Shear lag solution for tuning ultrasonic piezoelectric wafer active sensors with applications to Lamb wave array imaging

Lingyu Yu*, Giola Bottai-Santoni, Victor Giurgiutiu

Department of Mechanical Engineering, University of South Carolina, 300 South Main St., Columbia, SC 29208, USA

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ABSTRACT

An analytical investigation of the interaction between piezoelectric wafer active sensor (PWAS), guided Lamb waves, and host structure is presented in this paper, supported with application examples. The analytical investigation assumes a PWAS transducer bonded to the upper surface of an isotropic flat plate. Shear lag transfer of tractions and strains is assumed, and an analytical solution using the space-wise Fourier transform is reviewed, closed-form solutions are presented for the case of both ideal bonding (i.e., load transfer mechanism localized at the PWAS boundary) and not ideal bonding (i.e., load transfer mechanism localized close the PWAS boundary). The analytical solutions are used to derive Lamb wave mode tuning curves which indicate that frequencies exist at which the A0 mode or the S0 mode can be either suppressed or enhanced. The paper further shows that the capability to excite only one desired Lamb wave mode is critical for practical structural health monitoring applications such as PWAS phased array technique (e.g., the embedded ultrasonics structural radar, EUSR) and the sparse array imaging. Extensive experimental tests that verify the tuning mechanism and prediction curves are reported. Examples of correctly tuned EUSR images vs. detuned cases illustrate the paramount importance of Lamb wave mode tuning for the success of PWAS based damage detection.

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1. Introduction

Piezoelectric wafer active sensors (PWAS) are a type of transducers operating on the piezoelectric principle and achieving direct transduction of electric energy into elastic energy and vice versa. In Lamb wave applications, this type of transducers couple their in-plane motion, excited through the piezoelectric effect, with the Lamb waves particle motion on the material surface, as either transmitters or receivers. PWAS differ from the conventional ultrasonic transducers in a fundamental way in that they are strongly coupled with the structure through an adhesive bonding layer and excite/sense Lamb waves in the structure directly through in-plane strain coupling. A comprehensive modeling of the interaction between the PWAS, the structure, and the Lamb waves traveling at ultrasonic frequencies through the structure is needed for fundamental study and as an essential prediction tool.

Crawley and De Luis [5] developed an analytical model of the coupling between piezoelectric wafer actuators and thin-wall structural members. The configuration studied was of two piezoelectric elements bonded on both sides of an elastic structure. They assumed that the strain distribution in the piezoelectric actuator was a linear distribution across the thickness (Euler–Bernoulli linear flexural or uniform extension) and developed a shear lag solution for the interfacial stress \( \tau \) between the PWAS and the structure. The shear lag parameter \( I \) was found to depend on modal repartition number \( \alpha \) which

* Corresponding author.
E-mail address: yu3@engr.sc.edu (L. Yu).
took the value $\alpha = 1$ for symmetric (i.e., axial) excitation and $\alpha = 3$ for antisymmetric (i.e., flexural) excitation. This initial analysis was further detailed by Crawley and Anderson [4]. Giurgiutiu [7] extended Crawley et al. [4,5] theory to the case of only one piezoelectric element bonded to the thin-wall structure by calculating the total effect as a superposition of symmetric and antisymmetric contributions and found the value of $\alpha$ for a single-sided PWAS excitation to be $\alpha = 4$. Refinements of Crawley et al. [4,5] approach have also been reported by other researchers. Luo and Tong [15] studied both static and dynamic solution of a piezoelectric smart beam and introduced the peel stress effect but still within the limitations of the Euler–Bernoulli theory of bending. Ryu and Wang [20] analyzed the interfacial stress induced by a surface-bonded piezoelectric actuator on a curved beam. They used the variational principle to derive the governing equations and the boundary conditions, but did not seem to go beyond axial–flexural combination.

In the work presented in this paper, we overcame the limitations of the shear lag model and derived a generic solution for the ultrasonic excitation transmitted between a PWAS and a thin-wall structure through an adhesive layer in the presence of multiple guided Lamb wave modes. For the case of two generic modes, we will derive a closed-form solution which is a direct extension to higher ultrasonic frequencies. The model we developed is used as a prediction tool for PWAS Lamb wave imaging applications using phased array and sparse array beamforming algorithms, showing that the capability to excite only one desired Lamb wave mode is critical for those practical applications.

2. Shear lag solution for low frequencies ranges (classic solution)

Fig. 1a shows how ultrasonic excitation would transmit from a PWAS into a thin-wall structure through an interfacial bonding layer; our aim is to understand how this excitation is distributed into the various guided Lamb wave modes existing in the structure. Assume the PWAS has thickness $t_a$, half-length $a$, elastic modulus $E_a$; the structure has thickness $t = 2d$, and elastic modulus $E$; the bonding layer has thickness $t_b$ and shear modulus $G_b$.

