LAMB WAVE TUNING BETWEEN PIEZOELECTRIC WAFER ACTIVE SENSORS AND HOST STRUCTURE: EXPERIMENTS AND MODELING

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Abstract
In structural health monitoring (SHM), a network of embedded sensors permanently bonded to the structure is used to monitor the presence and extent of damage. The sensors can actively interrogate the structure through ultrasonic waves. Among the ultrasonic waves, Lamb waves are quite convenient because they can propagate at large distances in plates and then interrogate a large area. Lamb waves in a plate can be produced with piezoelectric wafer active sensors (PWAS) that are small, inexpensive, unobtrusive transducers. PWAS can be surface-mounted on an existing structured or placed inside composite materials. PWAS sensors use the piezoelectric principle. An alternating voltage applied to the PWAS terminals produces an oscillatory expansion and contraction of the PWAS. An oscillatory expansion and contraction of the PWAS produces an alternating voltage at the PWAS terminals. PWAS are bonded to the structure through an adhesive layer; the coupling with the investigated structure is higher than conventional transducers. If the PWAS bonded to the structure is excited, it couples its in-plane motion with the Lamb wave particle motion on the material surfaces. In previous studies, the Lamb wave mode tuning between PWAS and isotropic plates has been observed experimentally and theoretically. Recently experiments have been performed to verify the presence of tuning between bonded PWAS and composite plates. In the present paper, it will be discussed a method, normal mode expansion (NME), for predicting the tuning frequencies of the PWAS-plate structure. This method can be used for both isotropic and non-isotropic material. Experimental values for the tuning frequencies in isotropic plates are compared with the theoretically data obtain with integral transform solution and NME.

INTRODUCTION
Structural health monitoring (SHM) addresses problems of aging structures, a major concern of the engineering community. Recently, damage detection through Lamb waves has proven to be an attractive option for SHM because it allows condition-based maintenance inspection instead of schedule driven inspections. Lamb waves are elastic perturbations that can propagate for long distances with very little amplitude loss in solid plate or shell structures with free surfaces. Through Lamb wave detection, fewer numbers of transducers can be used to monitor a structure and to actively investigate the health of a structure system. An embedded system of transducers is placed in the structural component and it will monitor the health of the material during its service. SHM techniques can be passive or active systems. Passive SHM are a network of sensors that “listen” to the structure to monitor whether the component is changing. Active SHM interrogate the structure health through active sensors. One way to excite Lamb waves into a structure is through strain-coupled transducers that are bonded to the structural surface or inserted between the layers of composite structures [1]. Piezoelectric wafer active sensors (PWAS) are strain-coupled transducers that are small, lightweight, and
relatively low cost. PWAS have been used as active transducers bonded to the structure. These transducers, through an electric excitation, can generate Lamb wave in the material and, at the same time, can convert strain stimulations in an electric signal. Before the advent of these small transducers, the size of the transducers were bulk and not of practical use for SHM in aeronautical applications. However, with these kinds of transducers, it was easy to set the desired parameters to excite the modes needed for the structural interrogation. PWAS theoretically excite in the structure all the modes existing at the given frequency-thickness product. Under electric excitation, the PWAS undergoes oscillatory contractions and expansions, which are transferred to the structure through the bonding layer and thus excite Lamb waves into the structure. In this process, several factors influence the behavior of the excited wave. The result of the influence of all these factors is the tuning of the PWAS with various Lamb wave modes in the material. This phenomenon has been studied recently by Giurgiutiu [2] and lately by Raghavan and Cesnik [3]. Bottai and Giurgiutiu [4] and Santoni et al. [5] presented significant comparisons between the theoretical curves and the experimental data for different kinds of PWAS, thickness of the structure, and material types. The theoretical model was developed for tuning of the PWAS with the isotropic host structure. In the present paper will be illustrated a theoretical model that can be used to predict the tuning for composite structures. The method will be compared to the integral transform solution developed by Giurgiutiu [2] for the case of isotropic material.

**PWAS LAMB-WAVE TUNING ON PLATES**

For SHM it is important to excite only one mode, hence, the study of the behavior of the interaction between PWAS and structure is quite important. The coupling between PWAS and structures has been studied by Crawley et al. [6][7], and Giurgiutiu [8].

