3$ (the lateral strains are contracted as the coefficient $d_{31}$ and $d_{32}$ have opposite sign to $d_{33}$). The strains experienced by PWAS are direct strains. Such an arrangement can be used to produce thickness-wise and in-plane vibration of the wafer.

![Figure 1](image1.png)

**Figure 1** PWAS elastic wave generation. (a) PWAS induced-strain responses; (b) PWAS elastic wave generation in 1-D model; (c) PWAS circular-crested elastic waves in 2-D model

### 2.2 PWAS Lamb Wave Generation:

In Lamb wave generation and sensing PWAS couple their in-plane motion with the particle motion of Lamb waves on the material surface, which is excited by the applied oscillatory voltage through the $d_{31}$ piezoelectric coupling. Circular PWAS excite omnidirectional Lamb waves that propagate in circular wave fronts. Unidirectional and omnidirectional Lamb wave propagation is illustrated in Figure 1b and Figure 1c. Omnidirectional Lamb waves are also generated by square PWAS, though their pattern is somewhat irregular in proximity to the PWAS. At far enough distance ($r>>a$), the wave front generated by square PWAS is practically identical with that generated by circular PWAS.

The interaction between the PWAS and the structure is achieved through the adhesive layer, in which the mechanical effects are transmitted through shear effects. Upon application of an electric voltage, the PWAS experiences an induced strain of $\varepsilon_{33} = d_{33}V/t_s$. The induced strain is transmitted to the structure through the boding layer interfacial shear stress ($\tau$). Because the PWAS ends are stress free, the build up of strain takes place at the ends, and it is more rapid when the shear stress is more intense. For large product of shear-lag parameter $\gamma$ and PWAS length $a$, the shear transfer process becomes concentrated towards the PWAS ends. In another word, a very thin bonding layer will produce a very rapid transfer that is confined to PWAS ends, indicating that in the limit, all the load transfer can be assumed to take place at the PWAS actuator ends. This leads to the concept of ideal bonding, also known as the pin-force model, in which all the load transfer takes place over an infinitesimal region at the PWAS ends, and the induced-strain action is assumed to consist of a pair of concentrated forces applied at the ends [4]. Using the shear-lag model, the energy transferred from PWAS to the structure can be found by analyzing either the elastic energy in the structure or work done by the shear stresses at the structural surface.

### 3. PWAS LAMB WAVE TUNING

#### 3.1 PWAS Tuning

Lamb waves are guided between two parallel free surfaces, such as the upper and lower surfaces of a plate. Lamb waves can exist in two basic types: symmetric ($S_0, S_1, \ldots$) and antisymmetric ($A_0, A_1, \ldots$). For each propagation type, a number of modes exist, corresponding to the solutions of the Rayleigh-Lamb equation. Lamb waves are highly dispersive and their speed depends on the product of frequency and the plate thickness.

The excitation of Lamb waves with PWAS is studied by considering the excitation applied by the PWAS through a surface stress $\tau = r_0(x) e^{i\omega t}$ applied to the upper surface of a plate in the form of shear lag adhesion stresses over the $(-a, +a)$ interval. Applying a space domain Fourier transform analysis of the basic Lamb wave equations to yield the strain wave and displacement wave solutions [4]

$$e_i(x,t)\big|_{x=0} = -i\frac{d\tau_0}{\mu} \left[ \sum_{\xi} \sin(\xi x/a) \frac{N_s(\xi^2)}{D_s(\xi^2)} e^{(\xi^2+\omega t)} + \sum_{\xi} \sin(\xi x/a) \frac{N_d(\xi^2)}{D_d(\xi^2)} e^{(\xi^2-\omega t)} \right]$$

with

$$N_s = \xi\beta\left(\xi^2 + \beta^2\right)\cos(\alpha d)\cos(\beta d)$$

$$N_d = \xi\beta\left(\xi^2 + \beta^2\right)\sin(\alpha d)\sin(\beta d)$$
\begin{align*}
D_S &= \left(\xi_S^2 - \beta^2\right)^2 \cos(\alpha d) \sin(\beta d) \\
&+ 4\xi_S^2 \alpha \beta \sin(\alpha d) \cos(\beta d) \\
D_A &= \left(\xi_A^2 - \beta^2\right)^2 \sin(\alpha d) \cos(\beta d) \\
&+ 4\xi_A^2 \alpha \beta \cos(\alpha d) \sin(\beta d)
\end{align*}

where \(\xi_S\) and \(\xi_A\) are the zeros of \(D_S\) and \(D_A\) respectively.