Crawley et al. [4,5] analyzed this situation under the assumption that only axial and flexural waves exist in the structure. Equilibrium of the infinitesimal PWAS element, shown in Fig. 1a, gives $t_a \sigma_a - \tau = 0$ where $\sigma_a = E_a (u_a - u) / t_a$ is the stress in the PWAS. The piezoelectrically induced strain, $u_{\text{ISA}} = d_{31} V / t_a$ (where, $V$ is the applied electric voltage and $d_{31}$ is the piezoelectric constant), is constant along the PWAS. Differentiation and substitution yields $t_a E_a u_{\text{ISA}} - \tau = 0$. Pure-shear analysis of the bonding layer yields:

$$\tau = G_b \gamma = G_b (u_a - u) / t_b$$

(1)

Double differentiation of Eq. (1) yields $\varepsilon_a'' = (t_a / G_b) \tau'' + \varepsilon'$. Substituting it into the equilibrium expression yields:

$$t_a E_a (t_a / G_b) \tau'' + t_a E_a \varepsilon' - \tau = 0$$

(2)

Eq. (2) contains both $\tau$ and $\varepsilon$. We can use equilibrium in the structure to express it only in terms of $\tau$. Consider an infinitesimal structural element of length $dx$ (Fig. 2), analyze the equilibrium between the external shear stress $\tau(x)$ and the stress resultants $N_x$ and $M_z$ defined as:

$$\begin{align*}
N_x(x) &= \int_{-d}^{d} \sigma(x, y) dy \\
M_z(x) &= \int_{-d}^{d} \sigma(x, y) y dy
\end{align*}$$

(3)

where $\sigma(x, y)$ is the total stress in the structure. Equilibrium of the infinitesimal element yields:

$$\begin{align*}
N_x + \tau &= 0 \\
M_z + \tau d &= 0
\end{align*}$$

(4)

Fig. 1. PWAS and structure interaction through the interface layer: (a) analysis model; (b) predicted interfacial shear stress, $\tau(x)$, for various interfacial thickness values.
In Crawley et al. analysis [4,5], it was assumed that the total stress in the structure is a superposition of symmetric (axial) and antisymmetric (flexural) stresses that have constant and linear displacement distributions across the thickness, respectively, i.e., \( \sigma(x, y) = \sigma_x(x, y) + \sigma_y(x, y) = \sigma_{ax}(x) + \sigma_{flex}(x) \) where \( \sigma_{ax}(x) \) and \( \sigma_{flex}(x) \) are the amplitudes of the axial and flexural waves at the structural surface. Substitution of Eq. (3) into Eq. (4) and use of stress expression yields \( t\sigma'_{ax}(x) + \tau(x) = 0 \) and \( t\sigma'_{flex}(x) + 3\tau(x) = 0 \). Addition of the two equations gives
\[
-t\sigma'(x, d) = \tau(x) + 3\tau(x) = 4\tau(x) = \alpha\tau
\]
where \( \sigma'(x, d) = \sigma'_{ax}(x) + \sigma'_{flex}(x) \). Using \( \sigma = E\varepsilon \) in Eq. (5) yields the structural strain rate at the upper surface as \( \dot{\varepsilon} = \alpha\tau/\varepsilon \); substitution into Eq. (2) yields the ODE for \( \tau \) only,
\[
\tau''(x) - \Gamma^2\tau(x) = 0
\]
where
\[
\Gamma^2 = (G_b/t_sE_a\psi)(\psi + \alpha)
\]
is the shear lag parameter and \( \psi = Et/E_a t_a \) is the stiffness ratio between structure and PWAS. Eq. (6) accepts solution of the form
\[
\tau(x) = c_1 \sinh \Gamma x + c_2 \cosh \Gamma x
\]
where constants \( c_1 \) and \( c_2 \) are to be determined from the boundary conditions. Since the strain at PWAS tips is \( \varepsilon_{ISA} \), whereas the strain in the structure at the same location is zero, and since the shear stress in the adhesive layer is \( \gamma = (u_a - u)/t_a \), it follows that the boundary conditions are \( \tau(z_a) = G_b\gamma(z_a) = G_b\varepsilon_{ISA}/t_a \). Solving the resulting algebraic system for \( c_1 \) and \( c_2 \) yields the shear lag solution
\[
\tau(x) = (G_b\varepsilon_{ISA}/t_a\Gamma a \cosh \Gamma a) \sinh \Gamma x
\]
The shear lag parameter \( \Gamma \) depends on \( \alpha \); the value of \( \alpha \) depends on how the wave modes are excited. If two PWAS are installed, as in Crawley and De Luis [5], and only axial wave is excited then \( \alpha = 1 \); if only flexural wave is excited then \( \alpha = 3 \). If a single PWAS is present (Fig. 1) then \( \alpha = 4 \) because both axial and flexural waves are equally excited [7]. Should the PWAS be placed at a different position within the thickness, then a different value of \( \alpha \) is possible. The interfacial shear stress concentrates about the PWAS edges as the bond layer becomes thinner (or stiffer) (Fig. 1b), this leads to the ideal-bonding model, in which the shear transfer is assumed to be localized at the PWAS edges, (i.e., represented by the Dirac \( \delta \) function).