![Figure 1: Modeling of layer interaction between the PWAS and the structure. [8]](image)

Figure 1 shows the PWAS bonded to the plate. The bonding layer acts as a shear layer in which the mechanical effects are transmitted through the shear layers. Figure 2 shows a characteristic plot of the interfacial shear stress along the length of the PWAS for different thickness of the bonding layers. For a thin layer of the bonding material (1μm), the shear stress is transmitted to the structure only at the ends of the PWAS. This is the ideal bonding solution, which can be considered true as a first approach to the problem. In this case, the shear stress in the bonding layer can be assumed to have the form

\[ \tau(x) = a \tau_0 \left[ \delta(x-a) - \delta(x+a) \right] \]

Figure 2: Variation of shear-lag transfer mechanism with bond thickness for a APC-850 PWAS attached to a thin-wall aluminum structure through a bonding layer. (normalized position covers a half-PWAS length from center outwards).

The PWAS coupled with the structure and the structure itself may extend, bend, and shear. The relatively importance of one of the three modes depends on the geometry and the stiffness of the structure, PWAS, and bonding layer. The bond between the PWAS and the structure is considered ideal. Only the interaction between the PWAS and the plate is of interest. The load is considered to be transferred at the end of the PWAS. To investigate the tuning of the two elements two different kind of methods can be used: the classical integral transform method as developed by Giurgiutiu [8] for 1-D PWAS and by Raghavan et al [9][10] for circular PWAS and 3-D PWAS; the modal normal expansion technique, as developed by Auld [9][11]. In the NME, the fields generated in the structure due to the application of the surface loading are expanded in the form of an infinite series of the normal modes of the structure itself. For both techniques, the velocity fields of the modes and the physical properties of the sources used to generate the waves must be known. In the following chapters, a brief overview of the integral
A transform technique is presented. It will be also introduced the NME solution and it will be compared with the integral transform technique for the case of isotropic plate.

**Classical Integral Transform Technique**

The classical integral transform technique for the case of infinite PWAS strip bonded with a finite adhesive layer to a plate was first developed by Giurgiutiu [8]. The strain solution is expressed as

\[
\varepsilon_s(x, d, t) = -\frac{i a \tau_0}{\mu} \left[ \sum_{i} \sin(\xi^S a) \frac{N_S(\xi^S)}{D_S(\xi^S)} e^{-i \xi^S x} + \sum_{i} \sin(\xi^A a) \frac{N_A(\xi^A)}{D_A(\xi^A)} e^{-i \xi^A x} \right] e^{i \omega t} \tag{2}
\]

where

\[
N_S = \xi \beta (\xi^2 + \beta^2) \cos(\alpha d) \cos(\beta d) \tag{3}
\]

\[
N_A = \xi \beta (\xi^2 + \beta^2) \sin(\alpha d) \sin(\beta d) \tag{4}
\]

\[
D_S = (\xi^2 - \beta^2)^2 \cos(\alpha d) \cos(\beta d) + 4 \xi^2 \alpha \beta \sin(\alpha d) \cos(\beta d) \tag{5}
\]

\[
D_A = (\xi^2 - \beta^2)^2 \sin(\alpha d) \cos(\beta d) + 4 \xi^2 \alpha \beta \cos(\alpha d) \cos(\beta d) \tag{6}
\]

\(\xi^S\) and \(\xi^A\) are the zeros of the Rayleigh-Lamb equations for symmetric and antisymmetric modes respectively,

\[
\beta^2 = \frac{\omega^2}{c_s^2} - \xi^2,
\]

\[
\alpha^2 = \frac{\omega^2}{c_p^2} - \xi^2,
\]

where \(c_s\) is the shear wave velocity, \(c_p\) is the longitudinal wave velocity, and \(d\) is the half plate thickness.

Figure 3 shows the strain of Eq. (2) at different frequencies. It is noted that the normalize strain curves follow the general pattern of a sine function, which hits zeros when the half-length of the PWAS match an odd multiple of one of the wavenumbers of the Lamb waves plate. As indicated in Figure 3, such zeros occurred at different frequencies for the A0 mode than for the S0 mode.