We can note that these are the solutions of the Rayleigh-Lamb equation. These results have been extended to the case of a circular transducer coupled with circular-crested Lamb waves by Raghavan and Cesnik [12] and corresponding tuning prediction formulae based on Bessel functions were proposed:

\begin{align*}
e_i(r, t) &= \frac{\tau_0 a}{\mu} e^{\alpha r} \left( \sum J_1(\xi_S a) \xi_S N_A(\xi_S) D_A'(\xi_S) H_i^{(2)}(\xi_S r) + \sum J_1(\xi_A a) \xi_A N_A(\xi_A) D_A'(\xi_A) H_i^{(2)}(\xi_A r) \right)
\end{align*}

Details of the Lamb waves theory can be found in the reference [4].

### 3.2 Experimental Verification:

When generated by PWAS, optimal Lamb wave excitation and detection can be obtained when the PWAS length is an odd multiple of the half wavelength of particle wave modes. The geometric tuning can be obtained through matching between their characteristic direction and the half wavelength of the excited Lamb wave mode. A comprehensive study of these prediction formulae in comparison with experimental results has recently been performed. Simulation plot of the equation (1) is presented in Figure 2a using a 7-mm square PWAS installed on 1.07-mm thick 2024-T3 aluminum alloy plate. Note the efficient PWAS length for a 7-mm PWAS has been verified as 6.4 mm Figure 2b.

Equation (1) contains the \(\sin(\xi a)\) behavior that displays maxima when the PWAS length \(L_e = 2a\) equals an odd multiple of the half wavelength, and minima when it equals an even multiple of the half wavelength. A complex pattern of such maxima and minima emerges, since several Lamb modes, each with its own different wavelength, coexist at the same time. The plot in Figure 2a shows that at 210 kHz, the amplitude of the A0 mode goes through zero, while that of the S0 is close to its peak. This represents an excitation “sweet spot” for S0 Lamb waves. Experimental results confirming this prediction are presented in Figure 2b.

### 3.3 Finite Element Modeling Verification:

Wave propagation excited by PWAS has been studied through finite element method (FEM). The investigation was carried on simple 1-D model of a narrow-strip aluminum beam specimen (914x8x1.6, in mm), as illustrated in Figure 3. The FEM model of the beam was constructed using four-node shell elements (SHELL63) that has six degrees of freedom at each node, displaying both bending and membrane capabilities. The excitation signal is a Hanning window smoothed 3-count sinusoidal signal.

In this research we simulated the PWAS transducer using coupled field elements. PWAS was excited with electrical signals and its electrical response to the wave motion was examined. The coupled field analysis therefore takes both mechanical motions and electrical characteristics into account, i.e., the direct interaction between mechanical motion and electrical potentials can be modeled in a nature way. Material properties, steel and piezoelectric material APC 850 are specified on the two objects. As shown in Figure 4a, bottom surface of the solid PWAS model was connected to the beam surface and thus permitted the mechanical motion
transfer between the two models. When PWAS being excited with the toneburst signal, it was the electrical energy that was directly fed onto the top surface of the PWAS, resembling the mechanics in the real application.

The simulation results are compared with experimental results, as given in Figure 4. It can be observed that the generated waves have incorporated both axial mode and flexural mode since PWAS was excited by electrical signals in the coupled field analysis and are similar to the experimental data. Also the waves were more complex than the mechanical excitation situation due to the reflection from the boundaries. Tuned excitation and other modeling modification are suggested in the future work to obtained higher quality signals.

![Figure 4](image)

**Figure 4** Coupled field analysis. (a) mesh design of the beam and coupled field modeled PWAS; (b) signals examined at PWAS B; (c) signals examined at PWAS C

4. **PWAS HEALTH MONITORING ON METALLIC STRUCTURES**

As active sensors, PWAS interact directly with the structure and find its state of health and reliability through the use of ultrasonic Lamb waves. Similar to the conventional ultrasonic transducers, PWAS can operate in pitch-catch, pulse-echo, or be wired into phased arrays to implement structural scanning.