Eq. (9) has been used by many investigators [1–3,6,10,13,14,17–19]. Although Eq. (9) is only valid at low frequency-thickness product values where the axial/flexural wave approximation holds, they have also used it at high ultrasonic frequencies. However, this shear lag model has several limitations. First, the theory supporting Eq. (9) assumes linear strain distribution across thickness (i.e., constant for axial waves and gradient for flexural waves). This assumption only applies for low values of the frequency-thickness product where S0 and A0 can be approximated with simple axial and flexural waves. The nonlinear strain distribution across thickness of actual S0 and A0 modes would lead to different values of \( \alpha \), but current analysis does not consider nonlinear strain distribution. Also, as frequency increases, more modes appear in the guided waves structure besides the S0 and A0 modes, but current theory cannot accommodate more than two wave modes.

3. Extension of shear lag solution to higher frequency

We have been able to develop a closed-form solution for the case a symmetric and an antisymmetric nonlinear modes and two generic guided wave modes. We have also been able to setup the general problem for an arbitrary number of nonlinear modes; in this case, a closed-form solution does not seem possible, hence an iterative solution methodology is needed in this study.

3.1. Extension to a symmetric and an antisymmetric nonlinear modes

Assume a symmetric nonlinear mode, \( \sigma_{as}(y) \), and an antisymmetric nonlinear mode, \( \sigma_{an}(y) \), acting together; the total stress is given by superposition
\[
\sigma(x, y) = \sigma_{as}(x)\sigma_{as}(y) + \sigma_{an}(x)\sigma_{an}(y)
\]
(10)
where \( a_S(x) \) and \( a_A(x) \) are \( x \)-dependent modal participation factors. At the upper surface the stress derivative with respect to \( x \) is

\[
\sigma'(x, d) = a_S'(x) \sigma_S(d) + a_A'(x) \sigma_A(d)
\]

Eq. (4) now becomes

\[
\begin{align*}
N_x(x) &= t A_S a_S(x) \\
M_z(x) &= td A_A a_A(x)
\end{align*}
\]

where

\[
A_S = (1/t) \int_{-d}^{d} \sigma_S(y) dy
\]

and

\[
A_A = (1/td) \int_{-d}^{d} \sigma_A(y) y dy
\]

Note that the axial force depends only on the symmetric mode and the bending moment depends only on the antisymmetric mode. Substitution of Eq. (12) into Eq. (4) yields after some rearrangement to

\[
\begin{align*}
t a_S'(x) + A_S^{-1} \tau(x) &= 0 \\
t a_A'(x) + A_A^{-1} \tau(x) &= 0
\end{align*}
\]

Multiplication of the first line of Eq. (15) by \( \sigma_S(d) \) and of the second line by \( \sigma_A(d) \) followed by addition and rearrangement of the terms yields an equation similar to Crawley’s Eq. (5) [5], i.e., \( -t \sigma'(x, d) = \alpha \tau(x) \) where

\[
\alpha = (A_S)^{-1} \sigma_S(d) + (A_A)^{-1} \sigma_A(d)
\]

Eq. (16) represents the extension of Crawley and De Luis’s solution [5] for the case on nonlinear symmetric and antisymmetric modes. It could be used to accurately calculate the shear lag parameter \( \Gamma \) of Eq. (6) at ultrasonic frequencies at which the linearity condition implied in Crawley and De Luis’s formulation no longer applies. Substitution of Eq. (16) into Eq. (6) yields

\[
\Gamma^2 = (C_G/t_b t_s E \psi) \left[ \psi + (\sigma_S(d)/A_S) + \sigma_A(d)/A_A \right]
\]

To ascertain the relative importance of Eq. (16), we have plotted it for the case of a symmetric S0 and an antisymmetric A0 modes acting together at ultrasonic frequencies (Fig. 3). For aluminum 2024, for frequencies below 782 kHz-mm, only the S0 and A0 modes exist and the theory presented in this section, i.e., Eq. (16) applies. Fig. 3a shows the Lamb wave mode shapes at 700 kHz-mm; it is apparent that the modes have become significantly nonlinear, (i.e., Crawley and De Luis’s initial assumptions no longer apply). Fig. 3b shows the variation of \( \alpha \) as calculated with Eq. (16). It is apparent that, at low frequencies, Crawley and De Luis’s value \( \alpha = 1 + 3 = 4 \) applies. As \( fd \) increases, \( \alpha \) also increases, from \( \alpha = 4 \) at low frequencies to \( \alpha \approx 5 \) at high frequencies. It is also interesting to observe that, at high frequencies, the relative contribution of S0 and A0 changes, with S0 contribution decreasing while A0 contribution increases.

![Fig. 3. Effect of non-linear stress distribution on the repartition parameter as frequency changes: (a) S0 and A0 Lamb wave modes at 700 kHz-mm for an Aluminum 2024 plate; (b) variation of \( \alpha \) with frequency-thickness product \( fd \).]
3.2. Extension to two generic guided wave modes

In the case of two generic modes, we will derive an expression for the shear lag parameter $I'$ to be used directly in the shear lag solution (9). Assume two generic guided wave modes; then, $\sigma(x, y) = a_1(x)\sigma_1(y) + a_2(x)\sigma_2(y)$, where $a_1(x)$ and $a_2(x)$ are $x$-dependent modal participation factors. At the upper surface, $y = d$, the stress derivative w.r.t. $x$ is

$$\sigma'(x, d) = a'_1(x)\sigma_1(d) + a'_2(x)\sigma_2(d)$$

(18)

Substituting Eq. (3) into Eq. (4) using the condition in Eq. (18) yields

$$\begin{align*}
N_0(x) &= t[A_1^0a_1(x) + A_2^0a_2(x)] \\
M_0(x) &= td[A_1^0a_1(x) + A_2^0a_2(x)]
\end{align*}$$

(19)

where

$$\begin{align*}
A_1^n &= (1/t) \int_{-d}^{d} \sigma_n(y)dy \\
A_2^n &= (1/td) \int_{-d}^{d} \sigma_n(y)dy
\end{align*}$$

(20)

Substitution of Eq. (19) into Eq. (4) yields to a system of two equations into two unknowns, $a'_1(x)$ and $a'_2(x)$. Using the solutions of the system and substituting Eq. (18) into Eq. (2) yields Eq. (6), but with a more general expression of $I'$ given by

$$I'^2 = \frac{G_b}{E_2t_bw} \left[ \psi + \frac{(A_2^2 - A_1^2)\sigma_1(d) + (A_2^1 - A_1^1)\sigma_2(d)}{A_1^1A_2^2 - A_1^2A_2^1} \right]$$

(21)

Eq. (21) can be applied to any two modes, at any frequency, and of arbitrary thickness distribution. It can be easily verified that Eq. (21) is asymptotically consistent with the low-frequency solution of Eq. (6). In fact, Eq. (21) reduces to Eq. (17) if the two generic modes are symmetric and antisymmetric modes respectively (with Eq. (20) yields $A_1^n = A_2^n = A_1^0 = 0$, and $A_2^n = A_2^0$). Moreover, we have already shown that Eq. (17) is asymptotically consistent with the Crawley and De Luis’s [5] low-frequency solution.

3.3. Extension to $n$ generic guided wave modes

In this section, we will tackle the situation of $N$ generic nonlinear guided wave modes. It will be shown that the closed-form solutions as achieved in previous sections are no longer possible. Assume the general solution to be a superposition of $N$ guided wave modes, i.e.,

$$\sigma(x, d) = \sum_{n=1}^{N} a_n(x)\sigma_n(d)$$

(22)

Substitution of Eq. (3) into Eq. (4) using the condition in Eq. (22) yields

$$\begin{align*}
N_n(x) &= t \sum_{n=1}^{N} A_n^n a_n(x) \\
M_n(x) &= td \sum_{n=1}^{N} A_n^n a_n(x)
\end{align*}$$

(23) and

(24)

where $A_n^n$, for $n = 1, \ldots, N$, are defined in Eq. (20). Eq. (4) becomes

$$\begin{align*}
\frac{t}{N} \sum_{n=1}^{N} A_n^n a'_n(x) + \tau(x) &= 0 \\
\frac{t}{N} \sum_{n=1}^{N} A_n^n a''_n(x) + \tau(x) &= 0
\end{align*}$$

(25)

For two generic modes ($N = 2$), the linear system (25) can be solved for $a'_n(x)$, $n = 1, 2$ as we showed previously. For $N$ generic modes, Eq. (25) has two equations with $N$ unknowns, i.e., it is $N$-2 indeterminate. We use the normal mode expansion method to derive the interfacial shear stress. Recall the generic expression of a forward (+) and backward (−) moving guided waves under surface shear excitation $\tau(x)$, $-a \leq x \leq a$ in [8]

$$a_{n}^{\pm}(x) = \pm \left( \vec{v}_n^{\pm}(d)e^{i\xi_n x}/4P_{nm} \right) \int_{-a}^{x} e^{i\xi_n x} \tau(x)dx$$

(26)

where $\vec{v}_n$ is the wave number, $\vec{v}_n(y)$ is the velocity vector of the $n$th mode, tilde (≈) signifies complex conjugate, and $P_{nm}$ is the power flow in $n$th mode. Substitution of Eq. (26) into Eq. (22) and division by $E$ yields
\[
\dot{\epsilon}(x) = E^{-1} \left[ \sum_{n=1}^{N} a_n^+(x)\sigma_n(d) + \sum_{n=1}^{N} a_n^-(x)\sigma_n(d) \right]
\]  
(27)

Substitution of Eq. (27) into Eq. (2) yields the integro-differential equation for \( \tau(x) \), i.e.,

\[
\tau''(x) - T^2 \tau(x) - i(G_b/t_bE) \sum_{n=1}^{N} \xi_n [a_n^+(x) + a_n^-(x)] = 0
\]

(28)

where

\[
I^2 = (G_b/t_bE)|\psi - t \sum_{n=1}^{N} (\bar{\nu}_n(d)\sigma_n(d)/4P_{nm})|
\]

(29)

The integro-differential Eq. (28) needs to be solved by numerical methods. Solution of Eq. (29) is reported in reference [11].

At frequencies where the axial and flexural approximations is valid, the integro-differential Eq. (28) reduces to Eq. (6) and the shear lag in Eq. (29) becomes equal to the shear lag derived in the classic solution (Eq. (7)). This is because as the frequency decreases, the wavenumber approaches zero (\( \xi_s = \xi_A \rightarrow 0 \)) and hence the third term in Eq. (28) goes to zero, and the integro-differential equation is equal to Eq. (6).