**Normal Mode Expansion (NME) technique**

Liu et al. [12] has extended the integral transform solution derived for isotropic plates to the case of composite plates. The analytical solution of the inverse transform of the Fourier integral is to be solved numerically because it is not possible to obtain an analytical solution. Moreover, even if a numerical method is used, a proper treatment is needed, as the integrand goes to infinity at the poles on the integral axis.

A different method, NME, can be used to determine the transducer frequencies for any kind of plates. In NME the fields generated in the structure due to the application of the surface loading are expanded in the form of an infinite series of the normal modes of the structure itself. It has been used for application as hollow cylinders excited by wedge transducers where the dispersion curves have a second order of infinity. For complete mathematical rigor of the NME method, it must be proved that the field distribution of mode set is complete and orthogonal. We can assume that the set of acoustic waveguide mode functions is complete (Auld [9]). Following Auld [9] and Rose [13] we obtain that the acoustic field can be expressed as:

\[
v_m(y, z) = \sum_n a_{mn}(z) v_{mn}(y) \tag{7}
\]

\[
T_m(y, z) = \sum_n a_{mn}(z) T_{mn}(y) \tag{8}
\]

where

\[
a_{mn}(z) = \frac{e^{i b z}}{4 P_{mn}} \int_{-\infty}^{\infty} e^{i b \eta} t(\eta) d\eta
\]

\[
P_{mn} = \text{Re} \left[ \frac{1}{2} \int_{-\frac{b}{2}}^{\frac{b}{2}} (\mathbf{\tilde{v}}_{mn} \cdot \mathbf{T}_{mn}) \cdot dz \right]
\]

is the average power flow of the \(n\)th mode in the \(z\) direction per unit waveguide width (in the \(x\) direction).
Figure 4: Plate subject to a surface traction
and $t(\eta)$ is the external traction exerted by the actuator.

**Ideal Bonding Solution**

In the case of ideal bonding solution, the shear stress in the bonding layer is concentrated at the ends.

$$t(h/2,z) = \begin{cases} a \tau_0 \left[ \delta(z-a) - \delta(z+a) \right] z & \text{if } |z| \leq a \\ 0 & \text{if } |z| > a \end{cases}$$ (11)

Substituting (11) in (9), we obtain:

$$a_{az}(z) = BE_{az} \delta(z-a) \cos(\beta_z z)$$ (12)

Where

$$B = a \tau_0$$ (13)

$$E_{az} = \frac{\tilde{v}_{az}(b/2)}{4P_{na}}$$ (14)

$$F = \frac{1}{2a} \left[ \delta(\eta-a) e^{i\beta_z \eta} d\eta - \delta(\eta+a) e^{i\beta_z \eta} d\eta \right]$$ (15)

Function $B$ is a constant depending on the excitation.

Function $E$ is the excitability function of mode $v$, and $F$ is the excitation function.

Solving the integral (15), we obtain:

$$F = \pm 2i \sin \beta_z a .$$ (16)

Substituting this equation in (12), we obtain:

$$a_{az}(z) = i a \tau_0 \frac{\tilde{v}_{az}(b/2)}{2P_{na}} \sin \beta_z e^{i\beta_z z}$$ (17)

The NME technique developed is valid for isotropic and non-isotropic materials. In the case of non-isotropic material the Poynting vector integral must be solved numerically. We will further expand equation (17) for the case of isotropic materials and we will compare the results with the classical integral transform solution and the experimental results.

**Symmetric Poynting Vector.** For symmetric mode and isotropic material, displacements are:

$$u_{az} = Bi \xi_s \cos(\alpha_s y) + C \beta_z \cos(\beta_z y)$$

$$u_{ay} = -B \alpha_s \sin(\alpha_s y) - C i \xi_s \sin(\beta_z y)$$

Where

$$B = 2i \xi_s \beta_s \cos(\beta_s d)$$ and

$$C = \left( \xi_s^2 - \beta_s^2 \right) \cos(\alpha_s d)$$

For the range of interest of our experiments (0-1600 kHz mm), we can consider the ratio $c/c_s$ always.

