Experimental and theoretical studies of critical heat flux of flow boiling in microchannels with microbubble-excited high-frequency two-phase oscillations

Wenming Li, Fanghao Yang, Tamanna Alam, Jamil Khan, Chen Li*

Department of Mechanical Engineering, University of South Carolina, Columbia, SC 29210, United States

Abstract

Critical heat flux (CHF) during flow boiling in silicon microchannels (H = 250 μm, W = 200 μm, L = 10 mm) using self-excited and self-sustained high frequency two-phase oscillations is studied both experimentally and theoretically. Tests are performed on deionized water over a mass flux range of 200–1350 kg/m²s. An enhanced CHF of 1020 W/cm² is achieved experimentally at a mass flux of 1350 kg/m²s in the present study. Since no existing CHF models and correlations on parallel mini/microchannels considered high frequency two-phase oscillations, hence are not applicable to predict CHF in the present microchannel configuration. Adopting Helmholtz and Rayleigh instability theories and based on experimental study of liquid thin film dry-out phenomena in two-phase oscillations, a semi-theoretical CHF model is proposed. The proposed theoretical predictions show satisfactory agreement with experimental data with a reasonable low mean absolute error (MAE) of 25–32%.

1. Introduction

Critical heat flux (CHF) of flow boiling refers to the maximum heat flux just before the boiling crisis where a drastic decrease of heat transfer rate or a sudden increase of surface temperature occurs. Hence, this is one of the critical thermal limits in electronics cooling systems, heat exchangers and thermal-hydraulic system of nuclear power plants. Achieving an ultra-high CHF is desirable to increase the heat flux safety margin of thermal system. Additionally, high power electronics cooling becomes even more challenging as the transistor size keeps shrinking with an increasing power density. Therefore, it is of great interest to improve CHF of these miniature electronic devices. CHF can be triggered by boundary layer separation [1], bubble crowding [2] and sub-layer dryout on heated surface [3]. Many flow boiling enhancement techniques have been developed to enhance CHF in last decades. The main enhancement mechanisms are: regulating bubble slugs, suppressing flow instability, modifying surface characteristics, improving surface to volume ratio, and promoting liquid rewetting. For example, to suppress flow instability, inlet/outlet restrictors [4,5] and reentrant cavity [6,7] were introduced. Recently, nanofluid (e.g., Al₂O₃ nanoparticle in DI-water [8]) and nano/microscale coating (e.g., nanowire [9–11], nanotube [12] and nanoporous surface [13]) were developed to improve wettability. Meanwhile, surfactant can enhance CHF by reducing surface tension of fluids [14]. In addition, techniques, such as micro/mini-jets [15,16], tapered manifold [17], micromixer [18] and solution (e.g., aqueous n-butanol, TSP and boric acid solutions) [19,20] can enhance CHF as well. However, some enhancement techniques are at the cost of pressure drop. For example, inlet restrictors are considered as one of the most effective ways to improve CHF. However, significant additional pumping power is required for this technique. Furthermore, nano/microscale coating technique can drastically enhance CHF with penalty of pressure drop [9,21] as well. Pros and cons of existing CHF enhancement techniques are listed in Table 1.

Ultra high CHF (>30,000 W/cm²) was achieved at high mass fluxes (>38,111 kg/m²s) in microchannel flow boiling [28]. However, much more pumping power was required. Most recently, in our previous studies, a microbubble-excited actuation mechanism has been established to create intense mixing in the microchannels [16,29]. This new enhancement mechanism can effectively suppress flow boiling instabilities and significantly improve liquid rewetting without adding additional pressure drops [16]. In the present study, CHF of flow boiling in a multiple microchannel array with self-excited and self-sustained high frequency two-phase oscillations is experimentally studied. Experimental results show that an ultra-high CHF (up to 1020 W/cm²) can be achieved at a modest mass flux of 1350 kg/m²s on DI water.
To better understand this CHF enhancement mechanism, a semi-theoretical study in conjunction with experimental investigation is conducted based on fundamental thermal/fluid physics. To develop a semi-theoretical CHF model for flow boiling in microchannels with high frequency self-sustained two-phase oscillations, all essential thermo-physical properties, for example, thermal conductivity, specific heat, and latent heat of evaporation should be considered. In addition, the mass flux of subcooled liquid is critical to the direct condensation heat transfer at the vapor/liquid interface, which primarily determined bubble dynamics in the inlet manifold and CHF condition of subcooling flow boiling [38,39]. Especially, as the CHF is approaching, vigorous vapor generation resulting from thin film evaporation leads to reversal vapor flow, which is governed by the Helmholtz and Rayleigh instabilities. Therefore, it is critical to take account of the interfacial condensation [40].

It is challenging to couple the Helmholtz and Rayleigh instabilities [41] in a flow boiling CHF model. The Helmholtz critical velocity of the reversal vapor flow is determined by surface tension and hydraulic diameter of the auxiliary channel. The establishment of stable vapor columns in the auxiliary channel blocks the liquid supply to the heating areas, then results in the CHF conditions. The major challenges to develop theoretical CHF model are twofold: (a) the length of vapor columns varies with the subcooling liquid flow rate; and (b) the interfacial condensation heat transfer rate is difficult to determine. Visualization study of bubble dynamics at CHF conditions is conducted to address the former challenge. Additionally, three different interfacial heat transfer coefficient (h) correlations have been used to adopt the best suitable interfacial heat transfer coefficient model for this present condition. All related influences arising from local subcooling near vapor/liquid interface, conjugate heating, and mass flux are taken account into the model. Pressure and temperature oscillations have been discussed in our previous studies [16,29].