At low frequencies the stress across the thickness can be expressed as \( \sigma_{xy} = \sigma_{ax} = \text{const} \) and \( \sigma_{ty} = \sigma_{flex} = y/d \sigma_{flex} \). We also assumed that at the low frequencies \( \bar{\nu}_n''(y) = 0 \) and \( \sigma_{flex}''(y) = 0 \) accordingly to the Bernoulli–Euler assumption. For propagating modes, the average power flow for the nth mode is expressed as

\[
P_{nm} = -\Re \frac{1}{2} \int_{-d}^{d} [\bar{\nu}_n''(y)\sigma_n''(y) + \bar{\nu}_n''(y)\sigma_n''(y)]dy
\]

(30)

Substituting the expressions of the stresses at low frequency in the average power flow expression and integrating, we obtain \( P_{s} = -\Re \frac{w_{s0}^{\text{flex}}}{w_{s0}^{\text{flex}}} \) for the symmetric mode and \( P_{m} = -\Re \frac{w_{m0}^{\text{flex}}}{w_{m0}^{\text{flex}}} \) for the antisymmetric mode. Finally, the expression of the shear lag in Eq. (29) becomes

\[
I^2 = \frac{G_b}{t_bE|\psi - t \sum_{n=1}^{N} (\bar{\nu}_n(d)\sigma_n(d)/4P_{nm})|}
\]

(31)

### 3.4 Numerical validation

In the case of ideal bonding, the shear stress in the bonding layer is concentrated at the ends, i.e.,

\[
\tau(x) = a \tau_0 [\delta(x-a) + \delta(x+a)]
\]

(32)

The tuning formula for the case of ideal bonding becomes [8]

\[
\epsilon_s(x,t) = -\frac{a \tau_0}{\mu} \left[ \sin(\xi_A x) \frac{N_s(x)}{D_s(x)} e^{i(\xi_A x)} + \sin(\xi_A x) \frac{N_A(x)}{D_A(x)} e^{i(\xi_A x)} \right]
\]

(33)

For simplicity we have considered only the first symmetric and antisymmetric modes. \( N_s, N_A, D_s, \) and \( D_A \) are defined in Giurgiuțu [8].

In the case of a non-ideal bonding, the shear stress in the bonding layer can be expressed as

\[
\tau(x) = a \tau_0 \sinh l x
\]

(34)

where

\[
\tau_0 = \frac{G_b \rho \sigma_A}{t_b F^2 d_a^2}
\]

(35)

The tuning formula for the case of non-ideal bonding becomes

\[
\epsilon_s(x,t) = -\frac{a \tau_0}{\mu} \left[ \frac{\sinh(\xi_A x) \cos(\xi_A x) - \sin(\xi_A x) \cosh(\xi_A x)}{F^2 - (\xi_A^2)^2} \frac{N_s(x)}{D_s(x)} e^{i(\xi_A x)} + \frac{\sinh(\xi_A x) \cos(\xi_A x) - \sin(\xi_A x) \cosh(\xi_A x)}{F^2 - (\xi_A^2)^2} \frac{N_A(x)}{D_A(x)} e^{i(\xi_A x)} \right]
\]

(36)

Fig. 4a shows the tuning curves for both ideal and non-ideal bonding solution. In the case of non-ideal bonding solution the mode repartition number has been derived as in Eq. (16). At low frequencies, there is no significant difference between the two cases; however, as the frequency increases the tuning zeros are not coincident any more.

The results have been obtained for a bonding layer thickness of 1 \( \mu m \), if the theoretical bonding thickness is increased to 30 \( \mu m \), the calculated tuning values are closer to the experimental values (see Fig. 4b). The zeros matched exactly and the maxima locations are closer. It was previously demonstrated by Bottai et al. [3] that agreement between experimental data
and theory values through ideal bonding solution could be reached by adjusting the theoretical length of the PWAS. For the case of an aluminum plate of 1-mm thickness, the theoretical value for the PWAS length was 6.4 mm.

4. Applications

The use of tuned Lamb wave modes in structural health monitoring is very important because it permits the researchers to address the detection of specific defects with specific Lamb wave modes. We explored the use of PWAS arrays and tuned S0 Lamb wave mode to perform ultrasonic guided wave imaging that can image a large area from a single location or from a network of sensors. Although the tuning could be performed without ideal bonding assumption, we used the simpler formulation of ideal bonding since the results were not significantly different.

4.1. PWAS phased arrays

The phased array application of the PWAS transducers allows large structural areas to be monitored from a single location. The phased array application utilizes the beam steering concepts based on differentially firing various elements of the phased array such that constructive/destructive interference of all the transducers forms a wave beam in a certain direction. The phased-array concept was transitioned to PWAS-coupled Lamb waves by Giurgiutiu et al. [9] under the name “embedded ultrasonics structural radar” (EUSR). The main challenge in this application (as different from the electromagnetic radar) is the fact that Lamb waves are basically multimode and generally dispersive. However, these multimodal and dispersive challenges can be successfully alleviated through the use of Lamb wave tuning, as described in previous section.