Hence $\beta$ is always real.

$$B \in \begin{cases} \Re \text{ if } \beta_s \in \Re \\ \Im \text{ if } \beta_s \in \Im \end{cases}, C \in \Re$$

Case 1

$$\beta_s \in \Re \left\{ \begin{array}{l} u_z = \Re \left( V_z + W_r \right) \\ u_y = \Re \left( X_z + Y_r \right) \\ v_z = -i \omega V_z + i \omega W_r \\ v_y = i \omega X_z + i \omega Y_r \end{array} \right\}$$

Case 2

$$\beta_s \in \Im \left\{ \begin{array}{l} u_z = \Re \left( V_z + W_r \right) \\ u_y = \Re \left( X_z + Y_r \right) \\ v_z = -i \omega V_z - i \omega W_r \\ v_y = i \omega X_z - i \omega Y_r \end{array} \right\}$$

Case 2 is the same of case 1 but with the opposite sign:

$$\tilde{v}_z = \mp i \omega \left( V_z - W_r \right)$$

$$\tilde{v}_y = \mp i \omega \left( X_z - Y_r \right)$$

Where $V = Bi \xi_s \cos(\alpha_s y)$, $W = C \beta_s \cos(\beta_z y)$, $X = -B \alpha_s \sin(\alpha_s y)$, and $Y = -C i \xi_s \sin(\beta_z y)$

The Poynting vector can be written as:

$$P_{nm} = \Re \left[ -\frac{1}{2} \int_{-b/2}^{b/2} \left( \tilde{v}_z T_{xz} + \tilde{v}_y T_{zy} \right) dy \right]$$

$$P_{nm} = \Re \left[ +\frac{i \omega}{2} \int_{-b/2}^{b/2} \left( \left( X_z - Y_r \right) T_{xz} - \left( V_z - W_r \right) T_{zy} \right) dy \right]$$

where

$$T_{xz} = \mu \left( \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial y} \right)$$ (18)

$$T_{zy} = \left( \lambda + 2 \mu \right) \frac{\partial u_x}{\partial z} + \lambda \frac{\partial u_y}{\partial y}$$ (19)

Substituting the expression of the displacement, we obtain:

$$T_{xz} = \mu \left( C \xi_s^2 - \beta_s^2 \right) \sin(\beta_z y)$$

$$-2iB \xi_s \alpha_s \sin(\alpha_s y) e^{(i \xi_s \omega - \omega)}$$
The expression in brackets of the first term of the left hand side can be rearranged as:

\[
(\lambda + 2\mu)\xi^2_s + \lambda\alpha_s^2 = (\lambda + 2\mu)(\xi^2_s + \alpha_s^2) - 2\mu\alpha_s^2
\]

\[
= -2\mu\alpha_s^2 + (\lambda + 2\mu)\xi^2_s + (\lambda + 2\mu)\frac{\omega^2}{c_p^2} - (\lambda + 2\mu)\xi^2_s
\]

\[
= -2\mu\alpha_s^2 + (\lambda + 2\mu)\frac{\omega^2}{c_p^2}
\]

For antisymmetric displacements, Equations (18) and (23) become:

Equation (12) becomes:

\[
a_{as}(z) = \mp \frac{a}{\mu} \frac{B^2 c_s^2 \cos(\alpha_s d)}{-\alpha_s \cos(\beta_s d)} \sin(\beta_s a)e^{\pm j\beta_c z}
\]

The strain can be derived as:

\[
\varepsilon_z = \frac{\partial u}{\partial z} = \frac{\partial (v(z, y, t)dt)}{\partial z} = \frac{\partial a(z)v(y)}{\partial z} e^{-j\omega t}
\]

\[
\varepsilon_z = i\frac{\partial a(z)}{\partial z} v(y) e^{-j\omega t} = \pm \frac{\xi^2_s}{\omega} a(z)v(y)e^{-j\omega t}
\]

Antisymmetric Poynting Vector. For antisymmetric mode and isotropic material, displacements are:

\[
u_s(z, y, t) = (i\xi_s A \sin \alpha_s y - \beta_s D \sin \beta_s y)e^{j(\xi_s - \omega t)}
\]

\[
u_y(z, y, t) = (\alpha_s A \cos \alpha_s y - i\xi_s D \cos \alpha_s y)e^{j(\xi_s - \omega t)}
\]