Owing to the complexity of CHF mechanisms during flow boiling in microchannels, most of the existing CHF models or correlations are empirical and semi-empirical [30–32]. According to the recent review on CHF in mini/microchannels conducted by Roday and Jensen [33], experimental CHF results are inconsistent with existing models. For instance, there are disagreements with the influence of inlet subcooling degree, pressure, exit quality, and diameter on CHF in existing models [30,34–35]. Additionally, only a few theoretical CHF models are available for flow boiling in microchannels, for example, a theoretical CHF model based on interfacial waves [3,36], or local liquid thin film dryout [37]. However, none of these models considered two-phase oscillations; hence these models are not applicable to the present microchannel configuration.
2. Experimental study

2.1. Experimental apparatus and methods

Micro devices are designed and fabricated to enhance CHF of flow boiling in microchannels. Each micro device consists of four main channels \( (H = 250 \, \mu m, \ W = 200 \, \mu m, \ L = 10 \, mm) \) with an oriﬁce \( (H = 250 \, \mu m, \ W = 20 \, \mu m, \ L = 400 \, mm) \) placed at the inlet of each main channel to trap bubbles as shown in Fig. 1. Two auxiliary channels \( (H = 250 \, \mu m, \ W = 50 \, \mu m, \ L = 5 \, mm) \) are connected at the middle of each main channel through a 20 \( \mu m \) opening as highlighted in Fig. 2a. The auxiliary channels are also open to the inlet manifold. The design and fabrication of the microchannel device were detailed in our previous studies \[16,29\]. The experimental apparatus (including the test module and loop), experimental procedure, and the data reduction method established in our previous studies are adopted in this work.

In our previous study \[13\], it was shown that the temperature of inlet subcooling has negligible effects on CHF. Therefore, CHF data with an inlet fluid temperature of 25 \( ^\circ C \) is collected for varying mass fluxes from 200 to 1350 kg/m\(^2\) s. In addition, a uniform heat flux is applied to the bottom of the micro device.

2.2. CHF conditions

It is critical to learn from visualization data at CHF conditions in order to develop a high fidelity CHF model. In this study, CHF of flow

![Fig. 1. Schematic of CHF phenomena in microchannel. (a) architecture of the testing chip, (b) stable vapor column formed at the opening of auxiliary microchannel at \( G = 900 \, kg/m^2 \cdot s \).](image)

![Fig. 2. Schematic of vapor backflow in an auxiliary channel.](image)

![Fig. 3. Comparison between the present experimental CHF values and reported studies. #1, plain-wall microchannels in Qu and Mudawar’s study \[30\]. #2, plain-wall microchannels with inlet oriﬁces and #3, structured microchannels with inlet oriﬁces in Kuo and Peles’ study \[7\]. #4, plain-wall microchannels with pressure drop elements and #5, plain-wall microchannels without pressure drop elements in Kuan and Kandlikar’s study \[46\].](image)

Table 2

<table>
<thead>
<tr>
<th>CHF data by</th>
<th>The length of heating area (mm)</th>
<th>( T_{in} ) (°C)</th>
<th>( G ) (kg/m(^2) s)</th>
<th>( q_{00}^m ) (W/cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qu and Mudawar [30]</td>
<td>44.8</td>
<td>30</td>
<td>86–368</td>
<td>107.64–216.76</td>
</tr>
<tr>
<td>Kuan and Kandlikar [46]</td>
<td>63.5</td>
<td>25.4</td>
<td>46.4–171.9</td>
<td>20.6–54.5</td>
</tr>
<tr>
<td>Kuo and Peles [7]</td>
<td>10</td>
<td>22</td>
<td>83–303</td>
<td>161.2–645.3</td>
</tr>
<tr>
<td>Present Experimental study</td>
<td>10</td>
<td>25</td>
<td>200–1350</td>
<td>225.0–1020.0</td>
</tr>
</tbody>
</table>

main channels \( (H = 250 \, \mu m, \ W = 200 \, \mu m, \ L = 10 \, mm) \) with an oriﬁce \( (H = 250 \, \mu m, \ W = 20 \, \mu m, \ L = 400 \, mm) \) placed at the inlet of each main channel to trap bubbles as shown in Fig. 1. Two auxiliary channels \( (H = 250 \, \mu m, \ W = 50 \, \mu m, \ L = 5 \, mm) \) are connected at the middle of each main channel through a 20 \( \mu m \) opening as highlighted in Fig. 2a. The auxiliary channels are also open to the inlet manifold. The design and fabrication of the microchannel device were detailed in our previous studies \[16,29\]. The experimental apparatus (including the test module and loop), experimental procedure, and the data reduction method established in our previous studies are adopted in this work.

In our previous study \[13\], it was shown that the temperature of inlet subcooling has negligible effects on CHF. Therefore, CHF data with an inlet fluid temperature of 25 °C is collected for varying mass fluxes from 200 to 1350 kg/m\(^2\) s. In addition, a uniform heat flux is applied to the bottom of the micro device.