4.1.1. Frequency tuning in PWAS-EUSR phased array

Though Lamb wave consists of multiple dispersive modes, it is possible to confine the excitation to a particular Lamb wave mode of wave speed $c$ and wave length $\lambda$ through smoothed tone-burst excitation of carrier frequency $f_c$ and frequency tuning. Consider an example of crack detection on a thin-wall structure using Lamb waves. For a 1-mm wall thickness and an operation frequency range of 0–500 kHz, two Lamb wave modes (A0 and S0) exist simultaneously. However, as shown in Fig. 5a, there are frequency values where one of the two modes is nearly suppressed. At 300 kHz, A0 mode vibration is very small leaving the S0 mode dominates.

Group velocity vs. frequency plots of S0 and A0 modes are shown in Fig. 5b. We can see that within the 0–1000 kHz range, the S0 mode velocity is almost constant, i.e., the S0 mode is much less dispersive compared to the A0 mode. This observation is consistent with the fact that, usually in Lamb wave NDE/SHM applications, A0 mode is used for the detection of surface defects and disbonds/delaminations while S0 mode is used for through the thickness damage detection. Therefore, for crack detection using PWAS phased array, the S0 mode excitation is expected to give much better results. In addition, to obtain a high quality EUSR scanning image, we also need the response of the S0 Lamb wave mode to be sufficiently strong, thus carrying well the information about damage location and size. Another factor to be concerned during the frequency tuning is the ratio of $d/\lambda$. Our previous research has found out that this ratio should be smaller than or equal to 0.5 in order to observe the sampling theorem (otherwise a disturbing lobe called grating lobe may present). While within the effective range, larger $d/\lambda$ value is desired since it offers better directivity, known as thinner mainlobe width [22]. Scanning with a phased array having good directivity will give a more correct indication of the crack size. Based on the relation $\lambda = c/f$, we see that higher tuning frequency should be used for larger $d/\lambda$ values. Therefore, the objective of frequency tuning applied to the PWAS phased ar-

![Strain tuning for an Aluminum plate 1mm thick with a 7 mm PWAS bonded on the surface. Blue lines are S0 tuning curves, red lines are A0 tuning curves. Dots and plus are for experimental data; solid lines: ideal bonding solution with constant repartition number ($\alpha = 4$); Dashed lines: non-ideal bonding solution with varying repartition number (Eq.(16)). (a) Bond thickness at $t_b = 1$ mm; (b) Bond thickness at $t_b = 30$ μm. (For interpretation of color mentioned in this figure the reader is referred to the web version of the article.)](image-url)
rays is to find an excitation frequency high enough and at which the response signal will have high $S_0/A_0$ ratio (in terms of magnitude) as well as high signal-noise-ratio (SNR) while observing the sampling theorem.

4.1.2. PWAS-EUSR phased array detection with frequency tuning

The specimen used in the experiments was a 1220 mm square 1-mm thick panel of 2024-T3 Al-clad aircraft grade sheet metal stock. The crack is placed on a line midway between the center of the plate and its upper edge. The crack is 19 mm long, 0.127 mm wide. The crack is placed at broadside w.r.t. the phase array at coordinates $(0, 0.305 \text{ m})$, i.e., at $r = 305 \text{ mm}$, $\phi_0 = 90^\circ$ (Fig. 6a). The PWAS phased array was constructed from eight 0.2 mm thick 7-mm square piezoelectric wafers (American Piezo Ceramic Inc., APC-850) placed on a straight line in the center of the plate. The PWAS were spaced at pitch $d = 8 \text{ mm}$. The instrumentation consisted of an HP33120A arbitrary signal generator, a Tektronix TDS210 digital oscilloscope, and a portable PC with DAQ and GPIB interfaces. A LabVIEW™ computer program was developed to digitally control the signal switching, to record the data from the digital oscilloscope, and to generate the group of raw data files. Experimental setup is presented in Fig. 6b.

During the Lamb wave tuning procedure, both pitch-catch and pulse-echo methods were employed to find out the best tuning frequencies. The experimental voltage measurement obtained from pitch-catch experiment is shown in Fig. 7a. We see that at frequency 210 kHz, it yields the maximum $S_0/A_0$ ratio while at frequency 300 kHz the $S_0$ mode achieves maximum response. Therefore, both 210 and 300 kHz could be used as potential excitation frequencies. Corresponding theoretical and experimental $d/k$ values of the $S_0$ mode at various frequencies are listed in Table 1.

To further confirm the frequency tuning, pulse-echo tests were conducted using excitation at 210 and 300 kHz, respectively. Recorded received signals are shown in Fig. 7b and c. By comparing peak-to-peak value of crack echoes, we see that this echo is larger for the 300 kHz frequency (Fig. 7a, point A) than that of the 210 kHz frequency (Fig. 7a, point B). Since the crack echo is the indication of crack presence, the clearer (larger) one is desired. Hence, we choose the 300 over the 210 kHz.