Where

\[
A = 2i\xi_s \beta_s \sin(\beta_s d)
\]

\[
D = -(\xi^2_s - \beta^2_s) \sin(\alpha_s d)
\]

\[
\alpha_s \text{ is real if } \frac{\omega^2}{c_p^2} - \xi^2_s \geq 0, \text{ hence } \frac{\omega^2}{c_p^2} - \frac{\omega^2}{c} \geq 0
\]

\[
\frac{c^2 - c_s^2}{c^2 c_p^2} \geq 0 \text{ or } \frac{c^2}{c_p^2} \geq 1, \frac{c}{c_p} \geq 1
\]

For the first mode of propagation the ratio c/c_p is always less then 1 because c/c_p<1, hence \(\alpha\) is always imaginary; \(\beta\) is always imaginary.

\[
A \in \text{Im} \quad \forall \alpha_s, \beta_s, \quad D \in \text{Im} \quad \text{(if } \alpha_s \in \text{Im})
\]

For

\[
\alpha_s \in \text{Im} \quad \begin{cases}
\nu_z \in \text{Re} \quad v_z = -iou_z \in \text{Im} \quad \tilde{\nu}_z = iou_z \\
\nu_y \in \text{Im} \quad v_y = -iou_y \in \text{Re} \quad \tilde{\nu}_y = -iou_y
\end{cases}
\]

The Poynting vector can be written as:

\[
P_{sm} = \text{Re} \left[ -\frac{1}{2} \int_{-d}^{d} (\tilde{v}_s T_{zz} + \tilde{v}_t \hat{T}_{zz}) dy \right]
\]

Substituting (22) in (23) we have:

\[
P_{sm} = \text{Re} \left[ -\frac{1}{2} \int_{-d}^{d} (iou_s \nu_{zz} - iou_{zz} \nu_z) dy \right]
\]

or

\[
P_{sm} = \text{Re} \left[ -\frac{1}{2} i\omega \int_{-d}^{d} (u_s \nu_{zz} - u_{zz} \nu_z) dy \right]
\]

For antisymmetric displacements, Equations (18) and (19) become:

\[
T_{ss} = \mu \left[ \frac{2i\xi_s \alpha_s A \cos(\alpha_s y)}{(\xi^2_s - \beta^2_s)D \cos(\beta_s y)} \right] e^{j(\xi_s - \omega t)}
\]

(0.21)
Substituting (20) in the latter, we obtain:

\[ T_{zz} = -\mu \left( \xi_{d}^{2} + \beta_{d}^{2} - 2\alpha_{d}^{2} \right) A \sin(\alpha_{d}y) \left[ e^{i(\xi_{d}-\alpha_{d}t)} + e^{i(\xi_{d}+\alpha_{d}t)} \right] \]

Substituting (20) in the latter, we obtain:

\[ T_{zz} = -\mu \left( \xi_{d}^{2} + \beta_{d}^{2} - 2\alpha_{d}^{2} \right) A \sin(\alpha_{d}y) \left[ e^{i(\xi_{d}-\alpha_{d}t)} + e^{i(\xi_{d}+\alpha_{d}t)} \right] \]

The Poynting vector (24) becomes upon substitution and integration:

\[ P_{m} = -\frac{\alpha_{m}}{2} \text{Re} i \left[ \frac{i\xi_{a}A^{2}}{\beta_{a}} \sin(\beta_{a}d) \cos(\beta_{a}d) \right] + \frac{i\xi_{a}A^{2}}{\alpha_{a}} \sin(\alpha_{a}d) \cos(\alpha_{a}d) \]

\[ + \left( \alpha_{a} - \beta_{a} \right) \frac{3\xi_{a}^{2} + \alpha_{a} \beta_{a} + \beta_{a} \left( \alpha_{a} + \beta_{a} \right)}{\alpha_{a} - \beta_{a}} \frac{\sin(\alpha_{a} - \beta_{a})}{\alpha_{a} - \beta_{a}} \]

Equation (12) becomes:

\[ a_{m}(z) = \frac{\alpha_{m}}{\mu} \frac{u_{z}(d)}{P_{m}} \sin(\beta_{a}a) e^{i\beta_{a}z} \]