2.2. CHF conditions

It is critical to learn from visualization data at CHF conditions in order to develop a high fidelity CHF model. In this study, CHF of flow
As the flow is determined by Rayleigh instability \[41\]. Additionally, the critical vapor wavelength of vapor flow is induced by intense interface. The theory of Helmholtz instability is adopted to predict (Fig. 1b) due to equilibrium of heat transfer at the vapor/liquid interface. The objective of this comparison we made here is only to show efforts that have been done in last decade. CHF experimental results in the present microchannel configuration and results in literatures are compared and shown in Table 2. It can be seen from Table 2 that the present microchannel configuration made significant progresses using a passive enhancement technique.

### 3. Semi-theoretical CHF model

#### 3.1. Development of a semi-theoretical CHF model

A semi-theoretical CHF model provides insights into CHF mechanisms since it is built on a solid understanding of the CHF mechanisms from experimental studies. Moreover, it is also convenient to implement theoretical and semi-theoretical CHF models. In this present study, by incorporating interfacial heat transfer rate with a liquid thin film dryout model, a semi-theoretical CHF model is proposed based on an energy conservation analysis in a pair of neighboring auxiliary channels. All major thermo/fluidic properties and geometries are considered in this proposed model. The flow chart in Fig. 4 shows the relationship among key governing parameters of CHF. Heat in auxiliary channels is transported by high frequency bubble growth (from boiling) and collapse (from direct interfacial condensation) \[46\]. The direct condensation is determined by heat transfer at the vapor/liquid interface or by forces (such as surface tension, inertia forces) acting on the interface \[47,48\]. The heat transfer at the vapor/liquid interface with phase transition highly depends on local thermo-physical properties, such as liquid conductivity, specific heat, latent heat of vapor, density, viscosity and surface tension, etc.; therefore, it is a complex process.

In this study, CHF of flow boiling on the present microchannel configuration is assumed to be primarily governed by the Helmholtz instability \[44,45\]. The Helmholtz instability is induced by the fast vapor flow in auxiliary channels. Helmholtz critical velocity has a relationship with critical vapor wavelength. Hence, Rayleigh instability is applied to predict the critical vapor wavelength of vapor back flow in the auxiliary channels. At CHF, stable vapor columns are formed in front of auxiliary channels. At this condition, Helmholtz critical velocity should be equal to vapor jet velocity. The key characteristics of stable vapor column are expressed by following equations.

The vapor jet velocity is calculated as

\[
U_j = \frac{m_v}{\rho_v A_v} \tag{1}
\]

Helmholtz critical velocity \[42\] is given by

\[
U_c = \sqrt{\frac{2\pi \sigma}{\rho_v \cdot \lambda}} \tag{2}
\]

Rayleigh critical wavelength \[41\] is determined by

\[
\lambda = \sqrt{\frac{2\pi \cdot D_h}{\lambda}} \tag{3}
\]

boiling on deionized (DI) water is explored in a multiple microchannel array. Nucleation of bubble first starts on the bottom surface of the auxiliary channel. These nucleating bubbles confine quickly and then expand backwards (reversal vapor flow) in the auxiliary channels as shown in Fig. 2 due to the restriction of the nozzles. Direct condensation occurs at the vapor/liquid interface resulting in shrinkage of the bubble and eventually the bubble collapses in the auxiliary channels. This action then sucks liquid from inlet manifold and forms jetting flows in the main channels through the nozzles \[29\]. With further increase in heat flux to some extent, liquid supply through auxiliary channels is blocked due to stable vapor column and hence bubble collapse ends in auxiliary channels. In another word, CHF conditions approach when stable vapor columns are formed in front of auxiliary channels in the inlet manifold (Fig. 1b) due to equilibrium of heat transfer at the vapor/liquid interface. The theory of Helmholtz instability is adopted to predict critical vapor velocity \[42–45\]. Wave flow is induced by intense vapor flows. Additionally, the critical vapor wavelength of vapor flow is determined by Rayleigh instability \[41\].

### 2.3. Experimental CHF results

Significantly enhanced CHF has been experimentally demonstrated in the microchannels using self-excited and self-sustained microbubble oscillations mechanism \[29\]. In this study, experimental data of CHF is plotted in Fig. 3 for a wide range of mass fluxes from 200 kg/m² s to 1350 kg/m² s. The maximum CHF value of 1020 W/cm² is achieved at a mass flux of 1350 kg/m² s in this study. Existing CHF values are also compared in Fig. 3.

### 2.4. Comparisons of CHF experimental results

Such a comparison is hard to make due to the significant variations in the experimental conditions and dimensions of microchannels. The flow chart in Fig. 4 shows the relationship among key governing parameters of CHF. Heat in auxiliary channels is transported by high frequency bubble growth (from boiling) and collapse (from direct interfacial condensation) \[46\]. The direct condensation is determined by heat transfer at the vapor/liquid interface or by forces (such as surface tension, inertia forces) acting on the interface \[47,48\]. The heat transfer at the vapor/liquid interface with phase transition highly depends on local thermo-physical properties, such as liquid conductivity, specific heat, latent heat of vapor, density, viscosity and surface tension, etc.; therefore, it is a complex process.

In this study, CHF of flow boiling on the present microchannel configuration is assumed to be primarily governed by the Helmholtz instability \[44,45\]. The Helmholtz instability is induced by the fast vapor flow in auxiliary channels. Helmholtz critical velocity has a relationship with critical vapor wavelength. Hence, Rayleigh instability is applied to predict the critical vapor wavelength of vapor back flow in the auxiliary channels. At CHF, stable vapor columns are formed in front of auxiliary channels. At this condition, Helmholtz critical velocity should be equal to vapor jet velocity. The key characteristics of stable vapor column are expressed by following equations.
where, \( A_i \) is cross section area of auxiliary channel, \( m_v \) is the mass of vapor flow, \( \rho_v \) is density of vapor, \( r \) is surface tension, \( D_h \) is the hydraulic diameter of auxiliary channels. When \( U_c = U_v \), CHF conditions are approached.

To better describe the interfacial heat transfer process, the stable vapor column is divided into two distinct zones, i.e., the front zone and side zone, as shown in Fig. 5. For a given heat flux, the vapor column length in the side zone varies with mass fluxes. The vapor column in the front zone is assumed as a hemisphere with a diameter of \( 2D_h \) according to the experimental observations. Vapor in both front and side zones directly contacts with subcooled liquid and plays an important role in determining CHF of flow boiling on the present microchannel configuration.

According to the present experimental studies, two types of auxiliary channel dryout modes are found to exist. Out of these two modes, the partially liquid thin film dryout mode is observed at low mass fluxes; while the fully liquid thin film dryout mode occurs at high mass fluxes as illustrated in Fig. 6. In the partially liquid thin film dryout mode, only front surface of vapor column contacts with subcooled liquid. In contrast, the whole vapor column has direct contact with subcooled liquid at the fully liquid thin film dryout mode. Therefore, two CHF models are proposed based on these two distinct liquid thin film dryout modes.

### 3.1.1. CHF model based on fully developed liquid thin film dryout mode

Vapor jets merging from two neighboring auxiliary channels to form a stable vapor column were observed in CHF conditions. In case of the fully developed liquid thin film dryout mode (as shown in Fig. 6b), condensation occurs on the side surface and front surface of the vapor column. By incorporating direction condensation with the vapor back flow, a theoretical CHF model is developed using energy conservation principle:

\[
q_{\text{CHF}} \cdot A_w = h \cdot A_v \cdot \Delta T_{\text{sub}} + m_v \cdot h_{fg} \tag{4}
\]

where

\[
A_v = l \cdot H + D_h \cdot H \tag{5}
\]

\[
A_w = L \cdot d \tag{6}
\]

\( A_w \) is area of the heated surface, \( A_v \) is contact area between vapor column and subcooled liquid, and \( d \) is width of auxiliary channel respectively. \( q_{\text{CHF}} \) is the critical heat flux and \( h \) is the interfacial heat transfer coefficient. \( h_{fg} \) is latent heat of vapor. \( \Delta T_{\text{sub}} \) is the temperature difference between saturated vapor and incoming subcooled liquid. \( L \) is the length of heated surface. \( l \) represents the side length of vapor column entering subcooled liquid (Fig. 5b). Side length can be measured from flow visualization images. Therefore, a constant number \( (n) \) is introduced into the model related to side length of vapor column. \( n \) is obtained by:

\[
n = \frac{l}{\pi} \tag{7}
\]

The constant number \( (n) \) is an important parameter to determine the CHF value. Substituting Eqs. (5) and (6) into Eq. (4), the theoretical CHF model can be expressed as:

\[
q_{\text{CHF}} = \left[ h \cdot \Delta T_{\text{sub}} \left( n \cdot \sqrt{2} \cdot \pi + 1 \right) \cdot \frac{D_h}{\sigma} + h_{fg} \cdot \rho_v \cdot \sqrt{\frac{2 \cdot \sigma}{\rho_v \cdot D_h}} \right] \frac{H}{L} \tag{8}
\]

In Eq. (8), \( \Delta T_{\text{sub}} \) (the temperature difference between saturated vapor and incoming subcooled liquid) and \( h \) (heat transfer coefficient at vapor/liquid interface) are essential to predict CHF. However, it is extremely challenging to accurately determine these two parameters as they are significantly influenced by the
incoming subcooled liquid flow. The effect of mass fluxes on $\Delta T_{\text{sub}}$ is considered and $\Delta T_{\text{sub}}$ is estimated at any given axial location in the later section.

3.1.2. CHF model based on partially developed liquid thin film dryout mode

Equation (8) is not directly applicable to partially developed thin film dryout CHF condition. A modification is needed only for the front surface of vapor column contacted with the subcooled liquid (Fig. 6a). Moreover, a small portion of heated surface ($L$) is still covered by a thin liquid film. Therefore, the dryout surface in auxiliary channel and condensation area can be expressed by:

$$A_v = D_h \cdot H$$

(9)

$$A_w = \left(1 - \frac{nL}{T}\right) \cdot L \cdot d$$

(10)

where, $\frac{nL}{T} \ll 1$. $n \cdot L$ represents the length of liquid thin film in the auxiliary channel (Fig. 6a). Substituting Eqs. (9) and (10) into Eq. (4), the model based on the partially developed thin film dryout mode becomes:

$$q_{\text{CHF}}^{\text{PDLF}} = \left[\frac{h \cdot \Delta T_{\text{sub}}}{d} \cdot \frac{D_h}{d} + h_l \cdot \rho_v \cdot \sqrt{\frac{2 \cdot \sigma}{\rho_v \cdot D_h}} \cdot \frac{H}{L - n \cdot \sqrt{2 \cdot \pi^2 \cdot D_h}}\right]$$

(11)

Note that, when $n = 0$, it means no liquid exists in auxiliary channels.