After tuning, EUSR experiments are conducted at the tuning frequency 300 kHz and as well at other frequencies (210 and 450 kHz) for further understanding the significance of frequency tuning. EUSR scanning image results are shown in Fig. 8. At
the tuning frequency 300 kHz, we see that the broadside crack was clearly detected (Fig. 8a). The image not only indicates the existence of the crack, but also correctly indicates its position on the plate (note: polar coordinate is used in the imaging process). However, using exactly the same experimental setup, the EUSR-generated image at 210 kHz shown in Fig. 8b cannot clearly identify the existence of the crack as well as the image obtained at 300 kHz. At this frequency, we have lower $d/k$ value (see Table 1), which resulted in larger crack shade compared to the higher frequency at 300 kHz. Fig. 8c is the EUSR image at 450 kHz. Though a shade representing the crack image can be seen, it is still much weaker compared to the image obtained at the 300 kHz tuning frequency. By comparison, it can be concluded that phased array scanning at the 300 kHz tuning frequency offers the best image quality. However, the reader should be aware that for a different specimen and other PWAS types, the tuning frequency would be different, since the specimen material and the thickness, and the PWAS size and dimensions all contribute to the tuning process.

So far, the frequency tuning process has been concerned about the S0 mode for crack detection. For A0 mode, tuning plot (Fig. 5a) indicates that around 60 kHz, A0 reaches its peaks while S0 is almost suppressed. At this frequency, the corresponding A0 mode wave speed is about 1414 m/s, which results in a $d/k_A$ about 0.34. This is a feasible value for the application of PWAS phased array. Therefore, for other types of damage detection which needs A0 modes, such as corrosion detection, an A0 mode tuning at 60 kHz can be used for the PWAS phased array application.

4.2. PWAS sparse arrays

Pitch-catch method can also be used to detect the presence of crack through the sparse array imaging. Unlike phased arrays where sensors are physically close to each other, sparse arrays consist of a network of PWAS transducers spatially distributed and are used to scan the area covered by the array. In the network, one PWAS sends out interrogating guided wave. When the wave encounters damage, the wave gets scattered. From the comparison of the pristine and damaged wave signals, a scatter signal can be extracted. Analysis of the scattered signals between each pitch-catch pairs permits the correlation of the wave propagation in the structure with the damage progression.
4.2.1. Theory of sparse array imaging

A simple sparse array configuration consisting of four sensors is shown in Fig. 9a. Fig. 9b illustrates the triangulation principle used to locate the damage. Details of principles and applications of triangulation in damage detection can be found in many references and will not be described in this paper.

In triangulation approach, a minimum of three sensors are required in order to locate the damage. However, when the damage accidentally falls into the line between two sensors, the triangulation will be invalid and fail in locating the damage.

Fig. 8. EUSR inspection using frequency tuning. (a) EUSR mapped image at 300 kHz tuning frequency; (b) EUSR image at 210 kHz; (c) EUSR image at 450 kHz.

Fig. 9. Sparse array imaging: (a) a simple sparse array configuration; (b) triangulation principle.
Therefore, in sparse array application, usually a minimum of four sensors is required to correctly identify and localize the presence of damage.

The sparse array uses scatter signals rather than original signals to construct the interrogation image. When an elastic wave such as Lamb wave is transmitted and travels through the structure, wave scattering occurs in all directions where there is a change in the material properties due to damage, as shown in Fig. 10a. The scatter signal is defined as the difference between the measurement during the development of damage and the baseline signal at the initial stage. One advantage of using scatter signals is to minimize the influence caused by boundaries or other structural feature which would otherwise complicate the Lamb wave analysis.

The image construction of the sparse array is based on a synthetic time reversal concept [21] by shifting back the scatter signals at time quantities defined by the transmitter-receiver locations used in the pitch-catch mode. Fig. 10b c illustrates the imaging approach. Assuming a single damage scatter is located at point \(Z(x, y)\) in the structure, the scatter signal from transmitter \(Ti\) to receiver \(Rj\) contains a single wave packet caused by the damage (Fig. 10c). The total time of traveling \(\tau_Z\) is determined by the locations of the transmitter \(Ti\) at \((x_i, y_i)\), the receiver \(Rj\) at \((x_j, y_j)\), and \(Z(x, y)\), as

\[
\tau_Z = \frac{\sqrt{(x_i-x)^2 + (y_i-y)^2} + \sqrt{(x-x)^2 + (y-y)^2}}{c_g}
\]  

where \(c_g\) is the group velocity of the traveling Lamb wave, assuming constant. Using the time-reversal concept by Wang et al. [21], when a wave packet is shifted back by the quantity defined by the transducers and the exact position of the damage, i.e., \(\tau_Z\), ideally the peak will be shifted right back to the time origin. If the wave packet is shifted by a quantity defined with otherwise cases (such as \(\tau_i\) and \(\tau_o\)), the peak will not be shifted right at the time origin.