The strain can be derived as:

\[ \varepsilon_{z} = \frac{\partial u_{z}}{\partial z} = \frac{\partial}{\partial z} \int v_{z}(z,y,t) dt = \frac{\partial a(z)v(y)}{\partial z} e^{-i\omega t} \]

\[ \varepsilon_{z} = \frac{i}{\omega} \frac{\partial a(z)}{\partial z} v(y) e^{-i\omega t} = \pm \frac{\xi_{a}}{\omega} a(z)v(y) e^{-i\omega t} \]

\[ \varepsilon_{z} = \frac{\alpha_{m}}{\mu} \frac{\xi_{a} u_{z}(d)}{P_{m}} \sin(\beta_{a}a) e^{i\beta_{a}z} v(y) e^{-i\omega t} \]

COMPARISON BETWEEN INTEGRAL TRANSFORM METHOD, NME, AND EXPERIMENTS

Square PWAS

Experiments have been performed with square PWAS of 7 mm long, 0.2 mm thick (American Piezo Ceramics APC-850). The PWAS were bonded on a plate of aluminum Alloy 2024-T3, with 1.07 mm thickness, and 1222x1063 mm length (Figure 5). The PWAS were located at a distance of 250 mm from one another and at the center of the plate in order to avoid interference with the reflection from the boundaries.

Figure 5: Aluminum plate 2024-T3 1.07 mm with square, rectangular and round PWAS

Figure 6 shows the experimental data of the wave amplitude for the S0 and A0 modes. In the frequency range investigated, the A0 mode has two maxima at around 60 kHz and 400 kHz, and a minimum at 210 kHz. The S0 mode has one maximum at around the same frequency of the second maximum of the A0 mode.

Figure 7 shows the predicted values with integral transform solution of the wave amplitude for the S0 and A0 modes for a PWAS with effective length of 6.4 mm.
Figure 6: Experimental tuning data: Aluminum 2024-T3, 1.07 mm thickness, 7 mm square PWAS.

Figure 7: Predicted tuning with Eq. (2): Aluminum 2024-T3, 1.07 mm thickness, 7 mm square PWAS.

Figure 8: Predicted tuning with NME: Aluminum 2024-T3, 1.07 mm thickness, 7 mm square PWAS.

For this effective PWAS length value, we obtained the best agreement between experiments and predictions. In the development of the theory, it was assumed that there was ideal bonding between the PWAS and the plate. This assumption means that the stresses between the transducers and the plate are fully transferred at the PWAS ends. In reality, the stresses are transferred over a region adjacent to the PWAS ends (Figure 2). The experimental and theoretical values of the tuning were brought in good agreement by using the equivalent PWAS length value (Figure 6 and Figure 7).

Figure 8 show the predicted tuning frequencies with the normal mode expansion method. The minima of the A0 mode are in accordance with both the experimental values and the theoretical value derived with the integral transform solution. The S0 maximum is in good agreement with the experimental and theoretical data.

As the frequency decreases below the first A0 minimum, the value of the A0 strain increases. This behavior is different from the one found experimentally and theoretically with the integral transform solution. The second A0 maximum, derived with the NME technique, is around 350 kHz, 50 kHz below the experimental value. However, the relative amplitude between the S0 maximum and the A0 second maximum is represented more accurately by the NME method.

Rectangular PWAS

Rectangular PWAS of high aspect ratio were tested to examine the directional tuning of Lamb waves. Three rectangular PWAS of 25 mm × 5 mm size, and 0.15 mm thickness (Steiner & Martin) were used. The experiment configuration is shown in FIGURE 9.

FIGURE 9: Aluminum plate 2024-T3 1.07 mm thick with rectangular pwas

PWAS P1 was the transmitter and PWAS P2 and P3 were the receivers. One experiments is reported: transmitter P1, receiver P2.
Figure 10 shows the experimental data of the tuning. The experiment was conducted sweeping the frequency from 25 kHz to 250 kHz at steps of 3 kHz. Figure 10 shows the experimental values of the wave amplitude for frequency up to 250 kHz taken with steps of 3 kHz. The small steps we used to collect the data let us to detect the three maximum in the A0 mode that where not visible in the graph.