In summary, the above two CHF models are developed to predict the CHF for flow boiling in microchannels with high frequency self-excited two-phase oscillations. These two CHF models are investigated upon the energy conservation. It has been shown that $\Delta T_{\text{sub}}$ and $h$ are two critical parameters in determining CHF.

3.2. Estimating $\Delta T_{\text{sub}}$ and $h$

Since the local subcooling temperature varies significantly during flow boiling processes, it would be more convenient to represent $\Delta T_{\text{sub}}$ with a more stable parameter in the above two CHF models. It is discussed earlier that $\Delta T_{\text{sub}}$ is closely related to subcooled liquid mass flux, $G$, and can be represented as a function of $G$. The relationship between $G$ and $\Delta T_{\text{sub}}$ is studied using the control volume method. As shown in Fig. 7, a counter flow is observed at the vapor/liquid interface and used to model the interfacial heat and mass transfer. Based on the energy conservation principle, the heat from vapor interface is transferred to the incoming subcooled liquid by condensation through the contact area. The subcooled liquid surrounding the vapor bubble is then heated up by vapor via condensation and eventually, all the liquids are heated up by conduction and convection.

The heat transfer rate from vapor interface to liquid is computed as:

$$q_v = h \cdot A_v \cdot \Delta T_{\text{sub}}$$

(12)

where, $\Delta T_{\text{sub}} = T_{\text{sat}} - T_{\text{sub}}$.

The sensible heat of subcooled liquid is:

$$q_l = m_l \cdot c_p \cdot \Delta T$$

(13)

where, $m_l = A_v \cdot U_l$, the mass flux of subcooled liquid is $G = \rho_l \cdot U_l \cdot c_p$ is liquid specific heat and $\Delta T = T_{\text{sub}} - T_{\text{in}}$.

In the counter flow, heat transfer rate, $q_v = q_l$, based on energy conservation. $A_v$ is the cross sectional area of counter flow and can be represent ($dH$) in the simple form.

Conduction contribution to $T_{\text{sub}}$ through substrate in the channel length direction is ignored by assuming 1-D heat conduction through the bottom surface of auxiliary channels as well as due to high velocity of subcooled liquid flows. Therefore, the temperature of subcooled liquid ($T_{\text{sub}}$) around the interface is determined by:

$$T_{\text{sub}} = \left(\frac{n \cdot \sqrt{2 \cdot \pi + 1}}{n \cdot \sqrt{2 \cdot \pi + 1}}\right) \cdot D_h \cdot h \cdot T_{\text{sat}} + G \cdot c_p \cdot T_{\text{in}}$$

(14)

Then, the local subcooling, $\Delta T_{\text{sub}}$ can be expressed as

$$\Delta T_{\text{sub}} = \left(\frac{n \cdot \sqrt{2 \cdot \pi + 1}}{n \cdot \sqrt{2 \cdot \pi + 1}}\right) \cdot D_h \cdot h + G \cdot c_p \cdot d$$

(15)

The inlet temperature of DI-water is 25°C in this study. Substituting Eq. (15) into Eqs. (8) and (11), two CHF models in Eqs. (16) and (17) are expressed as a function of mass flux, $G$.

**Fully developed liquid thin film dryout model:**

$$q_{\text{CHF}}^{\text{FDLF}} = \left[\frac{h \cdot \frac{G \cdot c_p \cdot d \cdot (T_{\text{sat}} - T_{\text{in}})}{n \cdot \sqrt{2 \cdot \pi + 1}} - D_h \cdot h + G \cdot c_p \cdot d}{d}\right]$$

(16)

**Partially developed liquid thin film dryout model:**

$$q_{\text{CHF}}^{\text{PDLF}} = \left[\frac{h \cdot \frac{G \cdot c_p \cdot d \cdot (T_{\text{sat}} - T_{\text{in}})}{n \cdot \sqrt{2 \cdot \pi + 1}} - D_h \cdot h + G \cdot c_p \cdot d}{d}\right]$$

+ $\frac{h_l \cdot \rho_v \cdot \sqrt{\frac{2 \cdot \sigma}{\rho_v \cdot D_h}} \cdot \frac{H}{L - n \cdot \sqrt{2 \cdot \pi \cdot D_h}}}{d}$

(17)

In these two CHF models, most of critical parameters are included. However, the interfacial heat transfer coefficient in Eqs. (16) and (17) is still unknown, which is a key parameter to reflect the interfacial condensation heat transfer. This process is also called heat transfer controlled condensation [47]. Many correlations have been developed for condensation of vapor bubbles translating in subcooled liquid in previous studies. However, most of these correlations are developed by considering vapor bubbles rising in a stagnant subcooled liquid. Chen and Mayinger [38] studied condensation of vapor bubbles in slow flowing liquid and proposed a correlation to predict Nu as follows:

$$Nu = 0.185Re^{0.7}Pr^{0.5}$$

(18)