For an unknown damage, for a certain scatter signal with \(\tau_Z\), the possible locations of the damage is an orbit of ellipse with the transmitter and receiver as the foci (Fig. 10b). To locate the damage, ellipses from other scatters (or transmitter–receiver pairs in the network) are needed. For a given network of \(M\) transducers, a total of \(M^2\) scatter signals will be used if reciprocity is not considered. In our study, two algorithms, one based on summation process and the other based on the correlation process, have been developed for sparse array imaging. Using the summation algorithm, the pixel value at an arbitrary location \(Z(x, y)\) in the scanned plane is defined as [16]

\[
P_Z(t_0) = \sum_{i=1}^{M} \sum_{j=1}^{M} s_{ij}(\tau_Z), i \neq j
\]

where \(M\) is the total number of sensors in the sparse array; \(s_{ij}\) is the scatter signal obtained from \(i\)th transmitter and \(j\)th receiver. Using the correlation algorithm, the pixel value is defined as [12]

\[
P_Z(t_0) = \prod_{i=1}^{M} \prod_{j=1}^{M} s_{ij}(\tau_Z), i \neq j
\]

Fig. 10. PWAS sparse array imaging principle: (a) wave scattering; (b) damage localization; (c) wave back propagation.
4.2.2. Implementation of PWAS Lamb wave sparse array imaging

The sparse array imaging test was conducted on a 1-mm thick aluminum plate to image the development of a through-hole located at (328, 326 mm), as shown in Fig. 11a. Four PWAS were installed on the plate at random locations to build a 4-PWAS sparse array. The sensor locations are: P#1(190, 430 mm), P#2(170, 155 mm), P#3(510, 125 mm), P#4(475, 445 mm).

As pointed out previously, S0 Lamb wave mode is desired for this type of through-the-thickness hole damage detection. Meanwhile, to correctly shift the wave packet representing the damage back to the time origin, non- or less-dispersive signals are desired according to Eq. (37). Therefore, in the sparse array application, PWAS Lamb wave excitation needs to be tuned at a point where quasi nondispersive S0 mode can be obtained. For the material aluminum 2024 at the thickness of 1 mm in combination with round PWAS of 7 mm diameter, the tuning curves in Fig. 5 were used. The tuning results show that at 300 kHz the A0 mode is minimized leaving S0 mode dominates with an almost constant group velocity at about 5500 m/s.

The tuned S0 mode Lamb wave was then used to perform the sparse array imaging. After the baseline data was taken with the presence of a 2-mm diameter through hole at the coordinate (328, 326 mm), the hole was enlarged to 6 mm diameter. The baseline signals were collected in the initial condition at 2 mm (Fig. 11b top) while the measurements were taken at 6 mm (Fig. 11b middle). Scatter signals were obtained as the difference between the baseline and measurement signals (Fig. 11b bottom). The first arrival wave packet in the scatter signal uniquely represents the reflection caused by the enlargement of the hole carrying both damage localization (via time of arrival) and intensity (via amplitude) information. Scatter signals from each pitch-catch pairs in the network were then used to construct the image. To further smooth out the disper-
We now extended Crawley’s work to nonlinear guided wave modes. We obtained a closed-form solution for parameter guided wave modes are approximated by the axial and flexural wave with linear characteristic behavior across the thickness. The widely-cited seminal work by Crawley et al. which applicability is limited to low-frequency applications at which the through the adhesive layer and how it is distributed into the Lamb wave modes existing in the structure. We had used the principle that only one wave mode is present.

The first part of the paper studied how ultrasonic excitation is transmitted from a PWAS into a thin-wall structure through the adhesive layer and how it is distributed into the Lamb wave modes existing in the structure. We had used the widely-cited seminal work by Crawley et al. which applicability is limited to low-frequency applications at which the guided wave modes are approximated by the axial and flexural wave with linear characteristic behavior across the thickness. We now extended Crawley’s work to nonlinear guided wave modes. We obtained a closed-form solution for parameter $\alpha$ and observed that this parameter is no longer constant but varies with the frequency-thickness product. A graph was given of this variation up to the maximum $fd$ value for which only S0 and A0 modes exist. Changes in $\alpha$ of up to 25% were observed, along with redistribution of its S0 and A0 modes contributions. The theory was further extended to the case of two generic guided wave modes for which a closed-form solution was still possible. For the case of $N$ generic modes, a closed-form solution was no longer possible but an iterative approach has been considered and will be reported upon in a future communication.

The second part presented a couple of applications of the PWAS-Lamb wave tuning. In this paper we considered two SHM techniques that benefit greatly from PWAS-Lamb wave tuning: (1) the PWAS phased array and EUSR method and (2) the sparse array imaging. Our investigations have shown that the desired frequency for optimal through-the-thickness detection can be determined based on satisfaction of the basic criterion where the desired Lamb wave response is maximized. The PWAS phased-array method also needs a favorable $d/\lambda$ ratio for good phased-array performance with avoidance of perturbing side lobes. Using the basic and the additional criterion, theoretical and experimental tuning were conducted for the PWAS array application and it was found that the 300 kHz frequency was optimal for crack detection on a 1-mm thick aluminum plate using an eight 7-mm PWAS array. Comparison was performed among imaging results at 210, 450 kHz, and the tuned 300 kHz. The significance of frequency tuning was further confirmed by PWAS sparse array imaging where pixel image was constructed by shifting received scattered signals back to the time origin. In this scenario, S0 Lamb wave for through hole damage detection at a low dispersion region was desired and obtained through the PWAS tuning process. Combined with the dispersion smoothing process via Hilbert envelop extraction, we were able to generate clear images indicating the presence and development of hole damage on aluminum plate specimens.

Overall, this paper has shown that it is important to understand the interaction between PWAS and the propagating Lamb waves in the structures. PWAS Lamb wave tuning is a powerful tool in the implementation of PWAS based active SHM technology.

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References