Figure 10: Tuning on plate 2024-T3, 1.07 mm thick; rectangular PWAS (P1-P2); experimental data

Figure 11 shows the predicted values of the PWAS tuning as calculated with the integral transform solution.

Figure 11: Tuning on plate 2024-T3, 1.07 mm thick; rectangular PWAS (P1-P2); prediction with equation (2)

The first two maxima are in accordance with the experimental values, while the third is at a lower frequency than that of the experiments. The S0 maximum is in accordance with the experimental values. The value of the theoretical PWAS that best predicts the experimental behavior is 24.8 mm. It is interesting to note that when the receiver is along the line of the bigger dimension of the transmitter, the PWAS behaves as a square PWAS 25x25 mm long. Figure 12 and Figure 13 show the S0 and A0 mode tuning respectively. The S0 mode is agreement with both the experimental data and the integral transform solution. As for the square PWAS, the maxima derived with the NME is at the same frequency and strain value as the one derived with equation (2).

Figure 12: Predicted S0 mode tuning with NME: Aluminum 2024-T3, 1.07 mm thickness, 25x5 mm rectangular PWAS

The A0 mode tuning derived with the NME technique has the same minimum as the ones derived with the integral transform solution and the ones found experimentally. In this case, also the A0 mode maxima are in good agreement with the theoretical value derived with equation (2).

Figure 13: Predicted A0 mode tuning with NME: Aluminum 2024-T3, 1.07 mm thickness, 25x5 mm rectangular PWAS.

The strain values of the A0 maxima are quite different from the calculated values with the integral
transform solution and the experimental values. However, also in this case the NME strain plot describe the decreasing A0 energy level with the increasing frequency, hence the A0 strain maxima are lower with increasing frequencies.

CONCLUSIONS

The normal mode expansion technique has proved to be a valid method for the detection of the tuning frequency of the PWAS bonded to the structure. The method has been compared for the case of isotropic plates with the experimental data and theoretical values derived with the integral transform solution. The mathematical formulation of the strain with the NME methods is different from that derived by Giurgiutiu [8]. However, the plot of the strain is in agreement with the experiments in both cases for the minima of the mode present at that frequency – thickness range. The NME method seems to be less accurate in the determination of the frequency of the maxima of the modes present.

NME technique can be extended to the case of composite plates once the dispersion curves are known. The difference between the case of an isotropic plate and the case of a composite plate is that in the latter case the integral in the Poynting vector must be solved numerically. We have not developed the program that solve find the tuning frequency for composite plates, but we have presented a method that can easily be extended to anisotropic plates has Rose [13] has already done.

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Lamb Wave Tuning Between Piezoelectric Wafer Active Sensors and Host Structure: Experiments and Modeling

By: Giola Santoni and Victor Giurgiutiu


Review:

This paper purports to experimentally and analytically investigate the tuning of a piezoelectric actuator with a quasi-isotropic composite host structure. A normal mode expansion (NME) technique is developed to investigate the tuning of piezoelectric actuator to the host structure. Results using this method are then compared with previously published experimental results and an integral transform technique. The results showed that the NME method compared well with the experimental results and the integral transform method for an isotropic plate. Some issues with the paper are:

- The given development of the NME method is only valid for isotropic materials.
- Experimental results for the tuning of a piezoelectric actuator with a quasi-isotropic composite host structure are never given.
- Analytical results for the tuning of a piezoelectric actuator with a quasi-isotropic composite host structure are never given.

Only a preliminary discussion of the application of this method to composite plates is given. The results which were given for composite plates is a comparison of experimentally determined and analytically determined group velocity dispersion curves which are well known.

The paper does present an alternative theoretical formulation for the Lamb wave tuning of actuators with an isotropic host structure. In order for the paper to be acceptable either the discussion of tuning in a composite structure should be omitted or results for the composite plate should be added. Other general areas which need improvement are:

- There are numerous spelling and grammatical errors which can impede full understanding of the work.
- Parts of various equations on pages 4-6 are not complete.
- A key reference to Auld’s work is cited in the paper, but not given (properly) in the references.
Draft Recommendations/Comments

Comments
Thanks