4.1 **1-D PWAS Phased Array:** PWAS phased arrays have been developed for thin wall structures (e.g. aircraft shells, storage tanks, large pipes, etc.) that use Lamb waves to cover a large surface area through beam steering from a central location [13][14]. The embedded ultrasonic structural radar (EUSR) algorithm is a method and device for performing ultrasonic damage detection using PWAS phased array for the transmission and reception of guided waves and a signal post-processing methodology based on the beamforming process. In EUSR, excitation is implemented on only one PWAS at a time and a round-robin procedure is applied to measure a set of primitive signals that are stored in digital format and processed in virtual time after the data collection. This feature is specific to structural health monitoring implementations which allows for extensive signal processing over time. This feature permits inexpensive and light-weight implementation of phased array beamforming by avoiding complex multiplexing electronics.

The linear arrays were first implemented for damage detection as shown in Figure 5. The hardware includes a HP33120 function generator to send the excitation signal and a TDS210 digital oscilloscope to collect the reflection signals. An eight-PWAS array (7 mm round PWAS, spacing at 8 mm) was installed on a large aluminum plate (1200 mm square, 1 mm thick) (Figure 5b). A broadside crack was detected and represented as a dark shade in the EUSR scanned intensity image. However, due to geometrical limitation of linear arrays, their scanning ability was limited to 0°~180°. 360° scanning gives symmetric result about the array itself, as shown in Figure 5c.

![Figure 5](image)

**Figure 5** 1-D PWAS Phased Array: (a) PWAS phased array; (b) hardware include a HP33120 function generator to send the excitation signal and a TDS210 digital oscilloscope to collect the reflection signals; (c) a broadside crack was detected and represented as a dark shade in the EUSR scanned intensity image.
4.2 Linear Mini-Array: As any phased array method, EUSR has a near-field blind area which depends on the array aperture $D$, $D=Md$ ($M$-number of sensors; $d$-spacing). We have tested PWAS array of various sensor sizes and apertures; to date, we have been able to detect defects as close as 83 mm from the array center when using an eight-element array composing of 5-mm square PWAS with an aperture of 55 mm. The 5-mm PWAS array was installed along the boundary (as shown in Figure 6a). EUSR images obtained at scanning frequency of 402 kHz for broadside and offside cracks (10 mm in length) are given in Figure 6b. The results show that PWAS arrays have good potential for small area scanning and damage detection as well.

4.3 Planar Arrays: Planar arrays are constructed by positioning elements along various grids, such as cross grid, rectangular grid, or circular grid. Planar arrays have the advantages of providing more controls and being able to inspect all directions from a single location, i.e., covering the entire $0^\circ$~$360^\circ$ range. In this research, a densely populated square PWAS array consisting 8 by 8 elements was used. To compare with the linear array, the broadside crack detection was repeated by the square planar array except the crack is $45^\circ$ inclined. Scanning result is shown in Figure 7, indicating a unique crack image locating at $90^\circ$ in the structure. Note that the strongest reflection was found to be slightly away from right front side ($90^\circ$). This is due to the reason that we can not get specular reflection since the crack in this specimen is inclined.
5. PWAS GUIDED WAVE EXCITATION ON COMPOSITE MATERIALS

For PWAS applications on composite structures, it is important that either the dispersion curves of the Lamb modes or the tuning frequency of the desired mode can be predicted. Both analytical and mostly numerical approaches have been used for these predictions in laminated composite beams with PWAS.

5.1 PWAS Lamb Wave Dispersion Curves in Composite Materials: Different from the Lamb waves in metallic plate, it is not possible to find a close form solution of the dispersion curves in composite materials. However, there are different methods that can be used to solve the problem [3]. Consider a composite plate made of layers of unidirectional fibers, as shown in Figure 8a and use the Stiffness matrix in the form of

\[
C = T_1^{-1} C' T_2
\]

where \(T_1\) and \(T_2\) are the transformation matrix. In the case of PWAS transducers, the wave propagation direction is \(\theta = 90^\circ\). The total transfer matrix can be expressed as

\[
\begin{bmatrix}
\{u^+\} \\
\{\sigma^+\}
\end{bmatrix} =
\begin{bmatrix}
[A_{uu}] & [A_{us}] \\
[A_{su}] & [A_{ss}]
\end{bmatrix}
\begin{bmatrix}
\{u^-\} \\
\{\sigma^-\}
\end{bmatrix}
\]