Zeitoun et al. [49] showed that local subcooling and Jacob number have significant effects on the condensation heat transfer. In addition, very limited correlation is found in the literature for steam bubbles condensing in flowing subcooled water. Warrier et al. [50] developed a semi-empirical correlation for vapor bubbles condensation in subcooled flow boiling considering local subcooling:

$$Nu = 0.6Re^{2/3}Pr^{1/2} \left(1 - 1.2Ja^{10/10}Fo^{3/3}\right)$$

(19)
Kim and Park [51] proposed a correlation to estimate interfacial heat transfer of a large non-spherical condensing vapor bubble in subcooled turbulent flow on water as follows:

$$\text{Nu} = 0.2575\text{Re}^{0.7}\text{Pr}^{-0.4564}\text{Ja}^{-0.2043}$$

Thus, the interfacial heat transfer coefficient can be calculated as: $h = \text{Nu} \frac{k}{D_h}$. Vapor column Reynolds number is defined as, $\text{Re} = \frac{\rho l_u D_h}{\mu}$. Hydraulic diameter of auxiliary channel is: $D_h = \frac{4A_s}{P}$ and $U_{rel} = \sqrt{(U_v - U_l)^2}$. Where, $k_l$ is the conductivity of subcooled liquid, $\mu_l$ is dynamic viscosity of water, $A_s$ is the cross section area of auxiliary channel; $P$ is the perimeter of auxiliary channel. Prandtl number is defined by $Pr = \frac{c_p \mu}{k_l}$. Jacob number is:

$$\text{Ja} = \frac{\rho c_p (T_{sat} - T_{sub})}{\rho R_g}$$

Fourier number is a time dependent number and adopted from a simple correlation proposed by Chen and Mayinger [38].

$$\text{Fo} = 1.784\text{Re}^{-0.78}\text{Pr}^{-0.5}\text{Ja}^{-1.0}$$

In order to estimate local subcooling from Eq. (15), an expression to calculate $\text{Nu}$ is required. In addition, liquid subcooling is required to calculate $\text{Ja}$ and therefore, $\text{Nu}$. Hence, a trial and error method is suggested until the constant number $n$ in Eq. (15) is obtained.
process was adopted to estimate the local subcooling. All thermo-
physical properties are calculated according to local temperature.
Thermophysical properties of saturated water at atmospheric pres-
sure are used in present study.

### 3.3. Uncertainty analysis

The uncertainties of the measured values as shown in Table 3, are obtained from the manufacturer's specification sheets, while the uncertainties of the derived parameters are calculated using the method developed Kline and McClintock [52].

### 3.4. Mean absolute error (MAE)

Mean absolute error (MAE) is used to evaluate the difference between the experimental results and predicted CHF values from the developed semi-theoretical model:

\[
MAE = \frac{1}{M} \sum_{i=1}^{M} \frac{|q_{CHF,exp} - q_{CHF,semi-theoretical}|}{q_{CHF,exp}} \times 100\%
\]  

### 4. Results and discussion

#### 4.1. Correlation of constant number \((n)\) with mass flux

Constant number \((n)\), a dimensionless parameter related to the length of vapor column \((l)\) and critical wave length \((\lambda)\), is introduced in this study. According to the observation of CHF phenomenon, \(l\) varies with mass fluxes from 200 kg/m² s to 1250 kg/m² s and increases with the increase of mass flux, as illustrated in Fig. 8. Vapor column length, \(l\) is measured from flow visualization images at different mass flux conditions. Constant number, \(n\) is calculated using Eqs. (3) and (7). To quantitatively reflect the influence of \(l\) on \(n\), a correlation of \(n\) is expressed as a function of mass flux, \(G\), as shown in Eq. (24). This correlation is generated by interpolating experimental results based on four cases as plotted in Fig. 9 and shown in Table 4. The image resolution is 800 x 160, and sample rate is 22,857 fps. The side length of vapor column was calculated by counting the number of pixel (4.49 μm/pixel). The uncertainty of the side length of vapor column is ±10 μm (about ±2 pixels).

The correlation of constant number, \(n\), as a function of mass flux, \(G\), can be estimated as:

\[
n = -0.233 + 0.0011G - 9.7 \times 10^{-8}G^2
\]  

#### 4.2. Modeling direct condensation at the vapor/liquid interface

The CHF conditions are strongly influenced by the condensation of vapor column. The depth of vapor column entering subcooled liquid in the inlet manifold is determined by the boiling in auxiliary channels and the direct condensation. However, the condensation at the moving vapor/liquid interface is complex, especially when coupled with a non-uniform and highly dynamic temperature field. Direct condensation of vapor column in the subcooled liquid is governed by two different phenomena: (1) heat transfer at vapor/liquid interface, and (2) inertia force of the subcooled liquid. At a small/moderate local subcooling, direct condensation of vapor column in the subcooled liquid is governed by the vapor/liquid interfacial heat transport, whereas, at a higher subcooling, condensation process is controlled by inertia force of subcooled liquid. In this study, \(\Delta T_{sub}\) is obtained for all the mass fluxes and increases with increasing mass flux as shown in Table 5. Hence, it can be concluded that direct condensation is not only influenced by interfacial heat transfer, but also affected by inertia force of the liquid mass flux, \(G\), in this present condition.

### Table 5

<table>
<thead>
<tr>
<th>Mass flux, (G) (kg/m² s)</th>
<th>(\Delta T_{sub}) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>35.47</td>
</tr>
<tr>
<td>250</td>
<td>36.99</td>
</tr>
<tr>
<td>325</td>
<td>37.01</td>
</tr>
<tr>
<td>380</td>
<td>37.06</td>
</tr>
<tr>
<td>430</td>
<td>37.11</td>
</tr>
<tr>
<td>480</td>
<td>37.18</td>
</tr>
<tr>
<td>600</td>
<td>37.36</td>
</tr>
<tr>
<td>750</td>
<td>37.66</td>
</tr>
<tr>
<td>900</td>
<td>37.94</td>
</tr>
<tr>
<td>1037</td>
<td>38.24</td>
</tr>
<tr>
<td>1150</td>
<td>38.47</td>
</tr>
<tr>
<td>1250</td>
<td>38.70</td>
</tr>
<tr>
<td>1350</td>
<td>38.90</td>
</tr>
</tbody>
</table>

Fig. 10. The dependency of Jacob number, \(Ja\), on the local subcooling, \(\Delta T_{sub}\).
Studies showed that the interfacial heat transfer is greatly influenced by Jacob number \(Ja\) \([53]\). \(Ja\) represents the ratio of the sensible heat of liquid to the latent heat of vapor. It is a criterion for interfacial heat and mass transfer. Jacob number \(Ja\) is expressed in Eq. (21). The thermophysical properties of vapor and subcooled liquid are assumed to be constant at a given temperature. Hence, it can be seen from Eq. (21) that \(Ja\) depends on local subcooling, \(\Delta T_{sub}\). The dependency of \(Ja\) on \(\Delta T_{sub}\) plotted in Fig. 10. Figure shows that \(Ja\) increases with \(\Delta T_{sub}\). Previous experimental studies have shown that \(Ja\) is closely related to the thermal boundary layer thickness around the vapor column \([54]\). When \(Ja > 100\), the thermal boundary layer is thin and unstable due to local condensation effects \([38,47,53]\), resulting in an increased condensation heat transfer rate and liquid inertia is the dominant factor for this condensation. Conversely, at a small value of \(Ja\) \((Ja < 80)\), the thermal boundary layer becomes thicker, resulting in a decreased interfacial heat transfer rate \([47]\).

Therefore, the study of \(Ja\) (related to interfacial heat transfer) relationship to mass flux of subcooled liquid (related to inertia force) is critical to understand the condensation process. The relationship between \(Ja\) and \(G\) is investigated based on Eqs. (15) and (21), and plotted in Fig. 11. From figure, it can be seen that \(Ja\) increases from 108.05 to 118.71 for mass fluxes from 200 kg/m\(^2\) s to 1350 kg/m\(^2\) s.

4.3. Prediction of the present theoretical CHF model and comparisons with existing models/correlations

As shown in Fig. 8, the auxiliary channels under the CHF conditions are shown to be fully dryout at a mass flux range of 200 kg/m\(^2\) s to 1250 kg/m\(^2\) s. As a result, partially developed liquid thin film dryout model is not applicable to predict CHF on these conditions. According to the fully developed liquid thin film dryout model in combination with correlation of constant number, \(n\), CHF can be predicted. Three different interfacial heat transfer coefficient \((h)\) correlations have been used, namely, Chen and Mayinger \([38]\) correlation, Warrier et al. \([50]\) correlation and Kim and Park \([51]\) correlation to adopt the best suitable interfacial heat transfer coefficient model for this present condition. Additionally, the theoretical predictions are compared with experimental data at a mass flux range of 200 kg/m\(^2\) s to 1350 kg/m\(^2\) s as shown in Fig. 12. It can be seen from figure that the CHF model based on Chen and Mayinger \([38]\) correlation and Warrier et al. \([50]\) correlation over predict the experimental data with a \(MAE\) of 70% and 32% respectively, whereas, Kim and Park \([51]\) correlation under predict the experimental data with a \(MAE\) of 25%. This large deviation of CHF model based on Chen and Mayinger correlation with experimental data is due to the different operating condition and inaccuracy in local temperature measurement. Chen and Mayinger correlation was proposed based on bubble condensation in slow flowing subcooled liquid and bulk temperature was considered instead of local liquid temperature. Warrier et al. and Kim and Park correlation were proposed based on bubble condensation in subcooled flow boiling and local temperature (local subcooling) was incorporated in terms of \(Ja\). Therefore, higher accuracy is obtained in CHF model with experimental data due to the similarities in operating condition and measurement accuracy. Warrier et al. correlation was proposed at atmospheric pressure, whereas Kim and Park correlation was proposed at low pressure. Though, this present study is conducted at atmospheric pressure, higher deviation in Warrier et al. correlation may be due to the inaccuracy in predicting \(Fo\).

The proposed semi-theoretical CHF models are also compared to existing CHF models/correlations as summarized in Table 6 and plotted in Fig. 13. These three correlations are well recognized among numerous CHF correlations of flow boiling in mini/microchannels. As shown in Fig. 13, the semi-empirical correlation developed by Kosar et al. \([31]\) can be extended to the operating range of mass fluxes in present study. The Kosar’s correlation is shown to agree well with the experimental CHF data with \(MAE\) of 10.3% on DI-water. This prediction is even slightly better than our semi-theoretical model developed in this study. This could be a result of the similarities in the microchannel configurations and dryout mechanisms between the Kosar’s research and our present study. Specifically, inlet orifices are used in both microchannel

![Fig. 12. Comparison of CHF predictions with experimental CHF data.](image)

<table>
<thead>
<tr>
<th>Reference</th>
<th>Recommended channel size and fluid</th>
<th>Correlation of CHF</th>
<th>MAE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kosar et al. ([31])</td>
<td>(D_p = 0.227) mm water</td>
<td>(q_{\text{CHF}} = -0.740\text{G}_0\text{h}<em>0\text{W}</em>{e}^{-0.12} \text{W}^{-0.54})</td>
<td>10.3</td>
</tr>
<tr>
<td>Bowers and Mudawar ([32])</td>
<td>(D_0 = 0.51–2.54) mm R-113</td>
<td>(q_{\text{CHF}} = -0.160\text{G}_0\text{h}<em>0\text{W}</em>{e}^{-0.19} \text{W}^{-0.54})</td>
<td>117.2</td>
</tr>
<tr>
<td>Qu and Mudawar ([30])</td>
<td>(D_0 = 0.38–2.54) mm R-113 and water</td>
<td>(q_{\text{CHF}} = -33.43\text{G}_0\text{h}<em>0\text{W}</em>{e}^{-0.21} \text{W}^{-0.36})</td>
<td>76.3</td>
</tr>
<tr>
<td>Current theoretical study (1)</td>
<td>(D_0 = 0.083) mm water</td>
<td>(q_{\text{CHF}} = \frac{h}{(\text{G}<em>0\text{h}<em>0\text{W}</em>{e}^{-0.21})} \left(\sqrt{\frac{2}{\pi}} + 1\right) \frac{\text{h}}{\rho L} + \rho_s \sqrt{\frac{2\pi}{\rho_c h</em>{\text{fg}}}} \text{q}[\text{h is calculated based on Kim and Park [51] Correlation}]</td>
<td>25</td>
</tr>
<tr>
<td>Current theoretical study (2)</td>
<td>(D_0 = 0.083) mm water</td>
<td>(q_{\text{CHF}} = \frac{h}{(\text{G}<em>0\text{h}<em>0\text{W}</em>{e}^{-0.21})} \left(\sqrt{\frac{2}{\pi}} + 1\right) \frac{\text{h}}{\rho L} + \rho_s \sqrt{\frac{2\pi}{\rho_c h</em>{\text{fg}}}} \text{q}[\text{h is calculated based on Warrier et al. [50] Correlation}]</td>
<td>32</td>
</tr>
</tbody>
</table>

Table 6: Comparison of present CHF model and existing models/correlations.
configurations to regulate two-phase flows. CHF mechanisms and criteria behind these two configurations could be same, i.e., hydrodynamic instabilities. Surface tension force is dominant during flow boiling in microchannels [55]. Weber number that measures the relative importance of the fluid’s inertia to surface tension can well represent the critical role of surface tension force in governing CHF conditions. These reasons may lead to a great agreement between the Kosar’s correlation and our semi-theoretical model. It is noted that the operating range of mass fluxes (41–302 kg/m²s) and range of effective heat fluxes (28–445 W/cm²) in Kosar’s study is smaller compared to the present study.

Additionally, the correlation developed by Bowers et al. [32] overpredicts CHF values and Qu’s correlation [30] underpredicts CHF over the entire mass flux range with a large discrepancy. These large deviations may be attributed to differences in channel dimensions, applicable mass fluxes, heating area, etc. By comparing these two correlations to Kosar’s correlation, it is found that the heater size and channel hydraulic diameter were not considered in Kosar’s correlation. However, Bowers’ and Qu’s correlations employed a ratio of $L/d$. This ratio may be a possible reason for the resulting large deviations. Moreover, the ratio of liquid and vapor density was taken into account in Qu’s correlation.

5. Conclusions

In this study, CHF of flow boiling in microchannels with self-excited and self-sustained high frequency two-phase oscillations is experimentally and theoretically studied. A semi-theoretical CHF model is developed based on energy conservation principles and the Helmholz instability and Rayleigh instability theories. Previous CHF models and empirical CHF correlations are compared to the present theoretical CHF model. Major conclusions are summarized as follows:

1. A microbubble-excited actuation mechanism can significantly enhance CHF in microchannels during flow boiling in a passive way. Formation of stable vapor columns in auxiliary channels is the primary reason resulting in CHF conditions.
2. The direct condensation of vapor significantly alters the temperature of subcooled liquid. Jacob number increases with the increasing mass flux.
3. According to the observations of CHF phenomena, the length of vapor columns entering into subcooled liquid varies with mass fluxes (Fig. 8). Consequently, two theoretical CHF models are developed based on two possible liquid thin film dryout modes. The developed theoretical CHF models in this study can predict experimental data with a MAE of 25% and 32%.

4. Existing recognized CHF correlations in mini/micro-channels are also compared to present study. The semi-empirical correlation developed by Kosar shows a great agreement with the experimental data because of the similarities in the microchannel configuration and CHF mechanisms.

5. The theoretical CHF model developed in this study considers effects of thermo-physical properties, geometries, and flow conditions. The development of the theoretical CHF model provides insights into the CHF mechanisms. However, further investigation is needed to estimate accurate local subcooling and interfacial vapor to liquid heat transfer coefficient.

Conflict of interest

None declared.

Acknowledgements

This work was supported the US Department of Defense, Office of Naval Research under the Grant N000141210724 (Program Officer Dr. Mark Spector).

References