In order to obtain the dispersion curve, stress free at the upper and lower surface boundaries was imposed

\[
|A_{ss}| = 0
\]

A Matlab program has been developed to calculate the transfer matrix of each layer in the global coordinate system and the total transfer matrix for different values of velocity and frequency. The program determines the velocity at which, for a given frequency, the matrix determinant change sign and then, by the method of bisection, finds the velocity solution. A quasi-isotropic plate \([0/45/-45/90]_2s\) was studied. The layer material is A534/AF252. Figure 8b shows the theoretical data and the experimental values. The experimental and theoretical data for the A0 mode are in quite good agreement; the experimental and theoretical data for the SH and S0 modes are also in good correlation with each other for low frequencies. At higher frequencies, where the two waves are closer, it is difficult to determine experimentally the wave location due to their superposition and dispersion.

5.2 PWAS Tuning Experiments on Composite Plates: For isotropic plates, the PWAS transfers the oscillatory contractions and expansion to the bonded layer and the layer to the metal surface. In this process several factors influence the wave behavior: thickness of the bonding layer, geometry of the PWAS, thickness, and material of the plate. The result of the influence of all these factors is the tuning of the PWAS with the material [3]. For interaction between PWAS and the composite plates, Xi has obtained the solution using the integral transform solution developed for isotropic plates [11]. Another method, normal mode expansion (NME), can also be used to determine the transducer frequencies for any kind of plates [15]. It has been used for application as hollow cylinders excited by wedge transducers where the dispersion curves have a second order of infinity [9][15]. Yet, no application for composite plates has been found.

PWAS tuning on composite plates has been investigated experimentally. The pitch-catch experiments were performed on a composite plate using ten round PWAS (American Piezo Ceramics, APC-850, 7 mm diameter, 0.2 mm thickness) as shown in Figure 9. PWAS #1 was used as transmitter while the other nine used as receivers. Here the data received by PWAS #3 on 90º fiber direction are analyzed. Note that clay dam was used to attenuate the reflection from the boundary. During the experiments two waves were detected, S0 and A0 modes. The SH0 mode was not possible to be located. The experiments were performed without an amplifier. The S0 amplitude was always quite low and the wave amplitude was of the order of magnitude of the noise at the frequency above 140 kHz.

Figure 10a shows the group velocity curve for the S0 and A0 modes. The NME theory was used to derive the theoretical prediction of the curves. The experimental values are in good agreement with the theoretical results. Beyond 185 kHz, the wave packets
were not possible to be separated from the noise. The experimental data were then used to construct the tuning curves, as shown in Figure 10b. Both theoretical A0 and S0 mode data were scaled of the same amount to compare those experimental data. It can be seen that the A0 mode tuning curve predicts with good accuracy the frequency at which A0 amplitude reaches a maximum. For the S0 mode, the theoretical and experimental results are not in agreement. Further investigation on different transmitter-receiver distance and direction with the reference to the fiber direction needs to be investigated in order to determine the validity of the method presented in this paper.

6. CONCLUSION
In this paper, an in-situ SHM technology has been discussed by using the piezoelectric wafer active sensor (PWAS) Lamb waves. The in-situ SHM is enabled by the use of embeddable PWAS transducers that can be permanently attached to metallic and composite structures.

Although remarkable progress has been made with in-situ PWAS SHM, considerable work remains to be done. To increase the acceptance of this emerging technology, the refining of the theoretical analysis and calibration against well-established experiments is needed, especially for applications on composite structures. In addition, to implement the in-situ SHM technique, several hurdles have to be overcome. In particular, the operational and environmental variations of the monitored structure need to be addressed. It shows signals will change due to varying operational and environmental conditions of the structure such as temperature, humidity, etc. Also, the bonding layer between the PWAS and structure is often the durability weak link for SHM applications, leading to loss of contact with the structure. The bonding layer may also include acoustic impedance mismatch with detrimental effects on damage detection. Better durability may therefore be expected from a built-in sensor that is incorporated into the structure, such as the in-situ fabricated PWAS. Last, but not least, a chip-level PWAS device possessing local processing ability and wireless communication ability must be developed to achieve in-field in-situ SHM applications.

7. ACKNOWLEDGEMENT
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8. REFERENCE:


