STRUCTURAL HEALTH MONITORING OF ADHESIVELY BONDED JOINTS WITH PIEZOELECTRIC WAFER ACTIVE SENSORS

by

Adrian Cuc

Bachelor of Science
“Politehnica“ University of Timisoara, Romania 1996

Master of Science
University of South Carolina, 2002

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University of South Carolina

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Accepted by:
Victor Giurgiutiu, PhD, Major Professor
Yuh Chao, PhD, Committee Member
Anthony Reynolds, PhD, Committee Member
Sarah Gassman, PhD, Committee Member
Tim Mousseau, PhD, Interim Dean of the Graduate School
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ABSTRACT

The present research proposes a new approach to structural health monitoring of adhesively bonded joints using small, unobtrusive piezoelectric wafer active sensors (PWAS) which are permanently affixed on the surface of the structure. PWAS can be placed in restrictive spaces, like in built-up aerospace structures. The surface-bonded PWAS can produce guided waves traveling parallel to the surface and could detect damage that would escape some ultrasonic methods. The major focus of this research was directed towards the electromechanical impedance method for disbond detection. The electromechanical impedance (EMI) method was modeled extensively with analytical and finite element method; the theoretical predictions were compared with carefully conducted experiments on geometrically tractable specimens. Other damage detection methods, i.e. pitch-catch and pulse-echo, were also introduced but used only experimentally.

The transfer matrix method was used to develop the analytical model for a uniform beam using the Euler-Bernoulli beam theory (shear deformation and rotary inertia were not considered). Next, the method was expanded to address the more complex case of a multi-layer adhesively bonded beam. The length of the beam was divided in smaller segments, and for each segment the state vectors (displacements and internal forces) were calculated using the field transfer matrices and the point transfer matrices. If damage (disbond) is present in the structure the model becomes more
complicated and the beam is now divided in branches. For each branch, the mathematical formulation for the equivalent material properties, field and point transfer matrices were developed. Finally, the state vectors, hence the frequency response function at any location on the beam was calculated. Using the frequency response function the electromechanical impedance was calculated.

A finite element analysis (FEA) was also performed for comparison with the analytical results. A conventional FEA analysis in which a harmonic force applied at the nodes to simulate the PWAS two ends was performed first, using the commercial code ANSYS. Then, a coupled-field FEA was performed using the couple-field option of the PLANE13 element in ANSYS. Both analytical and FEA results were compared against experimental results from two simple beam specimens: one pristine and one damaged with artificially simulated disbond.

A set of damage indices were developed to detect the presence of disbond damage in the structure. From the results, it was shown that by using only one damage index it may be difficult to have positive damage detection. Different damage indices respond in different ways to changes in the impedance spectrum.

The last part of this research addressed the possibility of using PWAS transducers for damage detection on real world specimens. Experiments were conducted on (a) a fabricated large scale adhesively bonded lap-joint; (b) a spacecraft simulated panel specimen, and (c) on a full scale helicopter blade. The experiments successfully proved the possibility to excite and receive guided Lamb waves in real world specimens despite the attenuation and dispersion encountered while travelling in the structures. Detection of
damage was achieved with several methods (pitch-catch, pulse-echo, electromechanical impedance), which were critically compared.

The dissertation ends with conclusions and suggestions for future work.
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Chapter 1
Introduction

The driving force of the industry is to make parts, assemblies, and machinery lighter and more efficient. Aluminum as well as composite materials are being used more and more due to their good properties as an alternative to ferrous materials. In order to use such materials, they need to be joined in assemblies and structures. One way to attach aluminum and composite materials together is to permanently join them using different type of adhesives. Thus, the need to inspect adhesive joints has increased. One way to inspect adhesive joints is to use ultrasonic nondestructive evaluation (NDE) methods that allow examining and assessing the health of an adhesively bonded structure without damaging the structural integrity of the tested specimen.

Ultrasonic NDE evaluation is important to several industries. Even though ultrasonic inspection has been used for several decades, the demands of the actual economy have pushed the development of NDE methods further and further. New ultrasonic methods are being developed, and there is a transition from the conventional ultrasonic methods using coupled transducers, to more advanced technologies using embedded ultrasonic sensors. These embedded sensors generate guided ultrasonic waves, which travel within the tested subject, and provide information about the existence and location of possible flaws.
The beneficiary of such modern detection technologies will be many industries such as aerospace and automotive. Today’s aircraft fleet is an aged fleet that needs more and more tedious examination for cracks, corrosions, and delaminations. This examination is done mainly manually using visual inspection, which is expensive, time consuming and makes the aircraft inoperable for a long time. The use of automated NDE methods will enable semi and full automation of the entire inspection process, reduce the time of inspection and move the maintenance approach from a scheduled maintenance to an on-demand maintenance. Early detection of such flaws may prevent catastrophic failures and save human lives. Automotive industry is also going to benefit from such modern technologies. Weight is an important issue and a factor that affects the performances of a car as well as the price. That is why aluminum, non-ferrous alloys and composite materials are used to create new assemblies with reduced weight and increased strength against corrosion and collisions. Such materials cannot be welded or joined using bolted connections. The only alternative is to use structural adhesives to permanently join them together. The highly automated process of car manufacturing requires an automated process of inspecting such joints. The NDE methods have the capabilities of real time automated inspection.

With an increasing use of adhesively bonded joints in the industry, there is a correspondingly growing need for the inspection of adhered joints in two aspects. First, it is necessary to detect defects in the adhesive materials such as voids, inclusions, or the possibility of chemical misformulation or miscure of the adhesive. Second, in situations of automated assembly typical of mass manufacture, it is important that the dimensions of the adhesive bond line are both measurable and within tolerance. Ultrasonic inspection by
bulk wave scanning of adhesive bond lines requires physical access to the joint region, and it is time consuming and expensive.

In aerospace applications, it is important that the size and the location of the damage that exist in the structure can be detected as accurately as possible. There has been extensive work done by many researchers on NDE used for damage detection and damage localization in aerospace applications. Conventional ultrasonic methods for damage detection in metallic plates and composite materials using traditional ultrasonic transducers have been used by various researchers. However, guided waves NDE has shown encouraging results and are becoming more popular in the NDE field versus conventional through-the-thickness bulk wave ultrasonic methods. A large number of nondestructive inspection (NDI) techniques have been developed to identify local damage and detect incipient failure in aerospace structures. In the past years, some of these techniques have been tested in the automotive industry as well. Among them, ultrasonic inspection based on elastic wave propagation is well established and has been used in the engineering community for several decades (Krautkramer, 1990). Also used is the mechanical impedance method (Cawley, 1984).

1.1 Motivation

In-service disbondings and delaminations have become a real concern in many applications. Consider for example the adhesively bonded helicopter blade shown in Figure 1. This blade is subject to in-service disbond due to blade vibrations. So far, the disbond detection technology has been limited to visual inspection techniques (“follow the dew line during an early morning visual inspection”). Although the maintenance
crews have become very innovative in their inspection procedures, the process remains, as a whole, a manual labor-intensive job. In lack of good detection technology, the repair limits are frequently exceeded and the blades are returned to depot where they may sit for months in quarantine status, and may be sometimes even destroyed. The overall cost associated with this problem equates to millions of dollars, significant loss of aircraft availability and serious safety concerns. An automated, early-damage detection system for bonded mechanical systems would be invaluable to the maintainers of a helicopter fleet. This system could be used for both military and civilian helicopters.

![Figure 1 Typical helicopter rotor blade; detail of the blade cross-section indicating the adhesive joining of C-sections](image)

1.2 Research scope, goal and objectives

The scope of the present research is to propose a new approach to the nondestructive evaluation of bonded joints using small, unobtrusive piezoelectric active sensors permanently affixed on the surface of the structure that are an enabling technology for structural health monitoring (SHM). The piezoelectric wafer active sensors (PWAS) methodology bears substantially on the experience accrued with conventional ultrasonic techniques. However, major differences exist between conventional ultrasonics and active-sensor methods. Drawbacks of the ultrasonic
techniques are the bulkiness of transducers and the need for a normal (perpendicular) interface between the transducer and the test structure. The former limits the access of ultrasonic transducers to restricted spaces. The latter influences the type of waves that can be easily generated into the structure.

In contrast with conventional ultrasonic, the embedded active sensors methods use wafer-like transducers (PWAS) that are permanently bonded to the structural surface. These active sensors are small, thin, unobtrusive, and non-invasive. They can be placed in very restrictive spaces, like in built-up aerospace structures. The surface-bonded active sensors can easily produce guided waves traveling parallel to the surface and could detect damage that would escape an ultrasonic method. Additionally, the ultrasonic probes must be moved across the structural surface through manual or semi-automated scanning, whereas PWAS are permanently wired at predetermined locations. They can be remotely scanned through electronic switching.

The goal of this research is to develop a methodology to detect disbonds in adhesively bonded structures using PWAS transducers. We will develop a computational model to predict the response of PWAS transducers in the presence of structural disbonds. Both analytical and finite element models will be considered. For analytical predictions, we will develop a transfer matrix approach, which goes beyond conventional modal analysis because it can deal with disbanded structures. In the finite element analysis, we use coupled field finite elements that can provide directly how electrical signals are affected by structural disbonds. Our predictive modeling is compared with results from carefully conducted experiments on geometrically tractable specimens. The developed methodology is then experimented on representative specimens such as (a) a
large adhesive lap-joint specimen; (b) a spacecraft-like panel specimen; and (c) a full scale helicopter blade. In the process of developing the computational model, we developed a novel analytical model to predict the vibration response of adhesively bonded structures under PWAS excitation and calculated the electromechanical impedance based on the transfer matrix approach and compared two cases: (a) pristine and (b) damaged (disbond between two adhesively bonded aluminum strips).

The objectives of this research are:

1. To derive the frequency response function for simple uniform beam under PWAS excitation using conventional modal analysis.
4. Perform conventional and couple-field finite element analysis (FEA) for uniform beam and multi-layer beam with disbond damage.
5. Validate analytical model against FEA and experimental results
6. To experimentally demonstrate feasibility of PWAS based SHM methods to detect damages in real specimens.

1.2.1 Organization of dissertation

The dissertation is structured in three parts. Part I addresses the fundamentals of wave propagation based SHM and covers four chapters: Chapter 2, Chapter 3, Chapter 4, and Chapter 5.
In Chapter 2, a review of the vibration and wave principles is presented. The general equation of motion for Lamb waves, the symmetric and anti-symmetric and the dispersion of Lamb waves are discussed.

Chapter 3, covers the state of the art for Lamb waves based non-destructive evaluation for crack, corrosion and disbond/delamination detection.

In Chapter 4, the principles of piezoelectric wafer active sensors (PWAS) are discussed. The coupling between the PWAS and the host structure as well as the generation of axial, flexural and Lamb waves are presented.

Chapter 5, is the last chapter in Part I and covers methods used for damage detection for SHM using standing wave methods (i.e. electromechanical impedance method) and wave propagation methods (i.e. pitch-catch and pulse-echo).

Part II of the dissertation addresses the development of the analytical model and validation process. It includes Chapter 6 through Chapter 10.

In Chapter 6, the frequency response function and the electromechanical impedance of a uniform beam under PWAS excitation are derived. The analytical results are compared against experimental results performed on a uniform beam.

In Chapter 7, the transfer matrix method for vibration analysis of pristine and disbanded multi-layer beams is developed. Detailed derivations are presented for the calculations of the electromechanical impedance,

In Chapter 8, the analytical results obtained using the transfer matrix method are validated against the experimental results.
In Chapter 9, a conventional and couple-field finite element analysis (FEA) is performed and the results are compared against the analytical and the experimental results.

In Chapter 10, three damages indices (root mean square deviation, mean absolute percentage deviation and correlation coefficient deviation) are developed in order to be able to make prognostics about changes in the structural response of complex structures due to the presence of various damages.

Part III of the dissertation addresses experimental results obtained by using PWAS based SHM methods for damage detection on real specimens. It will cover three chapters: Chapter 11, Chapter 12 and Chapter 13.

In Chapter 11, a bonded lap-joint specimen was fabricated with disbonds artificially introduced in the structure. The wave propagation methods as well as standing waves methods were used to detect the disbonds.

Chapter 12 will present experimental results on a more complex spacecraft panel specimen containing cracks, disbonds, and corrosion.

In Chapter 13, the possibility to use small PWAS transducers permanently attached to a full scale helicopter blade to successfully excite and propagate Lamb waves is presented.
PART I

Fundamentals
Chapter 2
Review of vibration and waves principles

The study of propagating waves has preoccupied many researchers. Early work was carried out by luminaries such as Lord Rayleigh (1885) and Lamb (1917). Because of the mathematical simplicity, the wave propagation in strings and rods was studied first and the equations of motion were developed. Later, the results were used for more complicated cases of waves propagating in infinite or semi-infinite media such as plates and membranes. In the last few decades, the guided wave propagation and the equations of motion was the focus of many researchers: Achenbach (1973), Graff (1975) Nayfeh (1995), Rose (1995), etc. The properties of guided waves have made them suitable to be used in the inspection techniques of metallic structures and, in the last few years, they become predominant in the NDE and SHM applications.

2.1 Review of wave propagation theory

Waves are disturbances that travel, or propagate, from one region of space to another. A classification of different types of waves propagating in elastic solids is presented in Table 1.
### Table 1 Waves in elastic solids

<table>
<thead>
<tr>
<th>Wave Type</th>
<th>Particle Motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure (longitudinal, compressional, dilatational, P-wave, axial)</td>
<td>Parallel to the direction of wave propagation</td>
</tr>
<tr>
<td>Shear (transverse, distortional, S-wave)</td>
<td>Perpendicular to the direction of wave propagation</td>
</tr>
<tr>
<td>Flexural</td>
<td>Elliptical; plane sections remain plane</td>
</tr>
<tr>
<td>Rayleigh (bending)</td>
<td>Elliptical; amplitude decays quickly with depth</td>
</tr>
<tr>
<td>Lamb (guided, plates)</td>
<td>Elliptical; free-surface conditions satisfied at the top and bottom surface</td>
</tr>
</tbody>
</table>

#### 2.1.1 Pressure waves

Pressure waves are also called compressional, axial, dilatational, longitudinal, or P-waves. For this type of wave motion, the particle displacement is parallel to the direction of propagation. The particle displacement in a pressure wave can be written as:

\[ u_x(x,t) = u_0 \cdot e^{i(kx - \omega t)} \]  

(1.1)

where \( k \) is the wave number, and \( \omega \) is the angular frequency. The wave number can be written in terms of the wave speed \((c)\) and the angular frequency \((\omega)\) as:

\[ k = \frac{\omega}{c} = \frac{2\pi}{\lambda} \]  

(1.2)

The particle displacement can be also written in terms of the wavelength \( \gamma \).

\[ u_x(x,t) = u_0 \cdot e^{i\omega \left(\frac{x}{c}\right)} = u_0 \cdot e^{i2\pi \left(\frac{x}{T}\right)} \]  

(1.3)

where \( \gamma = cT \), \( T = 1/f = 2\pi/\omega \) is the period of oscillation. The wave speed \((c)\) is calculated using various formulas, depending on the specific assumptions and the boundary conditions. For a three-dimensional free solid, the wave speed for a pressure wave is:

\[ c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \]  

(1.4)
where \( \lambda = \frac{\nu E}{(1-2\nu)(1+\nu)} \) and \( \mu = \frac{E}{2(1+\nu)} \) are the Lamé constants, \( E \) is the Young’s modulus, \( \nu \) is the Poisson ratio, and \( \rho \) is the density. For a two-dimensional plate with free top and bottom surfaces, the longitudinal wave speed takes the form:

\[
c_p = \sqrt{\frac{1}{1-\nu^2} \frac{E}{\rho}} \tag{1.5}
\]

For a one-dimensional slender bar the longitudinal wave speed can be calculated as:

\[
c = \sqrt{\frac{E}{\rho}} \tag{1.6}
\]

### 2.1.2 Shear waves

Shear waves are also called transverse waves, distortional waves, or S-waves. For this type of wave motion, the particle displacement is perpendicular to the direction of wave propagation. The particle displacement can be written as:

\[
u_s(x,t) = \bar{u}_0 \cdot e^{i(k_s x - \omega t)} \tag{1.7}
\]

where the wave number \( k_s = \frac{\omega}{c_s} \)

In terms of the Lamé constants the wave speed of the shear waves can be written as:

\[
c_s = \sqrt{\frac{\mu}{\rho}} \tag{1.8}
\]

### 2.1.3 Flexural waves

Flexural waves appear due to bending action. The Bernoulli-Euler beam theory assumes that plane sections remain plane after deformation. The Kirchhoff plate theory assumes that straight normal planes to the mid-surface remain straight after deformation.
Both theories imply a linear distribution of axial displacement across the thickness. The axial and vertical displacements $u_x$ and $u_y$ can be calculated as:

$$u_x = -yu_y$$ (1.9)

and assuming an harmonic expression for the vertical displacement

$$u_y(x, y, t) = u_0 \cdot e^{i(k_F x - \omega t)}$$ (1.10)

the expression for the in-plane displacement field across the thickness becomes:

$$u_x(x, y, t) = y \frac{\partial}{\partial x} u_0 \cdot e^{i(k_F x - \omega t)} = i k_F u_0 \cdot e^{i(k_F x - \omega t)}$$ (1.11)

The flexural wave number $k_F$ can be expressed in terms of the geometric and materials properties:

$$k_F = \sqrt{\frac{\omega}{a}}$$ (1.12)

Where, for a beam,

$$a = \sqrt{\frac{EI}{\rho A}} = \sqrt{\frac{E \cdot bh^3}{\rho \cdot 12bh}} = \frac{h}{2} \sqrt{\frac{1}{3} E \rho}$$ (1.13)

The flexural wave speed can be calculated using:

$$c_F = ak_F = \sqrt{\omega \rho}$$ (1.14)

The above equation shows an important property, that the wave speed is frequency dependant. For zero frequency, the wave speed is also zero; as the frequency increases, the flexural wave speed also increases, following the $1/\sqrt{f}$ rule. When the wave speed is a function of frequency the wave is called dispersive.
2.1.4 Rayleigh wave

Rayleigh waves are also known as surface waves. They propagate close to the body surface, with the motion amplitude decreasing rapidly with depth. The polarization of the Rayleigh waves lies in a plane perpendicular to the surface. The effective depth of penetration is less than a wavelength.

The two components of the Rayleigh wave $u_x$ and $u_y$ can be calculated using an approximated equation for the wave velocity:

$$ c_R = c_S \left( \frac{0.87 + 1.12\nu}{1 + \nu} \right) $$  \hfill (1.15)

The Rayleigh wave number is $k_R = \omega / c_R$ and the components $u_x$ and $u_y$ are:

$$ u_x(x, y, t) = u_0 k_R \left( e^{-\nu y} - \frac{2qs}{k_R^2 + s^2} e^{-\nu y} \right) e^{i(k_R x - \omega t)} \hfill (1.16) $$

$$ u_y(x, y, t) = iu_0 q \left( e^{-\nu y} - \frac{2k_R^2}{k_R^2 + s^2} e^{-\nu y} \right) e^{i(k_R x - \omega t)} \hfill (1.17) $$

where $q = \sqrt{k_R^2 - k_P^2}$ and $s = \sqrt{k_R^2 - k_S^2}$

2.1.5 Lamb waves

Lamb waves are guided waves between two parallel free surfaces, such as the upper and lower surfaces of a plate. The displacements $u_x$ and $u_y$ are calculated as:

$$ u_x = i \cdot \left[ \xi \left( A \sin \eta_x y + B \cos \eta_x y \right) + \eta_x \left( C \cos \eta_x y - D \sin \eta_x y \right) \right] \cdot e^{i\psi} \hfill (1.18) $$

$$ u_y = \left[ \eta_y \left( A \cos \eta_y y - B \sin \eta_y y \right) + \xi \left( C \sin \eta_y y + D \cos \eta_y y \right) \right] \cdot e^{i\psi} \hfill (1.19) $$

where $\psi = \xi x - \omega t$. The two non-zero stresses are:
\[ \sigma_{yy} = (\lambda + 2\mu) \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) - 2\mu \left( \frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 H_z}{\partial x \partial y} \right) \]  
\[ (1.20) \]

\[ \sigma_{xy} = 2\mu \left( \frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} \right) \]  
\[ (1.21) \]

Lamb waves can exist in two basic types: symmetric and antisymmetric. For each propagation type, a number of modes exist, corresponding to the solutions of the Rayleigh-Lamb equation. The symmetric modes are designated S_0, S_1, …, while the antisymmetric modes are designated A_0, A_1, ….. The particle motion for the two cases, symmetric and antisymmetric is presented in Figure 2.

Symmetric motion

Antisymmetric motion

Figure 2  Symmetric and antisymmetric particle motion across the plate thickness

### 2.2 General equation of motion for Lamb waves

Bulk waves typically refers to wave propagation in infinite media. Guided waves are the waves that propagates in bounded media and can be surface waves (aka Rayleigh waves), Lamb waves, and interface waves.

The early developments of wave propagation in plates were carried out by Rayleigh (1885) and Lamb (1917). They considered the propagation of continuous,
straight crested waves in an infinite plate with traction-free surfaces. Lamb waves are guided ultrasonic waves between two parallel traction-free surfaces; they are a combination of pressure (P) wave and shear vertical (SV) waves, and they can be of two types: symmetric and antisymmetric.

To derive the Lamb wave equations we assume a traction-free surface plate with thickness $2d$ (Figure 3) in which straight crested Lamb waves are propagating.

To start the derivation the following assumption are made:

- **z-invariant motion** (the motion of particles in the $z$-axis does not change)
  
  $$u_z = 0 \text{ (P+SV waves only)}$$
  
  $$u_x \neq 0; \; u_y \neq 0$$

- **traction-free surface**
  
  $$\sigma_{zz} = \sigma_{yz} = 0$$
  
  $$\sigma_{xx} \neq 0; \; \sigma_{yy} \neq 0; \; \sigma_{zz} \neq 0; \; \sigma_{xy} \neq 0$$

The potentials of the pressure (P) and shear vertical (SV) waves are:

$$\phi(x, y, t) = f(y) \cdot e^{i(\xi x - \omega t)} \quad (1.22)$$

$$H_z(x, y, t) = i \cdot h(y) \cdot e^{i(\xi x - \omega t)} \quad (1.23)$$
where \( f(y) \) describes the motion in the \( y \)-direction and \( i(\xi x - \omega t) \) describes the motion in the \( x \)-direction.

The wave equation in terms of the two potentials can be written as (Graff, 1975)

\[
\nabla^2 \phi = \frac{1}{c_p^2} \ddot{\phi} \tag{1.24}
\]
\[
\nabla^2 H_z = \frac{1}{c_s^2} \ddot{H}_z \tag{1.25}
\]

where \( c_p^2 = (\lambda + 2\mu)/\rho \) is the speed of the pressure wave; \( c_s^2 = \mu/\rho \) is the speed of the shear vertical wave; \( \lambda, \mu, \rho \) are the two Lame constants and the density, respectively.

The displacement in the \( x \) and \( y \) directions are:

\[
u_x = \frac{\partial \phi}{\partial x} + \frac{\partial H_z}{\partial y} \tag{1.26}\]
\[
u_y = -\frac{\partial \phi}{\partial y} - \frac{\partial H_z}{\partial x} \tag{1.27}\]

After derivations and applying the boundary conditions, we arrive at the two differential equations:

\[
f''(y) + \eta_p^2 \cdot f(y) = 0 \tag{1.28}\]
\[
h''(y) + \eta_s^2 \cdot h(y) = 0 \tag{1.28}\]

The general solutions for the Equations (7) are:

\[
f(y) = A \sin(\eta_p y) + B \cos(\eta_p y) \tag{1.29}\]
\[
h(y) = C \sin(\eta_s y) + D \cos(\eta_s y) \tag{1.29}\]

where \( \eta_p^2 = \omega^2/c_p^2 - \xi^2 \), \( \eta_s^2 = \omega^2/c_s^2 - \xi^2 \), and \( \xi \) are the directional wave numbers.

Substituting Eq. (1.29) in Eq. (1.22) and Eq. (1.23), we obtain the two potentials as:

\[
\phi(x, y, t) = \left[ A \sin(\eta_p y) + B \cos(\eta_p y) \right] e^{i(\xi x - \omega t)} \tag{1.30}\]
\[ H_z(x, y, t) = \left[ C \sin(\eta_z y) + D \cos(\eta_z y) \right] \cdot e^{i(\xi x - \omega t)} \quad (1.31) \]

The displacements are calculated as:

\[ u_x = i \left[ \xi \left( A \sin \eta_p y + B \cos \eta_p y \right) + \eta_i \left( C \cos \eta_i y - D \sin \eta_i y \right) \right] \cdot e^{i\psi} \quad (1.32) \]

\[ u_y = \left[ \eta_p \left( A \cos \eta_p y - B \sin \eta_p y \right) + \xi \left( C \sin \eta_i y + D \cos \eta_i y \right) \right] \cdot e^{i\psi} \quad (1.33) \]

where \( \psi = \xi x - \omega t \)

The two non-zero stresses are:

\[ \sigma_{yy} = \left( \lambda + 2\mu \right) \cdot \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) - 2\mu \cdot \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 H_z}{\partial x \partial y} \right) \quad (1.34) \]

\[ \sigma_{xy} = 2\mu \cdot \left( \frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} \right) \quad (1.35) \]

After calculation they become:

\[ \sigma_{yy} = \mu \cdot \left[ \left( \xi^2 - \eta_i^2 \right) \left( A \sin \eta_p y + B \cos \eta_p y \right) + 2\xi \eta_i \left( C \cos \eta_i y - D \sin \eta_i y \right) \right] \cdot e^{i\psi} \quad (1.36) \]

\[ \sigma_{xy} = i\mu \cdot \left[ 2\xi \eta_p \left( A \cos \eta_p y - B \sin \eta_p y \right) + \left( \xi^2 - \eta_i^2 \right) \left( C \sin \eta_i y + D \cos \eta_i y \right) \right] \cdot e^{i\psi} \quad (1.37) \]

### 2.3 Symmetric Lamb modes

Symmetrical Lamb modes are similar to the axial waves and the displacement of particles is presented in Figure 4.
We note in Eq. (1.32) and Eq. (1.33) that the displacements contain symmetric and antisymmetric components (cos and sine).

For the symmetric Lamb modes, only the terms in $B$ and $C$ give symmetric displacements with respect to $y = 0$ for $u_x$. Same for the displacement $u_y$, only the $B$ and $C$ terms give symmetric displacement. Hence, we can write: $A = D = 0$.

From Eq. (1.32), (1.33), (1.36), and (1.37) applying the boundary conditions at $y = \pm d: \sigma_{yx} = \sigma_{xy} = 0$ we arrive at a homogeneous system of linear equations. The characteristic equation is:

$$
\left( \zeta^2 - \eta_s^2 \right) \tan(\eta_s d) + 4\eta_\rho \zeta^2 \tan(\eta_\rho d) = 0 \quad (1.38)
$$

From the characteristic Equation (1.38) the Rayleigh-Lamb frequency equation can be derived in the form (Graff, 1975):

$$
\frac{\tan \eta_\rho d}{\tan \eta_s d} = -\frac{4\eta_\rho \zeta^2}{\left( \zeta^2 - \eta_s^2 \right)^2} \quad (1.39)
$$
2.4 Antisymmetric Lamb modes

Antisymmetrical Lamb modes are similar to the flexural waves and the displacement of particles is presented in Figure 5.

![Figure 5 Displacement of particles for an antisymmetric Lamb wave](image)

Similar to the symmetric mode, the displacements $u_x$ and $u_y$ for the antisymmetric Lamb mode contain symmetric and antisymmetric terms (cos and sin). Hence the only terms in Eq. (1.32) and Eq. (1.33) that will give antisymmetric displacements with respect to $y = 0$ are $A$ and $D$. That means $B = C = 0$.

Following the same algorithm, from Eq. (1.32), (1.33), (1.36), and (1.37) and applying the boundary conditions at $y = \pm d : \sigma_{yx} = \sigma_{xy} = 0$

we obtain a homogeneous system of linear equations. The characteristic equation is:

$$
\left( \xi^2 - \eta_s^2 \right) \tan (\eta_p d) + 4\eta_s \eta_p \xi^2 \tan (\eta_s d) = 0
$$

(1.40)

The Rayleigh-Lamb frequency equation for the antisymmetric mode is (Graff, 1975):

$$
\frac{\tan \eta_p d}{\tan \eta_s d} = -\frac{\left( \xi^2 - \eta_s^2 \right)^2}{4\eta_s \eta_p \xi^2}
$$

(1.41)

Equation (1.39) and Equation (1.41) can be combined in a single equation given by:
\[ F(\xi, \omega) = \frac{\tan \eta_d}{\tan \eta_d} + \left[ \frac{4\eta_s \eta_p \xi^2}{(\xi^2 - \eta_s^2)^2} \right]^{\pm 1} = 0, \quad \pm 1 = \text{symmetric} \]

\[ = -1 = \text{antisymmetric} \quad (1.42) \]

For a given frequency, \( \omega \) the wave numbers can be determined \( \xi_0, \xi_1, \xi_2, \ldots \) and \( \xi^A_0, \xi^A_1, \xi^A_2, \ldots \), that satisfies the Rayleigh-Lamb equation. Then, using the relationship \( \omega = \xi c \) the wave speed of the propagating waves can be calculated \( c^S_0, c^S_1, c^S_2, \ldots \) and \( c^A_0, c^A_1, c^A_2, \ldots \).

### 2.5 Dispersion of Lamb waves

Wave dispersion may occur when the wave speed varies with frequency. Lamb waves are dispersive in nature, that means the wave speed depends on the product of frequency and thickness of the plate in which the Lamb waves are traveling (Figure 6).
The physical phenomenon can be explained considering a wave packet or a tone burst as shown in Figure 7a. The tone burst consists of a carrier frequency (tone) and has a short duration in time (burst). An example of a tone burst is presented in Figure 7a where the signal is a 5-count sinusoidal tone burst, meaning that the initial signal is a
sinusoidal having five complete cycles. In the frequency domain, there is a dominant frequency $f_c$, and other side frequencies (Figure 7b).

For non-dispersive wave propagation, the shape of the wave is preserved and the wave speed is constant (Figure 8). All frequency components in the wave packet travel with the same speed and the packet keeps its shape.

For dispersive wave propagation, each frequency component travels with a different speed (wave speed is a function of frequency $c = c(f)$ hence the wave packet spreads out, it disperse (Figure 9). The degree of dispersion depends on the spectrum bandwidth.
Figure 9      Dispersive wave propagation, $c = c(f)$
Chapter 3
State of the art for Lamb waves based damage detection

Ultrasonic inspection of structures using conventional transducers is a time consuming process. Due to the advantages that guided waves (Lamb waves) offer, they can be a better alternative to ultrasonic inspection. Lamb waves can travel over large distances with little loss in amplitude, they are sensitive to different types of flaws, and they are capable to propagate along curved structures and reach in closed or hidden areas. For these reasons, researchers have used Lamb waves for detection of cracks, corrosion, and disbonds/delaminations in various structures (aerospace panels, pipes, tanks, etc).

3.1 Lamb waves for crack detection

The interaction of Lamb waves with cracks in aluminum plates or aerospace metallic structures was investigated by numerous researchers in the last years. Early work to find how Lamb waves interact with damage was carried out by Alleyne and Cawley (1992). They used finite element analysis to study the interaction of individual Lamb waves with simulated damages (represented by notches). The results showed an excellent agreement between theory and experimental investigation. In addition, they reported that
the sensitivity of Lamb waves to particular notches is dependent on the frequency-thickness product, the mode type (symmetric or antisymmetric), the mode order, and the geometry of the notch. The transducers used in the experiments were conventional wideband ultrasonic immersion transducers.

The propagation of Lamb waves and the interaction with damages can be modeled using various simulation techniques such as finite difference equations (FDE), finite element analysis (FEA), boundary element methods (BEM), spectral element methods (SEM), and local interaction simulation approach (LISA). Much attention was given to the scattering of Lamb waves from defects.

Tomographic image techniques can be used to extract the location, shape, and the extent of flaws in structures. However, the scattering of Lamb waves from severe defects introduces artifacts that make the tomographic reconstruction difficult. McKeon and Hinders (1999) used the Mindlin’s higher order plate theory to explain the scattering of S₀ Lamb waves from a circular through hole in an aluminum plate. The theoretical model predicted that for three holes with increasing radii, the lobes with maximum amplitude are located at 180°, and 90° and 270°. The experimental results using a “point source” solution to combine the displacements of realistic S₀ beams incident on holes of various sizes, showed a good correlation with the theoretical model, and confirmed that scattering is important when the hole size is larger than the S₀ beam size.

Valle et al. (2001) examined the propagation of guided circumferential waves in a metallic hollow cylinder and the interaction of the guided waves with a simulated crack. They used guided waves to both locate and size the crack. First, the crack is sized using a scattering formula developed by Auld (1990) and modified to analyze transient signals.
Second, the crack is located using the backscattered signal and applying a time-frequency digital signal processing technique (reassigned spectrogram). The processed signal is then compared against signals obtained from a cylinder without a crack (pristine). The authors used a commercial FEM code, ABAQUS, to model the scattering of guided circumferential waves caused by a crack and to calculate the transient response of the wave guides. They showed that the results for both crack sizing and crack locating are dependent on the frequency of the input signal. For multiple cracks to be detected, both methods, Auld’s formula and the reassigned spectrogram, must be used.

For a better identification of the location and the size of defects, efficient numerical methods faster than the finite element based models are required. Clezio et al. (2002) used a modal decomposition method to solve direct problems of Lamb wave scattering. Their work presents the interaction of the first symmetric Lamb mode, S₀, with vertical cracks in an aluminum plate placed in vacuum. The cracks are symmetric with respect to the middle plane of the aluminum plate. The reflection and transmitted coefficients and the crack motion were predicted and compared against finite elements calculations and experimental data. In the experiments, an 8 mm-thick aluminum plate was used with three double notches of 0.7 mm width and height varying from 25%, 50%, and 75% of the plate thickness. The frequency of the excitation (incident mode S₀) was 0.14 MHz, so that the frequency-thickness product was 1.12 MHz, below the S₁ mode cut-off frequency. In this way, and because of the symmetry of the problem, no mode conversion was obtained. The results showed very good agreements between the predictions and the experimental data. Also, the results using modal decomposition
method were very similar to those obtained using the FE approach but the computational time was about 100 times smaller when the modal decomposition method was used.

Grondel et al. (2002) used Lamb waves to inspect riveted aluminum lap-joints during cyclical loading. They excited and received Lamb waves using piezoelectric transducers coupled with the aluminum plates. The metallic specimens used were symmetrical aluminum multi-riveted plates 750x300x2 mm fastened by six rows of a total of 84 rivets (Figure 10). They were manufactured in such a way to ensure crack initiation from the central rivets.

![Symmetrical aluminum multi-riveted specimen](Figure 10)

The piezoelectric transducers were bonded symmetrically with respect to the middle strap joint. The excitation consisted of a 5 cycle sinusoidal tone burst at 400 kHz. The received signal was processed using the short time Fourier transform. The results of the time-frequency analysis confirmed the emergence of a 8 mm crack around the central rivet at 107,000 cycles. Variation in the distribution of the energy between 100,000 and 107,000 cycles was observed. Stronger variation is observed between 107,000 cycles and 180,000 cycles.

Due to the time consuming and labor-intensive procedures that current ultrasonic inspection techniques require, new fast low-cost embedded structural health monitoring
systems are needed. These systems will allow for inspection of the structure without interrupting the service of the aircraft or the machinery. Thus, will eliminate the labor-intensive inspection and the need to disassemble and re-assemble good structural parts, process during which accidental damage may be induced in a healthy structure. The new SHM systems must be low weight, change as little as possible the host structure, and have low cost and low maintenance requirements.


The crack detection scheme and the experimental set-up are shown in Figure 11. The experiments were carried on a 3.175 mm thick aluminum plate with a 0.2 mm wide notch cut. The length of the notch was increased and sensor measurements at different notch length were taken.

![Diagram](image)

Figure 11  (a) Crack detection scheme; (b) Experimental set-up (Ihn and Chang, 2004)

The excitation signal consisted of a 5-count sine burst signal. Lamb waves were generated in the structure by surface mounted piezoelectric actuators. The raw signals are
processed using MATLAB, and the envelope of the signal is extracted by applying the short time Fourier transform (STFT) to the initial signal. The time of flight (TOF) information is next extracted. When Lamb waves travel over damage areas, scattering occurs and the energy of the scattered signal can provide good information about the crack propagation. Based on these assumptions, Ihn and Chang (2004) developed a damage index (DI) defined as the ratio between the scatter energy contained in the $S_0$ mode wave-packet and the baseline energy contained in the $S_0$ mode wave-packet:

$$DI = \left( \frac{\int_{t_i}^{t_f} |S_{SC}(\omega_0, t)|^2 dt}{\int_{t_i}^{t_f} |S_b(\omega_0, t)|^2 dt} \right)^{\alpha} \left( \frac{\text{Scatter energy of } S_0 \text{ mode}}{\text{Baseline energy of } S_0 \text{ mode}} \right)^{1/2}$$

(2.1)

![Figure 12](image.png)

Figure 12  Damage index vs. crack growth (Ihn and Chang, 2004)

The results showed that the proposed damage index applied to the piezoceramic sensors signals was in good correlation with the actual fatigue crack growth obtained from visual inspection of the specimen (Figure 12). The disadvantage of the proposed
method is that it is sensitive to noise; hence, the signal to noise ratio of the received signals must be high.

3.2 Lamb waves for corrosion detection

Corrosion detection is one of the major concerns in the oil and chemical industries. Detection of hidden corrosion in aged pipes is particularly difficult because of the insulation layer that covers the pipes. Current technologies involving point-by-point inspection are expensive because of the need to remove this insulation layer. There is therefore an urgent need for the development of a quick and reliable method for detection of corrosion under the insulation layer.

Researchers started to look into the potential of ultrasonic guided waves for tubes inspection decades ago (Thomson et al., 1972; Silk and Bainton, 1979). Much of the work addressed inspection of heat exchanger tubing of 1-in diameter, and less work for larger diameter gas pipelines. However, in the past years relevant studies have been reported on the interaction of Lamb waves with defects in plates and pipe structures. Lowe et al. (1998) developed a method in which guided waves are propagated in the walls of thin pipes, and the reflected signals from defects are captured. They used the pulse-echo method. In parallel with experiments, they also conducted analytical and numerical studies. The testing scheme employed a pulse-echo arrangement from a single location on a circular pipe. The waves were excited and received using a ring transducer made of dry-coupled piezoelectric elements distributed around the pipe circumference.

The excitation signal used was 70 kHz corresponding to the longitudinal L(0,2) axially symmetric mode. The reason for this particular mode was that it is non-dispersive
over a wide bandwidth; it is also the fastest mode so any unwanted modes will arrive after it has been received; in addition, it is sensitive to internal or external defects. The experiments were performed on 40 steel pipe specimens (2.6 m long) with internal diameter of 76 mm and wall thickness 5.5 mm. A notch was machined approximately 2/3 of the distance along each pipe. The results showed a clearly reflection from the notch followed by the reflection from the end of the pipe.

Corrosion poses a threat not only in the oil and chemical industry but also in the aerospace industry. Hidden corrosion is a serious problem and can initiate on the inside or in the interface of an aircraft skin. In most cases, inspection of the aircraft skin is done from the outside using ultrasonic bulk-wave transducers. Conventional through-the-thickness ultrasonic methods for corrosion inspection of large surfaces using point-by-point examination can be tedious and time consuming. In addition, the disadvantage of existing methods is that the corrosion will create irregular surfaces and the echo signals might be unclear and hard to interpret. New ultrasonic inspection methods using guided waves (Lamb waves) have been developed. They have been proved to be more global in nature. In addition, they can provide more measurable features related to the interaction between guided waves and defects and hence contain more information about the flaws in structures.

Rose and collaborators did extensive work (2001, 2003) in the NDE and NDT for corrosion detection in pipes and plates using guided waves. Zhu et al. (1998) conducted an experimental study of hidden corrosion using guided waves, combined with a boundary element method (BEM) numerical simulation. The experimental work was carried out on both corrosion simulated specimens and real corrosion specimens. The
BEM method was used to simulate the guided wave scattering, mode cutoff, and conversion phenomena. Based on the BEM method, they proposed a quantitative technique to measure the hidden corrosion depth and compare the results with those obtained experimentally. Two types of aluminum corrosion specimens were used: plates with simulated corrosion prepared by machining away material; and real corrosion plates prepared by controlled electrochemical procedure (Figure 13).

Figure 13  (a) Simulated machined corrosion specimen; (b) real corrosion specimen (Zhu et al., 1998)

They used a pair of wedge transducers to produce and detect guided waves (Lamb waves); both through-transmission and pulse-echo methods were used. Plexiglass wedges were specially designed to decrease the effect of multireflection. A tone burst signal as well as a shock device were used to generate the different guided waves required in mode
selection and feature extraction. The time domain results showed a clear difference between the A3 Lamb mode in the pristine specimen (no corrosion) and the corroded specimen; similar difference was observed for the S5 Lamb mode between the pristine and corroded specimen. The transmission and reflection coefficients were calculated using the boundary element model (BEM) for different corrosion depth.

![Graph](image)

Figure 14  (a) Transmission coefficients for the S2, A2, and S3 incidence vs. corrosion depth; (b) Reflection coefficients for the S2, A2, and S3 incidence vs. corrosion depth (Zhu et al. 1998)

The features developed by Zhu et al. (1998) can be used to establish a vector feature to be used in pattern recognition analysis.
Rose and co-workers (Barshinger, Rose, and Avioli, 2002) also focused on the phase velocity, frequency wave resonance tuning, and mode selection. A mathematical model for the propagation of waves in hollow cylinders as well as numerical and experimental results were presented. They found, that tuning gives the possibility to improve penetration power through the coating layers and also to be able to detect difficult defect shapes.

### 3.3 Lamb waves for disbond/delamination detection

Ultrasonic testing of adhesively bonded joints using guided waves for both aerospace and automotive applications is gaining more and more attention. In the nondestructive evaluation of adhesively bonded joints of particular interest are the Lamb waves. Lamb wave methods have considerable potential for the inspection of adherent assemblies for two reasons: they do not require direct access to the bond region, and they are much more amenable to rapid scanning than are bulk-wave techniques. Lamb waves can be excited on one side of a bonded assembly, propagate across the joint region, and received on the other side of the assembly. Then, inspection of the joint would be based on the differences between the signals received on one side of the assembly compared to those transmitted on the other side.

The uniaxial, steady state time harmonic wave propagation in adhesively bonded joints was studied by Hanneman and Kinra (1991). They derived an exact formula for the transmission coefficient $H'(\omega)$ and studied the natural frequencies of the bond as a function of normalized adhesive thickness. To determine the transfer function of the bond, they considered a three-layer plate immersed in an elastic fluid (water). They found
that the sensitivity of the transfer function $H(\omega)$ is higher at odd resonance modes, thus
the odd resonances are more sensitive to changes in the material properties and thickness
of the adhesive layer than the even resonances.

Rose et al. (1995) used the ultrasonic guided waves for NDE of adhesively
bonded structures. They developed a double spring hopping probe (DHSP) to introduce
and receive Lamb waves. This method was used to inspect a lap splice joint of a Boeing
737-222 aircraft. A pair of variable angle beam transducers were used to excite Lamb
waves. Preliminary results showed the capability of through transmission for disbond
detection using the symmetric $S_0$ mode at 1.455 MHz frequency and the antisymmetric
$A_1$ mode at 3.525 MHz. Severe corrosion area was also identified using the DHSP hand
held.

Lowe and Cawley (1994) studied the applicability of plate wave techniques for
inspection of adhesive and diffusion bonded joints. The problems they addressed were
connected to (a) the measurement of the cohesive properties, that is the material
properties of the adhesive layer, and (b) the measurement of the adhesion properties, that
is the quality of bonding at the two interfaces between the adhesive and the metal parts.
They found that the Lamb wave techniques are limited by their strong sensitivity to the
material properties and the thickness of the adherents and that they are relative insensitive
to those of the adhesive layer.

Lee et al. (2003) studied the problem of wave propagation in a diffusion bonded
model using spectral elements (SE) and a new local interaction simulation approach
(LISA) for numerical modeling. The novelty of their work was that the sensor/actuator
configuration (Sonox P5) consisting of five different layers of materials with one
piezoceramic element generating a thickness mode vibration. The five layers were: two electrodes used for actuating and sensing, two copper layers, and in the middle a couplant layer. The experiments validated the numerical simulation, showing that the actuator/sensor configuration could operate either in $S_0$ or $A_0$ mode using an excitation frequency of 260.5 and 100 kHz, respectively. However, it was found that the coupling layer distorts the wave propagation due to its low impedance at the interface point and low speed within the couplant medium. The reliability of piezoelectric monitoring of adhesive joints was studied by Kwon et al. (2003). A single lap adhesively bonded tubular joint was tested during a torsional fatigue test. The results showed that the piezoelectric properties of the joint are related to the crack propagation. The measured electric flux density is a good estimator of the failure strain, and is sensitive to the maximum stress or strain in the layer rather than the average stress.

Repair patches are widely used in the aircraft industry for small repairs of the aircraft fuselage in order to extend the operational life of aging aircraft. Chiu et al. (2000) reported the development of a ‘perceptive repair’ or ‘smart’ system which provide information on the in-service performance of the repair and the associated structure. Their results showed the possibility to use piezoelectric elements to develop a ‘smart’ patch; they used the impedance measurements to determine the presence of damage. For impedance measurements, the sensor/actuator must be located close to the damaged area. Galea et al. (2001) described two in-situ health monitoring systems, one consisting of a piezoelectric polyvinylidene fluoride (PVDF) film, the second one consisting of an electrical–resistance strain gauge-based sensing system. The methods were tested on a bonded composite patch applied to an F/A-18 aircraft. The ‘smart patch’ approach was to
detect disbond growth in a safe life zone of the patch where disbonds are unacceptable, and to monitor the damage growth in a damage tolerant region. The method used to assess the “health” of the patch was to measure the load transfer in the safe-life zone. This was achieved by monitoring the ratio between patch strains and component strain during service life. Any decrease in this ratio was an indication of the disbonding of the patch. The results showed that the concept of “smart patch” approach for an autonomous health monitoring system is viable. However, more work needs to be done to minimize the power requirements of the system, and to develop other confidence building indicators.

More recent Koh and Chiu (2003) did a numerical study of the disbond growth under a composite repair patch. They used impedance method and the transfer function method to identify typical disbond growth shapes and sizes underneath the repair patch. The results showed that information about the location, type, and severity of disbond could be obtained using signal processing techniques and a strategic placement of the piezoelectric sensors.
Chapter 4
Piezoelectric Wafer Active Sensors

Piezoelectric wafer active sensors (PWAS) are inexpensive permanently-attached small transducers (Figure 15) that operate on the piezoelectric principle. The direct piezoelectric effect is manifested when the applied stress on the sensor is converted into electric charge. Conversely, the inverse effect, will produce strain when a voltage is applied to the sensor. In this way the PWAS can be used as both transmitter and receiver of elastic waves.

Figure 15  Array of PWAS transducers mounted on an aluminum plate

Piezoelectric actuators were used initially by Crawley et al. (1986, 1987) and Fuller et al. (1992). Tzou and Tseng (1991) and Lester and Lefevre (1993) modeled the piezoelectric sensor/actuator design for dynamic measurement and control. The use of
piezoelectric sensors for structural health monitoring and the Lamb waves for damage detection was pioneered by Chang and his collaborators (1998, 2001, 2002, 2004) who have studied the generation and the reception of elastic waves in composite materials. In their studies, passive reception of elastic waves was used for impact detection, and the pitch-catch method using low-frequency Lamb waves was used for damage detection. Other researchers have studied the propagation of Lamb waves and the use of different wave propagation methods (pitch-catch, pulse-echo) as well as standing wave methods (electromechanical impedance) for damage detection.

In the Laboratory for Active Materials and Smart Structures (LAMSS) at the University of South Carolina, extensive work has been done in developing embedded sensors for structural health monitoring (Giurgiutiu et al., 2002 - 2009). The team of students lead by Dr. Giurgiutiu proposed an embedded structural health monitoring (SHM) system using piezoelectric wafer active sensors (PWAS) that are lightweight, unobtrusive and permanent attached to the structure (Giurgiutiu et. al., Patent No. US7,174,255B2; Patent No. US7,024,315B2; Patent No. US6,996,480B2). The advantages of such sensors besides the lightweight and small geometry are that they can be used as passive sensors as well as active sensors. The same sensors are used for both exciting (actuating) and reading (sensing) the received signals. Other advantages of using piezoelectric wafer active sensors for structural health monitoring are:

- PWAS are small, non-intrusive, and inexpensive intimately affixed to the structure and can actively interrogating the structure (Giurgiutiu and Zahrai 2001; Giurgiutiu, Bao, and Zhao 2001).
• PWAS are non-resonant devices with wide band capabilities. They can be wired into sensor arrays that are connected to data concentrators and wireless communicators.

• PWAS have captured the interest of academia and industry due to their low cost and non-intrusive nature (Boller, 1999).

4.1 Principle of Piezoelectric Wafer Active Sensors

Piezoelectric wafer active sensors (PWAS) operate on the piezoelectric effect: (a) direct effect – applying stress on the surface of a piezoelectric material will generate an electric field; (b) indirect effect – an electric field applied to a piezoelectric material will cause the material to contract. PWAS can be manufactured in thin layers of lead zirconate titanate (PZT) material and can have various shapes (discs, rectangles, etc.) A piezoelectric wafer active sensor (PWAS) is permanently bonded on the surface of a structure through a layer of adhesive as shown in Figure 16.

![Piezoelectric wafer active sensor (PWAS)](image)

Figure 16  PWAS attached to the structure
The PWAS interacts with the structure through the adhesive layer. The adhesive layer acts as a shear layer and is transmitting the mechanical effects from the PWAS to the structure and vice versa through shear effects.

![PWAS interaction with structure: moments and forces (Giurgiutiu et al. 2004); circular crested Lamb waves generated in a two-dimensional structure](image)

The effect of such an interaction is the generation of elastic waves that travel through the structure to which the PWAS is affixed (Figure 17).

### 4.2 Coupling between PWAS and host structure

The transmission and reception of Lamb waves between the PWAS and the structure is achieved through the adhesive layer. The adhesive layer acts as a shear layer, in which the mechanical effects are transmitted through shear effects. Figure 18 shows a thin wall structure of thickness $t$ and elastic modulus $E$, with a PWAS of thickness $t_a$ and elastic modulus $E_a$ attached to its upper surface through a bonding layer of thickness $t_b$ and shear modulus $G_b$. 
The PWAS length is \( l_a \) while the half-length is \( a = l_a / 2 \). In addition, the definition \( d = t / 2 \) is used. Upon application of an electric voltage, the PWAS experiences an induced strain:

\[
\varepsilon_{IS4} = d_{31} \frac{V}{t}
\]  

(4.1)

The induced strain is transmitted to the structure through the bonding layer interfacial shear stress (\( \tau \)). For harmonic varying excitation, the shear stress has the expression \( \tau(x,t) = \tau_a(x)e^{j\omega t} \). The PWAS expansion is transmitted to the structure through the bonding layer which acts predominantly in shear. The shear stress intensity and distribution depend on the relative deformation of the PWAS and the structure. Crawley et al. (1986) developed a 1-D strain analysis using the Euler-Bernoulli hypothesis across the plate thickness, i.e., uniform displacement for axial motion, and linear displacement for flexural motion. The resulting interfacial shear stress can be written in terms of hyperbolic functions as:

\[
\tau(x) = \frac{l_a}{a} \cdot \frac{\psi}{\alpha + \psi} E_a \varepsilon_{IS4} \left( \Gamma a \frac{\sinh \Gamma x}{\cosh \Gamma a} \right)
\]  

(4.2)

where \( \psi = \frac{E_t}{E_a l_a} \), and the shear lag parameter is \( \Gamma^2 = \frac{G_{ab}}{E_a l_a} \cdot \frac{1}{t_b t_a} \cdot \frac{\alpha + \psi}{\psi} \).
The parameter $\alpha$ depends on the stress and strain distribution across the structural thickness. For low-frequency coupled axial-bending motion this parameter takes the value $\alpha = 4$. This value is changing as the frequency changes. The shear transfer along the PWAS is controlled by the product between the shear lag parameter, $\Gamma$, and the sensor half-length $a$. For low values of the $\Gamma a$ product, the shear transfer is distributed along the PWAS length, and the shear stresses have a relatively low intensity. For high values of $\Gamma a$, the shear transfer is localized towards the end of the sensor and the shear stress has high intensity.

4.3 Axial waves excited by PWAS

Assume a one-dimensional medium in which an external force induces an actuation strain, $\varepsilon_a(x,t)$. Such an actuation strain may be induced by surface mounted PWAS applied symmetrically to the top and bottom surfaces. The total strain is given by:

$$\varepsilon(x,t) = \frac{\sigma(x,t)}{E} + \varepsilon_a(x,t) \quad (4.3)$$

Considering an infinitesimal element of length $dx$; applying Newton’s law and considering a harmonic excitation, the strain becomes:

$$\varepsilon'' - \frac{\varepsilon^2}{\xi^2} \varepsilon = \varepsilon_a \quad (4.4)$$

where $\xi^2 = \omega^2/c^2$ is the wave number of axial waves in the one-dimensional medium.

Applying a space-domain Fourier transform on Eq. (4.4) yields:

$$\tilde{\varepsilon} = \frac{\xi^2}{\xi^2 - \xi_0^2} \tilde{\varepsilon}_a \quad (4.5)$$

Equation (4.5) represents the solution in the Fourier domain. Taking the inverse space-domain Fourier transform yields the solution in the space domain:
\[ \varepsilon(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\xi^2}{\xi^2 - \xi_0^2} \tilde{\varepsilon}_e(\xi) e^{i\xi x} d\xi \] (4.6)

For an ideally bonded PWAS, the induced strain is uniform over the PWAS length. The space-domain strain distribution is the rectangular pulse function:

\[ \varepsilon_e(x) = \begin{cases} \varepsilon_a, & |x| < a \\ 0, & \text{otherwise} \end{cases} \] (4.7)

Recalling Eq. (4.4) and taking the space-domain Fourier transform yields the solution:

\[ \tilde{\varepsilon} = \varepsilon_a \frac{2\xi}{\xi^2 - \xi_0^2} \sin \xi a \] (4.8)

Taking the inverse Fourier transform, the space-domain solution is:

\[ \varepsilon(x) = \frac{2\varepsilon_a}{2\pi} \int_{-\infty}^{\infty} \frac{\xi \sin \xi a}{\xi^2 - \xi_0^2} e^{i\xi x} d\xi \] (4.9)

The integral in Eq. (4.9) can be solved analytically using the residue theorem and a semicircular contour (C) in the complex \( \xi \) domain. The strain response due to a harmonically oscillating PWAS perfectly bonded to the structure has the form:

\[ \varepsilon(x,t) = i\varepsilon_a (\sin \xi_0 a) e^{i(\xi_0 x - \omega t)} \] (4.10)

From Eq. (4.10), it can be seen that the response amplitude follows a sinusoidal variation with respect to the parameter \( \xi_0 a \). Response peaks are observed at odd integer multiplies of \( \pi/2 \).

### 4.4 Flexural waves excited by PWAS

Consider the general equation of flexural vibrations under external moment excitation, \( M_e(x,t) \):

\[ E I v'' + \rho A \ddot{v} = M_e(x,t) \] (4.11)
Assuming harmonic variation in the time domain and considering the excitation curvature $\kappa_e$, the general equation of flexural vibrations becomes:

$$v'''' - \xi_F v'' = \kappa_e''$$

where

$$\kappa_e = \frac{M_e}{EI}, \quad \xi_F^4 = \frac{\omega^2}{d^2} \frac{DA}{EI}, \quad \alpha^2 = \frac{EI}{\rho A} = \frac{d^2}{\rho}$$

Applying a space-domain Fourier transform on Eq. (4.12) yields to:

$$(\xi^4 - \xi_F^4)\tilde{v} = -\xi^2 \tilde{\kappa}_e$$

The solution of Eq. (4.14) is:

$$\tilde{\xi} = \frac{-\xi^2}{\xi^4 - \xi_F^4} \tilde{\kappa}_e$$

Taking the inverse space-domain Fourier transform yields the solution in the space domain:

$$v(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\xi} e^{i\xi x} d\xi$$

The integral in Eq. (4.16) can be solved analytically using the residue theorem and a semicircular contour (C) in the complex $\xi$ domain. Adding the harmonic variation in the time domain yields the complete solution:

$$v(x,t) = i \frac{\tilde{\kappa}_e(\xi_F)}{4\xi_F} e^{(i\xi x - \omega t)} + \frac{\tilde{\kappa}_e(i\xi_F)}{4\xi_F} e^{-\xi_F x} e^{-i\omega t}$$

The first term in Eq. (4.17) represents a propagating wave, while the second term represents a vibration that is decaying fast with $x$. This term represents a local vibration that does not propagate and is called an evanescent wave. Retaining only the propagating wave part in Eq. (4.17) gives:
The strain solution can be derived as:

\[ \varepsilon_x(x, t) = y(i \xi_F) \frac{\tilde{\varepsilon}_e}{4} e^{i(\xi_F x - \omega t)} \] (4.19)

For an ideally bonded PWAS, the excitation moment is represented by a rectangular pulse function. The following expression for the strain at the material surface undergoing flexural wave excitation can be derived after mathematical calculations:

\[ \varepsilon(x, t) = i3\varepsilon_e (\sin \xi_F a) e^{i(\xi_F x - \omega t)} \] (4.20)

Response peaks are observed at odd integer multiples of \( \pi/2 \). Maximum excitation of flexural waves will occur when the PWAS length is an odd-integer multiple of the flexural half wavelength:

\[ l_a = (2m + 1) \frac{\lambda_F}{2} \] (4.21)

### 4.5 Lamb waves excited by PWAS

Consider a surface-mounted PWAS excited electrically with a time-harmonic voltage \( V e^{-i\omega t} \). As a result of the applied voltage the sensor will expand and contract, and a time harmonic interfacial shear stress, \( \tau_a(x) e^{-i\omega t} \) develops between the PWAS and the structure. The stress on the upper surface is given by:

\[ \tau_{yx} \bigg|_{y=h} = \tau_a(x) = \tau_0(x) \left[ H(x+a) - H(x-a) \right] \] (4.22)

where \( H(x) \) is the Heaviside step function. Using the space domain Fourier transformation yields:

\[ \tilde{f}(\xi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(x) e^{-i\xi x} dx, \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{f}(\xi) e^{i\xi x} d\xi \] (4.23)
where $\xi$ is the wavenumber. Applying the space domain Fourier to the excitation we obtain:

$$\tilde{\tau}_\alpha = \int_{-\infty}^{\infty} \tau_\alpha(x)[H(x+a)-H(x-a)]e^{-i\xi x}dx$$  \hspace{1cm} (4.24)

Recalling the wave equations in terms of the potential functions and the Lame constants:

$$\nabla^2 \phi = \frac{1}{c_p^2} \ddot{\phi}, \quad \nabla^2 \psi = \frac{1}{c_s^2} \ddot{\psi}$$  \hspace{1cm} (4.25)

where $c_p^2 = (\lambda + 2\mu)/\rho$ is the speed of the pressure wave; $c_s^2 = \mu/\rho$ is the speed of the shear vertical wave; $\lambda, \mu, \rho$ are the two Lame constants and the density, respectively.

The displacement in the $x$ and $y$ directions in terms of the two potentials are:

$$u_x = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}, \quad u_y = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}$$  \hspace{1cm} (4.26)

Applying the space domain Fourier transform to the wave equations, displacements, stresses and strains we obtain:

$$\frac{d^2 \ddot{\phi}}{dy^2} + c_p^2 \ddot{\phi} = 0, \quad \frac{d^2 \ddot{\psi}}{dy^2} + c_s^2 \ddot{\psi} = 0$$  \hspace{1cm} (4.27)

$$\tilde{u}_x = i\xi \ddot{\phi} + \frac{d\ddot{\psi}}{dy}, \quad \tilde{u}_y = \frac{d\ddot{\phi}}{dy} - i\xi \ddot{\psi}$$  \hspace{1cm} (4.28)

$$\tilde{\tau}_{xx} = \mu\left(2i\xi \frac{d\ddot{\phi}}{dy} + \xi^2 \ddot{\psi} + \frac{\partial^2 \ddot{\psi}}{\partial y^2}\right), \quad \tilde{\tau}_{yy} = \lambda\left(-\xi^2 \ddot{\phi} + \frac{d^2 \ddot{\phi}}{dy^2}\right) + 2\mu\left(-\xi^2 \ddot{\phi} - i\xi \frac{d\ddot{\psi}}{dy}\right)$$  \hspace{1cm} (4.29)

The general solution for Eq. (4.27) has the form:

$$\ddot{\phi} = A_1 \sin(py) + A_2 \cos(py)$$  \hspace{1cm} (4.30)

$$\ddot{\psi} = B_1 \sin(qy) + B_2 \cos(qy)$$

where $p^2 = \omega^2/c_p^2 - \xi^2$, $q^2 = \omega^2/c_s^2 - \xi^2$, and $\xi$ are the directional wave numbers. The constants, $A_2, B_1$ and $A_1, B_2$ correspond to the two possible motions: symmetric and anti-symmetric (Figure 19).
4.5.1 Symmetric solution

The boundary conditions for the symmetric case are:

\[ \tilde{u}_x \bigg|_{y=d} = \tilde{u}_x \bigg|_{y=-d}, \quad \tilde{u}_y \bigg|_{y=-d} = -\tilde{u}_y \bigg|_{y=d} \quad (4.31) \]

\[ \tilde{\tau}_{yx} \bigg|_{y=d} = -\tilde{\tau}_{yx} \bigg|_{y=-d}, \quad \tilde{\tau}_{yy} \bigg|_{y=-d} = \tilde{\tau}_{yy} \bigg|_{y=d} = 0 \quad (4.32) \]

After substitutions and further calculations, we arrive at the following linear system of equations:
\[
(-2i\xi p\sin pd)A_x + \left[(\xi^2 - q^2)\sin qd\right]B_i = \frac{1}{2\mu}\bar{\tau}_x
\]
\[
[(\xi^2 - q^2)\cos pd]A_x + (2i\xi q\cos qd)B_i = 0
\]

(4.33)

Solving the system of equations yields the strain at the upper surface for the symmetrical motion as:

\[
\bar{\varepsilon}_x^s \bigg|_{id} = -i\frac{\bar{\tau}_x N_s}{2\mu D_s}
\]

(4.34)

where \(N_s = \xi q(\xi^2 + q^2)\cos pd \cos qd\), \(D_s = (\xi^2 - q^2)^2\cos pd \sin qd + 4\xi^2 \sin pd \cos qd\).

4.5.2 Anti-symmetric solution

To solve for the strain at the upper surface for the anti-symmetrical motion we follow the same procedure as for the symmetrical motion. The boundary conditions are:

\[
\bar{u}_x\bigg|_{-d} = -\bar{u}_x\bigg|_{d}, \quad \bar{u}_y\bigg|_{-d} = \bar{u}_y\bigg|_{d}
\]

(4.35)

\[
\bar{\tau}_{yx}\bigg|_{-d} = \bar{\tau}_{yx}\bigg|_{d} = 0, \quad \bar{\tau}_{yy}\bigg|_{-d} = -\bar{\tau}_{yy}\bigg|_{d} = \frac{\bar{\tau}_y}{2}
\]

(4.36)

The strain at the upper surface for the anti-symmetrical motion is:

\[
\bar{\varepsilon}_x^A \bigg|_{id} = -i\frac{\bar{\tau}_x N_A}{2\mu D_A}
\]

(4.37)

where \(N_A = \xi q(\xi^2 + q^2)\sin pd \sin qd\), \(D_A = (\xi^2 - q^2)^2\sin pd \cos qd + 4\xi^2 \cos pd \sin qd\).

4.5.3 Total solution

The complete response to the PWAS excitation is obtained by combining the symmetric and anti-symmetric responses:

\[
\bar{\varepsilon}_x \bigg|_{id} = -i\frac{\bar{\tau}}{2\mu}\left(\frac{N_s}{D_s} + \frac{N_A}{D_A}\right)
\]

(4.38)
Applying the inverse Fourier transform and adding the harmonic time behavior we move from the wave number domain back into the space domain:

$$\tilde{\varepsilon}_i|_t = \frac{1}{2\pi} \frac{-i}{2\mu} \int_{-\infty}^{\infty} \left( \tilde{\tau}_N \frac{\tilde{N}_S}{D_S} + \tilde{\tau}_N \frac{\tilde{N}_A}{D_A} \right) e^{i(\xi x - \omega t)} d\xi$$ \hfill (4.39)

The integral in Eq. (4.39) is singular at the roots of $D_S$ and $D_A$ which are the symmetric and anti-symmetric eigenvalues of the Rayleigh-Lamb equations, i.e., $\xi_{0S}, \xi_{1S}, \xi_{2S}, \ldots$ (symmetric motion) and $\xi_{0A}, \xi_{1A}, \xi_{2A}, \ldots$ (anti-symmetric motion). At low frequencies, i.e. $\omega \to 0$, only two eigenvalues exist, $\xi_{0S}$ and $\xi_{0A}$. At higher frequencies several other eigenvalues exist.

To solve Eq. (4.39) the residue theorem is used and a contour consisting of a semicircle in the upper half of the complex $\xi$ plane and the real axis (Figure 20). The total solution for the strain at the upper surface is:

$$\varepsilon_x(x,t) = \frac{1}{2\mu} \left( \sum_{\xi S} \tilde{\tau}(\xi S) N_S(\xi S) e^{i(\xi x - \omega t)} + \sum_{\xi A} \tilde{\tau}(\xi A) N_A(\xi A) e^{i(\xi x - \omega t)} \right)$$ \hfill (4.40)

Integration with respect to $x$ yields the displacement:

$$u_x(x,t) = \frac{1}{2\mu} \left( \sum_{\xi S} \frac{1}{\xi S} \tilde{\tau}(\xi S) N_S(\xi S) e^{i(\xi x - \omega t)} + \sum_{\xi A} \frac{1}{\xi A} \tilde{\tau}(\xi A) N_A(\xi A) e^{i(\xi x - \omega t)} \right)$$ \hfill (4.41)

![Figure 20 Contour for evaluating the inverse Fourier transform integral.](image-url)
In this chapter, the principles of piezoelectric wafer active sensor (PWAS) and the possibility to use such small permanently attached low cost transducers to excite guided waves were discussed. The coupling between the PWAS and the structure through the adhesive layer was studied. The adhesive layer acts as a shear layer, in which the mechanical effects are transmitted to the structure through shear effects. For low values of the product between the shear lag parameter and the length of the PWAS, the shear stress is relatively low. For ideal bonding and high values of the product between the shear lag parameters and the length of the PWAS, the shear transfer is localized towards the two ends of the PWAS and the shear stress has high intensity (this is also known as the pin-force model).

The axial, flexural, and guided Lamb waves excited by PWAS were also studied and the solution to the response of the PWAS excitation was obtained.

It has been shown that small PWAS transducer electrically excited can generate through shear effects Lamb waves which propagates in the structure over relatively large distances and can be received by other PWAS transducers.
Chapter 5
PWAS-based damage detection methods

A large number of nondestructive inspection (NDI) techniques have been developed to identify local damage and detect incipient failure in aerospace structures. In the past years some of these techniques have been tested in the automotive industry as well. Among them, ultrasonic inspection based on elastic wave propagation is well established and has been used in the engineering community for several decades (Krautkramer, 1990). Also used is the mechanical impedance method (Cawley, 1984). The piezoelectric wafer active sensors (PWAS) methodology bears substantially on the experience accrued with conventional ultrasonic techniques.

However, major differences exist between conventional ultrasonics and active-sensor methods. Drawbacks of the ultrasonic techniques are the bulkiness of transducers and the need for a normal (perpendicular) interface between the transducer and the test structure. The former limits the access of ultrasonic transducers to restricted spaces. The latter influences the type of waves that can be easily generated in the structure. In contrast with conventional ultrasonics, the embedded active-sensors methods use wafer-like transducers (PWAS) that are permanently bonded to the structural surface.

These active sensors are small, thin, unobtrusive, and non-invasive. They can be placed in very restrictive spaces, like in built-up aerospace structures. The surface-
bonded active sensors can easily produce guided waves traveling parallel to the surface and could detect damage that would escape an ultrasonic method. Additionally, the ultrasonic probes must be moved across the structural surface through manual or semi-automated scanning, whereas embedded active sensors are permanently wired at predetermined locations. They can be remotely scanned through electronic switching.

5.1 Standing wave methods

The impedance method is a damage detection technique complementary to the wave propagation techniques. Ultrasonic equipment manufacturers offer, as options, mechanical impedance analysis (MIA) probes and equipment (Staveley NDT). The mechanical impedance method consists of exciting vibrations of bonded plates using a specialized transducer that simultaneously measures the applied normal force and the induced velocity. Cawley (1994) studied the identification of local disbonds in bonded plates using a small shaker. Though phase information was not used in Cawley’s analysis, present day MIA methodology uses both magnitude and phase information to detect damage.

The electro-mechanical (E/M) impedance method (Giurgiutiu et al. 1997; Park et. al. 2003; Bois and Hochard, 2004) is an emerging technology that offers distinctive advantage over the mechanical impedance method. While the mechanical impedance method uses normal force excitation, the E/M impedance method uses in-plane strain excitation. The mechanical impedance transducer measures mechanical quantities (force and velocity/acceleration) to indirectly calculate the mechanical impedance, while the
E/M impedance PWAS transducer measures the E/M impedance directly as an electrical quantity. The principles of the E/M impedance technique are illustrated in Figure 21.

![Figure 21 Electro-mechanical coupling between the PWAS and the structure.](image)

The effect of a PWAS transducer affixed to the structure is to apply a local strain parallel to the surface that creates stationary elastic waves in the structure. The structure presents to the active sensor the drive-point impedance,

\[
Z_{str}(\omega) = \frac{i \omega m_c(\omega) + c_e(\omega) - ik_e(\omega)}{\omega}.
\]

Through the mechanical coupling between the PWAS and the host structure, on one hand, and through the electro-mechanical transduction inside the PWAS, on the other hand, the drive-point structural impedance is directly reflected into the effective electrical impedance as seen at the PWAS terminals.

The apparent electro-mechanical impedance of the PWAS transducer coupled to the structure is:

\[
Z(\omega) = \left[ i \omega C \left( 1 - \kappa_{31}^2 \frac{Z_{str}(\omega)}{Z_{PWAS}(\omega) + Z_{str}(\omega)} \right) \right]^{-1}
\]

(5.1)

where \(Z(\omega)\) is the equivalent electromechanical admittance as seen at the PWAS terminals, \(C\) is the zero-load capacitance of the PWAS, \(\kappa_{31}\) is the electromechanical cross coupling coefficient of the PWAS \((\kappa_{31} = d_{13} / \sqrt{S_{11} e_{33}})\), \(Z_{str}\) is the impedance of the structure, and \(Z_{PWAS}\) is the impedance of the PWAS. The electromechanical impedance method is applied by scanning a predetermined frequency range in the hundreds of kHz band and recording the complex impedance spectrum. By comparing the impedance
spectra taken at various times during the service life of a structure, meaningful information can be extracted pertinent to structural degradation and the appearance of incipient damage. It must be noted that the frequency range must be high enough for the signal wavelength to be significantly smaller than the defect size.

In our work, the PWAS is permanently bonded to the structural surface. In this way, the PWAS is part of the structure and will act as an actuator generating both axial and flexural vibrations in the structure. The PWAS will dynamically expand and contract when an alternating electric field is applied. In this way, stationary elastic waves are introduce in the structure and can be scanned over a frequency range, typically in the kHz band. The complex impedance spectrum is recorded. The frequency spectrum is then analyzed for new features like frequency shift of existing peaks; increase in peak amplitudes; appearance of new peaks. For a bonded structure, we expect the frequency spectrum to be the same for measurements taken where a good bond exists. In the presence of disbonds, the electromechanical impedance of the structure will change and those changes should be reflected in the frequency spectrum. The spectrum should present the types of features described above. This method will be used for the near-field detection of disbonds.

5.2 Wave propagation methods

Ultrasonic methods rely on elastic wave propagation and reflection within the material, and identify the field inhomogeneities due to local damage and flaws. Ultrasonic testing involves one or more of the following measurements: time of wave transit (or delay), path length, frequency, phase angle, amplitude, impedance, and angle
of wave deflection (reflection and refraction). The most common detection methods using propagating waves are pitch-catch and pulse-echo.

**5.2.1 Pitch-catch method**

The pitch-catch method detects damage from the changes that Lamb waves undergo when traveling through a damaged region. The method uses the transducers in pairs, one as transmitter, the other as receiver. In the embedded pitch-catch method (Figure 22), the transducers are either permanently attached to the structure or inserted between the layers of composite layup.

![Embedded Pitch-catch method](image)

**Figure 22** Embedded ultrasonics damage detection techniques with pitch-catch method

Adhesively bonded lap joints are often used in high performance structures such as airframes. Periodic inspection of these joints can be time consuming. Figure 23 shows the inspection of a lap splice joint specimen (other configurations of adhesively bonded joints are also possible). Using PWAS, wave energy is generated on one side of the joint, transmitted through the adhesive bond line, and received on the other side of the joint.

![PWAS # 1 and PWAS # 2](image)

**Figure 23** Pitch-catch method for joint inspection: (a) pristine joint carries the signal well from PWAS # 1 to PWAS # 2 through “leakage”; (b) Disbonded joint cannot carry well the signal resulting in degradation of signal received at PWAS # 2
The amplitude of the transmitted signal is a measure of the bond quality. In order to find the best transmission mode across the joint, tuning is necessary. Tuning the frequency allows inspection of bonded joints having various thickness, structural configurations, and even joints having more than two layers. For a healthy bond the amplitude of the received signal is large. If disbond has occurred, than there will be a decrease in amplitude of the received signal proportional with the severity of the disbond.

Pitch-catch method can also be used to detect cracks. Cracks in metallic structures typically run perpendicular to wall surface. A fully developed crack will cover the whole thickness (through-the-thickness crack) and will produce a tear of the metallic material. In conventional NDE, metallic-structure cracks are detected with ultrasonic or eddy current probes that have point-wise capabilities. Intensive manual scanning around the hot-spots is required for crack detection. The aim of embedded pitch-catch NDE is to detect cracks in metallic structures using guided waves transmitted from one location and received at a different location. The analysis of the change in guided wave shape, phase, and amplitude should yield indications about the crack presence and extension.

Another application for the pitch-catch method is the detection of damage in composite materials. Composite structures are typically resistant to through-the-thickness cracks due to the inherent crack-resistant effect of fiber reinforcement. However, in layered composite structures, cracks can easily propagate parallel to the wall surface, typically at the interface between layers. These cracks can be initiated by fabrication imperfections or low-velocity impact damage; subsequently, they propagated by cyclic fatigue loading.
In conventional NDE, composite cracks and delaminations are detected with ultrasonic probes that can sense additional echoes due to through-the-thickness P waves being reflected by the delamination. Area coverage is achieved with surface scanning (C scans) using manual means or mechanical gantries. The aim of embedded pitch-catch NDE is to detect cracks and delamination in composite structures using guided waves transmitted from one location and received at a different location. The disbond/delamination produces signal diffraction and mode conversion that can be analyzed in comparison with the pristine signals. Analysis of the change in the guided wave shape, phase, and amplitude should yield indications about the crack presence and extension. In addition, the sensor network built into the structure can be also used for the detection of low-velocity impacts that may be the cause of composite damage.

5.2.2 Pulse-echo method

In conventional NDE, the pulse echo method has traditionally been used for through-the-thickness testing. For large area inspection, through-the-thickness testing requires manual or mechanical moving of the transducer over the area of interest, which is labor intensive and time consuming. It seems apparent that guided-wave pulse echo seems more appropriate, since wide coverage could be achieved from a single location. Disbond detection can be also performed using the pulse-echo technique which is able to detect surface cracks and subsurface cracks within the detection depth.

The use of Lamb-wave pulse echo methods with embedded PWAS follows the general principles of conventional Lamb-wave NDE. A PWAS transducer attached to the structure acts as both transmitter and detector of acoustic guided waves traveling in the structure. The wave sent by the PWAS is partially reflected at the crack. The echo is
captured at the same PWAS acting as receiver (Figure 24). For the method to be successful, it is important that a low-dispersion Lamb wave is used. The selection of such a wave, e.g., the $S_0$ mode, is achieved through the Lamb-wave tuning methods (Giurgiutiu, 2003).

Figure 24  Pulse-echo method for damage detection

This wave reflection may be sensed as an echo at the transmitter sensor (PWAS). The echo time of flight (TOF) is proportional with twice the distance between the transmitter sensor and the disbond. In order to distinguish the echo from the background noise, a differential signal method, using historically recorded baseline signatures can be utilized.
PART II

Development and validation of analytical model
Chapter 6
Modal analysis method for electromechanical impedance

The interaction between the PWAS and the host structure and how changes in the mechanical impedance of the structure shows in the electrical impedance of the PWAS have been studied by Giurgiuțiu and Zagrai (2001) and Giurgiuțiu (2008). Their theoretical results were based on the classic vibration theory for axial and flexural vibrations for the free-free boundary conditions of a beam using modal analysis.

6.1 Forces and moments for PWAS attached to structure

Following Giurgiuțiu (2008), we consider a PWAS transducer attached to a 1-D structure as shown in Figure 25. By applying a voltage to the PWAS of length $l_p$ located at $x_1$ and $x_2$ respectively from the left end, the PWAS will expand and contract by an amount $\varepsilon_p$. The strain created by the PWAS will generate a reaction force $F_p$ from the beam onto the PWAS which in turn will create an opposite reaction force from the PWAS onto the beam.
At the neutral axis of the beam, the force created by the sensor $F_p$ induces an axial force $N_F$ and a bending moment $M_F$. Harmonic excitation of the PWAS with a high frequency signal induces in the structure standing or travelling waves that will travel and return back to the PWAS after being reflected at boundaries. To derive the axial force and the bending moment we start from the expression of the force generated by the PWAS:

$$F_p = A e^{io\omega t} = A e^{jo\omega} \cdot e^{io\omega} = \hat{F}_p e^{io\omega}$$  
(6.1)

where $\hat{F}_p$ is the complex amplitude. The axial force and bending moment at locations $x_1$ and $x_2$ are written as:

$$N_p = F_p = \hat{N}_p e^{io\omega}$$  
(6.2)

$$M_p = \frac{h}{2} F_p = \frac{h}{2} \hat{F}_p e^{io\omega} = \hat{M}_p e^{io\omega}$$  
(6.3)

We used the Heaviside function:

$$H(x - y_1) - H(x - y_2) = \begin{cases} 0 & x < x_1 \\ 1 & x_1 \leq x \leq x_2 \\ 0 & x > x_2 \end{cases}$$  
(6.4)

The axial force and bending moment at any location $x$ on the beam are given by:

$$N_p(x,t) = \hat{N}_p \left[ H(x - x_2) - H(x - x_1) \right] e^{io\omega}$$  
(6.5)

$$M_p = \hat{M}_p \left[ H(x - x_2) - H(x - x_1) \right] e^{io\omega}$$  
(6.6)
6.2 Axial vibrations

In this section, we are interested in finding the response under forced axial vibration; we will start the analysis with the free axial vibrations of a uniform elastic bar of length \( l \) cross-sectional area \( A \), mass \( m \), and axial stiffness \( EA \) as shown in Figure 26.

![Figure 26 Axial vibration of a uniform bar](image)

From the summation of forces in the x-axis the equation of motion can be derived:

\[
N(x,t) + dN(x,t) - N(x,t) = \rho A u_t(x,t)
\]

(6.7)

with \( N = \sigma A = EA \frac{\partial u}{\partial x} = EA u' \) the equation of motion for free axial vibrations can be written as:

\[
EAu''(x,t) - \rho A u_t(x,t) = 0
\]

(6.8)

To find a solution to Eq. (6.8) the method of separation of variables can be used; we assume :

\[
u(x,t) = U(x) T(t) = U(x) e^{i\omega t}
\]

(6.9)

We are interested in the steady-state solution \( U(x) \). Substituting Eq. (6.9) into Eq. (6.8) and differentiating, we obtain a second order differential equation:

\[c^2 U''(x) + \omega^2 U(x) = 0, \quad c^2 = E / \rho\]

(6.10)

Eq. (6.10) admits the general solution in the form:

\[U(x) = C_1 \sin \gamma x + C_2 \cos \gamma x\]

(6.11)
where \( \gamma = \frac{\omega}{c} \); since Eq. (6.11) represents a harmonic oscillation similar to a wave, \( \gamma \) is also called the wavenumber. The two constants \( C_1 \) and \( C_2 \) are determined from the boundary conditions (i.e. free-free, clamped-free, etc.). For the free-free case the boundary conditions are:

\[
U'(0) = 0 \\
U'(l) = 0
\]  
(6.12)

After differentiating Eq. (6.11), substituting in Eq. (6.12), and solving the characteristic equation for the eigenvalues \( \gamma_j \), the general solution will yield the mode shapes in the form:

\[
U_j(x) = C_j \cos \gamma_j x, \quad \gamma_j = j \frac{\pi}{l}, \quad j = 1, 2, 3, \ldots
\]  
(6.13)

where \( C_j \) is calculated as \( C_j = \sqrt{\frac{2}{ml}} \) and the mode shapes satisfy the orthogonality property:

\[
\int_0^l U_m(x) U_n(x) \, dx = \delta_{mn} = \begin{cases} 
1 & , m = n \\
0 & , m \neq n
\end{cases}
\]  
(6.14)

For forced vibrations, the equation of motion takes the form

\[
EAu''(x,t) - \rho A\ddot{u}(x,t) = N'_p(x,t)
\]  
(6.15)

Next, we assume the modal expansion

\[
u(x,t) = \sum_j C_j U_j(x) e^{i\omega t}, \quad j = 1, 2, 3, \ldots
\]  
(6.16)

Here, the constants \( C_j \) are the modal participation factors and \( U_j(x) \) are the modes shapes satisfying Eq. (6.10). Substituting the modal expansion (6.16) into the equation of motion (6.15) we arrive at the expression:
\[-\omega^2 \rho A \sum_j C_j U_j (x) e^{i\omega t} - EA \sum_j C_j U_j' (x) e^{i\omega t} = N'_p (x,t) \quad (6.17)\]

From Eq. (6.5), \(N'_p (x,t)\) can be derived and substituted into Eq. (6.17) to give

\[-\omega^2 \rho A \sum_j C_j U_j (x) - EA \sum_j C_j U_j' (x) = \hat{N}_p \left[ \delta (x-x_2) - \delta (x-x_1) \right] \quad (6.18)\]

From the Equation (6.10),

\[U''_j (x) = - \frac{\rho \omega^2}{E} U (x) \quad (6.19)\]

Substitution of Eq. (6.19) in Eq. (6.18) and a few calculations yields

\[\sum_j \left[ C_j (\omega_n^2 - \omega^2) U_j (x) \right] = \frac{\hat{N}_p}{\rho A} \left[ \delta (x-x_2) - \delta (x-x_1) \right] \quad (6.20)\]

Multiplying with \(U_k (x)\) on both sides and taking the integral over the length of the beam of Eq. (6.20) we can write

\[\int \int U_k (x) \sum_j \left[ C_j (\omega_n^2 - \omega^2) U_j (x) \right] dx = \int \int U_k (x) \frac{\hat{N}_p}{\rho A} \left[ \delta (x-x_2) - \delta (x-x_1) \right] dx \quad (6.21)\]

rearranging the terms

\[\sum_j C_j (\omega_n^2 - \omega^2) \int U_j (x) U_k (x) dx = \int U_k (x) \frac{\hat{N}_p}{\rho A} \left[ \delta (x-x_2) - \delta (x-x_1) \right] dx \quad (6.22)\]

Using the orthogonality property of mode shapes (6.14) and the property of the delta function

\[\int f(x) \delta (x-a) dx = f(a) \quad (6.23)\]

Equation (6.22) becomes

\[\sum_j C_j (\omega_n^2 - \omega^2) = \frac{\hat{N}_p}{\rho A} [U_k (x_2) - U_k (x_1)] \quad (6.24)\]

From Eq. (6.24), the modal participation factors \(C_j\) can be expressed as
Using Eq. (6.16) the axial vibration response becomes

\[ u(x,t) = \frac{\hat{N}_p}{\rho A} \sum_j \frac{U_j(x_2) - U_j(x_1)}{\omega_j^2 - \omega^2} U_j(x) e^{j\omega t} \]  

(6.26)

For a damped system with the hysteretic damping coefficient \( g \), Eq. (6.26) becomes

\[ u(x,t) = \frac{\hat{N}_p}{\rho A} \sum_j \frac{U_j(x_2) - U_j(x_1)}{(1 + ig)\omega_j^2 - \omega^2} U_j(x) e^{j\omega t} \]  

(6.27)

### 6.3 Flexural vibrations

We now consider the flexural displacement \( w \) for the flexural vibration of a uniform elastic bar of length \( l \), mass \( m \) and flexural stiffness \( EI \). We will start from analyzing a portion of a beam undergoing flexural vibrations under the excitation of an external force per unit length of the beam \( f(x,t) \) as shown in Figure 27.

![Figure 27: Flexural vibration of a uniform bar](image)

From the free-body diagram, we can write the equation of motion in the \( z \) direction and the moment equation about the \( y \) axis through the point \( O \) as:

\[ V(x,t) + f(x,t)dx - [V(x,t) + dV(x,t)] = m \ddot{w}(x,t) \]  

(6.28)

\[ f(x,t) - V'(x,t) = \rho A(x) \ddot{w}(x,t) \]  

(6.29)
\[ M(x,t) + dM(x,t) + f(x,t) \frac{dx}{dx} - \frac{1}{2} \left[ V(x,t) + dV(x,t) \right] dx - M(x,t) = 0 \quad (6.30) \]

neglecting the terms involving second order in \( dx \)

\[ M'(x,t) - V(x,t) = 0 \Rightarrow V(x,t) = M'(x,t) \quad (6.31) \]

From the Euler-Bernoulli elementary beam theory the relation between bending moment and deflection is:

\[ M(x,t) = EI(x)w''(x,t) \quad (6.32) \]

Using Eq. (6.32) and Eq. (6.31) and substituting in Eq. (6.29) the equation of motion for a uniform beam undergoing flexural vibrations under a force \( f(x,t) \) can be expressed as:

\[ \rho A \ddot{w}(x,t) + EIw''''(x,t) = f(x,t) \quad (6.33) \]

For free vibrations, \( f(x,t) = 0 \), and the equation of motion becomes

\[ \rho A \ddot{w}(x,t) + EIw''''(x,t) = 0 \quad (6.34) \]

A solution to Eq. (6.34) can be found using the separation of variables method; assuming the general solution in the form

\[ w(x,t) = W(x)T(t) = W(x)e^{i\omega t} \quad (6.35) \]

Substituting Eq. (6.35) into Eq. (6.34) we obtain a fourth order differential equation

\[ W''''(x) - \gamma^4 W(x) = 0, \quad \gamma = \left( \frac{m}{EI} \right)^{1/4} \sqrt{\omega} \quad (6.36) \]

Eq. (6.36) admits the general solution in the form

\[ W(x) = Ce^{i\omega x} \quad (6.37) \]

Substituting Eq. (6.37) into Eq. (6.36) after calculation the solution can be expressed as:

\[ W(x,t) = C_1 \sin \gamma x + C_2 \cos \gamma x + C_3 \sinh \gamma x + C_4 \cosh \gamma x \quad (6.38) \]

where \( C_1, C_2, C_3, C_4 \) are constants to be found from the boundary conditions.
To calculate the total flexural displacement for a uniform bar under a moment excitation $M_p$ the equation of motion can be written as

$$\rho A \ddot{w}(x,t) + Elw'''(x,t) = -M_p''(x,t)$$  \hspace{1cm} (6.39)

where the bending moment $M_p$ is given by Eq. (6.6). We will assume the modal expansion

$$w(x,t) = \sum_j C_j W_j(x)e^{i\omega t}$$  \hspace{1cm} (6.40)

where $W_j(x)$ are orthonormal mode shapes satisfying the free vibration differential equation (6.34). Differentiating $w(x,t)$ and $M_p$, we get

$$\ddot{w}(x,t) = -\omega^2 \sum_j C_j W_j(x)e^{i\omega t}$$

$$w'''(x,t) = \sum_j C_j W_j'''(x)e^{i\omega t}$$  \hspace{1cm} (6.41)

$$M_p''(x,t) = \hat{M}_p \left[ \delta'(x-x_2) - \delta'(x-x_1) \right] e^{i\omega t}$$  \hspace{1cm} (6.42)

Substituting Eq. (6.42) and (6.41) into Eq. (6.39)

$$\rho A \ddot{w}(x,t) + Elw'''(x,t) = -\hat{M}_p \left[ \delta'(x-x_2) - \delta'(x-x_1) \right]$$  \hspace{1cm} (6.43)

Multiplying Eq (6.43) by $W_k(x,t)$ and taking the integral over the length of the beam

$$\int_0^l W_k(x) \left[ \rho A \ddot{w}(x,t) + Elw'''(x,t) \right] dx = -\hat{M}_p \int_0^l W_k(x) \left[ \delta'(x-x_2) - \delta'(x-x_1) \right] dx$$  \hspace{1cm} (6.44)

Using the property of the $\delta$ function

$$\int_a^b f(x) \frac{d}{dx} \delta(x-g) dx = -\int_a^b \delta(x-g) \frac{df(x)}{dx} dx = -\frac{d}{dx} f(g)$$  \hspace{1cm} (6.45)

the right term of Eq. (6.44) becomes

$$-\hat{M}_p \int_0^l W_k(x) \left[ \delta'(x-x_2) - \delta'(x-x_1) \right] dx = -\hat{M}_p \left[ W_k'(x_1) - W_k'(x_2) \right]$$  \hspace{1cm} (6.46)
and knowing that the mode shapes $W_k(x)$ satisfy the free flexural vibration equation (6.34) we get

$$EIW'''(x) - \alpha_k^2 \rho AW_k(x) = 0, \quad EIW'''(x) = \omega_k^2 \rho AW_k(x)$$ (6.47)

the left term of Eq. (6.44) becomes

$$\int_0^l W_k(x) \left[ \rho A \ddot{w}(x,t) + EIw'''(x,t) \right] dx = \sum_j C_j (\omega_j^2 - \omega^2) \rho A \int_0^l W_k(x) W_j(x) dx$$ (6.48)

where $\int_0^l W_k(x) W_j(x) dx = 1$ according to the orthogonality property of the mode shapes.

Using Eq. (6.48) and Eq. (6.46) and substituting in Eq. (6.44)

$$\sum_j C_j (\omega_n^2 - \omega^2) \rho A = - \dot{M}_p \left[ W_k'(x_1) - W_k'(x_2) \right]$$ (6.49)

the modal participation factors $C_j$ can be expressed as

$$C_j = - \frac{\dot{M}_p}{(\omega_j^2 - \omega^2) \rho A} \left[ W_k'(x_2) - W_k'(x_1) \right]$$ (6.50)

Substituting Eq. (6.50) into Eq. (6.40) the flexural response of the beam becomes

$$w(x,t) = - \frac{\dot{M}_p}{\rho A} \sum_j \frac{W_j'(x_2) - W_j'(x_1)}{(\omega_j^2 - \omega^2)} W_j(x) e^{i\omega t}$$ (6.51)

For a damped system with the hysteretic damping coefficient $g$ Eq. (6.51) becomes

$$w(x,t) = - \frac{\dot{M}_p}{\rho A} \sum_j \frac{W_j'(x_1) - W_j'(x_2)}{(1 + ig\omega_j^2 - \omega^2)} W_j(x) e^{i\omega t}$$ (6.52)

### 6.4 Frequency response function

In general, for the vibration of a uniform beam the steady-state term of the vibration response can be expressed as the force divided by a frequency dependent term.
The frequency depended term is often referred to as the frequency response function \( FRF(\omega) \).

\[
u(\omega) = \frac{F}{\Omega(\omega)}
\]

or

\[
u(\omega) = F \cdot FRF(\omega)
\]  

(6.53)

where \( FRF(\omega) = \frac{1}{\Omega(\omega)} \)

and from Eq. (6.53) the frequency response function for a given excitation can be calculated as

\[
FRF(\omega) = \frac{u(\omega)}{F}
\]  

(6.54)

In order to calculate the frequency response function we need to find the total in-plane displacement \( u_T \) on the top of the beam where the PWAS is located. The total displacement on the top of the beam is the superposition of the axial displacement \( u \) and the rotation \( w' \), i.e.,

\[
u_T = u \pm \frac{h}{2} w'
\]  

(6.55)

where the \( \pm \) sign refers to either tension or compression of the top surface.

For a PWAS attached to a beam, the differential displacement between two points A and B (Figure 25) representing the ends of the PWAS is

\[
u_p(x) = u_s(x) - u_d(x) = u_s(x_2) - u_d(x_1)
\]  

(6.56)

Using Eq. (6.55), Eq. (6.56) becomes

\[
u_p = u(x_2) - u(x_1) - \frac{h}{2} [w'(x_2) - w'(x_1)]
\]  

(6.57)
The axial displacement $u$ and the rotation $w'$ at the two locations $x_1$ and $x_2$ can be calculated using Eq. (6.27) and Eq. (6.52) respectively. Substituting these values into Eq. (6.57) it becomes

$$u_p = \frac{\hat{N}_p}{\rho A} \left[ \sum_j \frac{U_j(x_2) - U_j(x_1)}{(1 + ig) \omega_j^2 - \omega^2} U_j(x_2) - \sum_j \frac{U_j(x_2) - U_j(x_1)}{(1 + ig) \omega_j^2 - \omega^2} U_j(x_1) \right] e^{i\omega t}$$

$$+ \frac{h \hat{M}_p}{2 \rho A} \left[ \sum_j \frac{W_j'(x_2) - W_j'(x_1)}{(1 + ig) \omega_j^2 - \omega^2} W_j(x_2) - \sum_j \frac{W_j'(x_2) - W_j'(x_1)}{(1 + ig) \omega_j^2 - \omega^2} W_j(x_1) \right] e^{i\omega t}$$

(6.58)

After further calculations and simplifications, the total axial displacement can be written as

$$u_p = \frac{F_p}{\rho A} \left[ \sum_j \frac{\left[U_j(x_2) - U_j(x_1)\right]^2}{(1 + ig) \omega_j^2 - \omega^2} + \left(\frac{h}{2}\right)^2 \sum_j \frac{\left[W_j'(x_2) - W_j'(x_1)\right]^2}{(1 + ig) \omega_j^2 - \omega^2} \right]$$

(6.59)

And the frequency response function $FRF(\omega)$ can be expressed as

$$FRF(\omega) = \frac{1}{\rho A} \left[ \sum_j \frac{\left[U_j(x_2) - U_j(x_1)\right]^2}{(1 + ig) \omega_j^2 - \omega^2} + \left(\frac{h}{2}\right)^2 \sum_j \frac{\left[W_j'(x_2) - W_j'(x_1)\right]^2}{(1 + ig) \omega_j^2 - \omega^2} \right]^{-1}$$

(6.60)

The dynamic stiffness of the structure can be expressed as the one over the FRF

$$k_{ar}(\omega) = \frac{F_p}{u_p(\omega)} =$$

$$= \frac{1}{\rho A} \left[ \sum_j \frac{\left[U_j(x_2) - U_j(x_1)\right]^2}{(1 + ig) \omega_j^2 - \omega^2} + \left(\frac{h}{2}\right)^2 \sum_j \frac{\left[W_j'(x_2) - W_j'(x_1)\right]^2}{(1 + ig) \omega_j^2 - \omega^2} \right]^{-1}$$

(6.61)
6.5 Impedance and admittance

Once the dynamic stiffness of the structure is know, we can relate the mechanical impedance of the structure to the electrical impedance of the sensor. According to Giurgiutiu (2008), the frequency-dependent stiffness ratio is defined as the ratio

\[ r(\omega) = \frac{k_{\text{str}}(\omega)}{k_{\text{PWAS}}} \] (6.62)

where \( k_{\text{str}} \) is the frequency dependent dynamic stiffness of the structure and \( k_{\text{PWAS}} \) is the PWAS static stiffness calculated as

\[ k_{\text{PWAS}} = \frac{A_p}{s_{11}^{E} l_p} \] (6.63)

According to Giurgiutiu (2008), the electrical admittance for a constrained PWAS can now be expressed as

\[ Y = i\omega C \left[ 1 - k_{31}^{2} \left( 1 - \frac{1}{\varphi \cot \varphi + r} \right) \right] \] (6.64)

The electrical impedance measured at the PWAS terminals is

\[ Z = \frac{1}{Y} = \frac{1}{i\omega C} \left[ 1 - k_{31}^{2} \left( 1 - \frac{1}{\varphi \cot \varphi + r} \right) \right]^{-1} \] (6.65)

where \( \varphi = \frac{\gamma_p l_p}{2}, \quad \gamma_p = \omega \rho_s e_{11} \)
Chapter 7
Transfer matrix method for electromechanical impedance of disbonded structures

In this chapter the transfer matrix method for the vibration analysis of complex structures i.e., adhesively bonded materials with structural damages in the form of disbonds or cracks is, developed. The principles of the transfer matrix method are discussed first. Then, we will present the analysis of free and forced vibration of a uniform homogeneous beam. Next, we continue with a detailed presentation of using the transfer matrix method to predict the vibration of a damaged multi-layer beam.

7.1 State of the art

For many years the analysis of continuous systems was the focus of many researchers. Although the beam model is very simple it provides good information when used to investigate the response of more complex structures. A thorough review of the vibration methods for cracked structures was provided by Dimarogonas (1996).

Ritchie et. al. (1975) incorporated the transfer matrix method into a simple finite-element technique to calculate the spectrum of flexural and torsional resonant frequencies
of specimens of material of orthotropic symmetry. In their calculations they accounted for the torsion-flexure coupling and used the Timoshenko beam corrections for flexure. They reported that the transfer matrix technique can be used to predict the free-free resonant frequencies for rectangular composite material beams of orthotropic symmetry. They also showed that the coupling torsion-flexure produces measurable perturbations of the resonant frequencies and gross distortion of the vibration shapes.

Subrahmanyam and Garg (1997) developed a computer code to calculate the frequencies and mode shapes for uncoupled flexural vibrations of straight beams for different types of boundary conditions using the transfer matrix method. In their model they included the shear deformation effects, rotary inertia and variable axial loading. The field transfer matrix and the point transfer matrix were calculated. To demonstrate the capabilities of using the transfer matrix method for uniform beam with shear deformation and rotary inertia they considered a rectangular cross-section beam of length \( L = 1 \) m divided into 100 equal segments. The mass and mass moment of inertia were lumped at the centroid of each segment. The exact solution for the pinned-sliding case was produced using the Timoshenko beam theory. Also, the exact solution following the Euler-Bernoulli theory was used to compare the results. Based on the results, they concluded that excellent agreement exists between the results obtained using the code and those obtained by using the exact solutions.

Lee (2000) introduced a general approach to the spectral transfer matrix method (STMM) by combining the features of the spectral element method (i.e., high accuracy) with the features of the transfer matrix method (i.e., high analysis efficiency). He applied this method to address the vibration problems of large periodic lattice structures. For the
analytical case he considered a plane lattice structure consisting of four beam-like lattice substructures and one single lattice cell, joined together at the center. He applied then the STMM method to a simple Euler-Bernoulli beam to calculate the dynamic response under a unit impulse moment. The same method was then applied to a large plane lattice structure. In both cases he was able to predict the dynamic response for each structure.

Ellakany et. al. (2004) used a combination of between the transfer matrix method and the analog beam method (TMABM) to study the vibration of a composite beam composed of an upper slab and a lower beam, connected at the interface by shear transmitting studs. The analog beam theory includes the coupling between the bending and torsional modes of deformation. Using this method, the real beam is replaced by an analogue beam where all the shear deformation is concentrated in a thin horizontal layer called the shear layer. They used the following assumptions: (a) each sub-beam behaves as a simple Euler-Bernoulli beam (no shear effects); (b) the vertical displacements of the sub-beams are the same (the shear layer is transversely rigid). The method was applied to compute the natural frequencies of a simple supported beam with uniformly distribute mass. The results were compared against the classical solution for simple supported beams. They reported good agreement between the TMABM method and the classical method.

Bilello and Bergman (2004) used the transfer matrix method to solve the eigenvalue problem for a one-dimensional system with non-uniform mechanical properties, namely the vibration of a damaged beam under a moving mass. They discretized the beam into N segments of constant linear mass density, flexural stiffness and length. The cracked beam was modeled using the “rotational spring model” which
takes into account the damage by using the local compliance, which quantifies the relation between the load and the strain in the vicinity of the crack. To get the dynamic response the modal expansion method was used. From the analytical results and the experiments they observed a good agreement between the two and that the presence of damage produced larger perturbation to the dynamic response of a moving load.

Bois et. al. (2004, 2007) used the transfer matrix method to calculate the frequency response of a composite beam with simulated delaminations. The analytical model for a delaminated beam was developed using the 3D constitutive laws for the piezoelectric material. Then, they expressed them in terms of the plane stress terms with uniaxial electric field. The expressions for the electric charge displacement, longitudinal displacement and transversal displacement were derived. The beam was then divided into several segments and at each segment the kinematic and continuity conditions were applied. They obtained an \( 3n_a \) linear system of equation where \( I_n \) is the number of nodes. By solving the system of equation the displacement fields in terms of the electric potential \( V^e \) were calculated. From the displacement fields, the current \( I \) was calculated and then the electrical admittance. Experimentally, they considered a laminate consisted of 16 unidirectional carbon/epoxy plies. Six piezoelectric transducers were bonded on the beam. Two delaminations were considered; one at one end and one located inside the beam. The frequency range was 0.1 kHz-30 kHz. For a damage size larger than 10% of the total beam length, the detection of the size and position of the defect was considered to be accurate.
7.2 Transfer matrix method

For complex beam geometries with various damages which add discontinuities in the mass and stiffness distribution, coupled with non-traditional boundary conditions, the differential equation of motion become more complex and the classical methods of solution (i.e. Galerkin method, finite difference method) become difficult to use. In these situations the transfer matrix method plays a significant role.

In this work, the transfer matrix method was used to model the dynamic response of a damaged Euler-Bernoulli beam (shear deformation and rotary inertia were not considered). The transfer matrix method (TMM) was introduced by Pestel and Leckie (1963). The principle behind this method is to break up a complicated structure into smaller components with simple elastic and dynamic properties which then can be expressed in a matrix form. Field transfer matrices and point transfer matrices are used to determine the state vectors at various locations in the structures. In this way, the eigenvalue and eigenvector problem can be easily solve and can be also incorporated in various computational software (i.e. MATLAB).

Following the method introduced by Pestel and Leckie (1963), a uniform beam of length $l$ can be divided in $N$ intervals as shown in Figure 28. We consider the beam to be under both axial and flexural load.
At each interval the displacements and corresponding internal forces can be grouped in a column vector $\mathbf{z}$, called the state vector:

$$
\mathbf{z}_i = \begin{bmatrix}
u \\
w \\
\varphi \\
N \\
M \\
V
\end{bmatrix}_{i}
$$

(7.1)

where the first three component of the state vector are displacements ($u$ is the axial displacement, $w$ is the flexural displacement, $\varphi$ is the rotation or slope) and the next three components are the internal forces ($N$ is the axial force, $M$ is the bending moment and $V$ is the shear force).
From equilibrium equations it is possible to show the relation between two consecutive state vectors as

$$z^L_i = F_i z^R_{i-1} \quad \text{(7.2)}$$

where the superscript $L$ or $R$ denotes the left or the right side of the beam segment (Figure 29).

![Diagram of state vectors, field and point transformation matrices for divided beam.](image)

The matrix $F_i$ is known as the field transfer matrix and relates the state vectors at the left and right end of one segment.

At each node, it is possible to express the state vector at the right of the node with respect to the vector at the left side of the node. From the equilibrium equation and the continuity conditions at each node we can write

$$z^R_i = P_i z^L_i \quad \text{(7.3)}$$

where matrix $P_i$ relating two adjacent state vectors is called the point transfer matrix.

Using Eq. (7.2) and Eq. (7.3) it is possible to write the following relations between adjacent state vectors

$$z^L_i = F_i z_0, \quad z^R_i = P_i z^L_i, \quad ..., \quad z^L_i = F_i z^R_{i-1}, \quad z^R_i = P_i z^L_i, \quad ..., \quad z_N = F_N z^R_{N-1} \quad \text{(7.4)}$$

and the state vector at the end of the uniform beam can relate to the state vector at the beginning of the beam through a transfer matrix $U$. 

80
\[ z_N = U z_0 \]  

(7.5)

where the transfer matrix \( U \) is

\[ U = F_N \cdot P_{N-1} \cdot F_{N-1} \cdot \ldots \cdot F_2 \cdot P_1 \cdot F_1 \]  

(7.6)

Eq. (7.5) allows for a convenient way to calculate the displacements and forces at one end knowing the state vector at the other end. Certain boundary conditions (i.e. free-free, clamped-clamped, clamped-free, etc) can be considered. Once the state vectors at the ends are fully determined, the state vectors at any location on the beam can be calculated by simply performing matrix operations.

Before we can proceed, the field transfer matrices \( F \) and the point transfer matrices \( P \) need to be determined. Let’s express the state vector as a matrix \( B \) multiplied by a constant column vector \( a \)

\[ z(x) = B(x) a \]  

(7.7)

where

\[ z = [u \ w \ \phi \quad N \quad M \quad V]^T, \quad a = [A_1 \ A_2 \ A_3 \ A_4 \ A_5 \ A_6]^T, \quad B = [6, 6] \]

Recall the general solutions for axial and flexural vibrations of a uniform beam

\[ u(x) = A_1 g_1(x) + A_2 g_2(x) \]  

(7.8)

\[ w(x) = A_3 f_1(x) + A_4 f_2(x) + A_5 f_3(x) + A_6 f_4(x) \]  

(7.9)

where
\[ g_1(x, \omega) = \sin[\gamma_a(\omega)x] \]
\[ g_2(x, \omega) = \cos[\gamma_a(\omega)x] \]
\[ f_1(x, \omega) = \frac{1}{2} \left\{ \cosh[\gamma_f(\omega)x] + \cos[\gamma_f(\omega)x] \right\} \]
\[ f_2(x, \omega) = \frac{1}{2\gamma_f(\omega)} \left\{ \sinh[\gamma_f(\omega)x] + \sin[\gamma_f(\omega)x] \right\} \tag{7.10} \]
\[ f_3(x, \omega) = \frac{1}{2\gamma_f^2(\omega)} \left\{ \cosh[\gamma_f(\omega)x] - \cos[\gamma_f(\omega)x] \right\} \]
\[ f_4(x, \omega) = \frac{1}{2\gamma_f^3(\omega)} \left\{ \sinh[\gamma_f(\omega)x] - \sin[\gamma_f(\omega)x] \right\} \]

\( \gamma_a \) and \( \gamma_f \) are the wavenumbers for the wave-like axial and flexural deformations

\[ \gamma_a(\omega) = \omega \sqrt{\frac{m}{EA}}, \quad \gamma_f(\omega) = \left( \frac{m}{EI} \right)^{1/4} \sqrt{\omega} \tag{7.11} \]

Using Eqs. (7.10), the dynamic matrix \( \mathbf{B} = \mathbf{B}(x, \omega) \) can be expressed as

\[
\mathbf{B} = \begin{bmatrix}
 g_1 & g_2 & 0 & 0 & 0 & 0 \\
 0 & 0 & f_1 & f_2 & f_3 & f_4 \\
 0 & 0 & \frac{d f_1}{dx} & \frac{d f_2}{dx} & \frac{d f_3}{dx} & \frac{d f_4}{dx} \\
 EA \frac{d g_1}{dx} & EA \frac{d g_2}{dx} & 0 & 0 & 0 & 0 \\
 0 & 0 & EI \frac{d^2 f_1}{dx^2} & EI \frac{d^2 f_2}{dx^2} & EI \frac{d^2 f_3}{dx^2} & EI \frac{d^2 f_4}{dx^2} \\
 0 & 0 & EI \frac{d^3 f_1}{dx^3} & EI \frac{d^3 f_2}{dx^3} & EI \frac{d^3 f_3}{dx^3} & EI \frac{d^3 f_4}{dx^3}
\end{bmatrix} \tag{7.12}
\]

where the relations between the flexural displacement, bending moment and shear force are

\[ M = EI \frac{d^2 w}{dx^2}, \quad V = EI \frac{d^3 w}{dx^3} \tag{7.13} \]

From Eq. (7.7) the state vector at the two ends of a segment of length \( l_i \) can be expressed as
\[ z_{i-1} = B(0)a \]  
(7.14)

\[ z_i = B(l_i)a \]  
(7.15)

From Eq. (7.14), the coefficients column vector \( a \) can be calculated

\[ a = B^{-1}(0)z_{i-1} \]  
(7.16)

Substituting Eq. (7.16) into Eq. (7.15) yields

\[ z_i = B(l_i)B^{-1}(0)z_{i-1} \]  
(7.17)

According to Eq. (7.2) we can rewrite Eq. (7.17) as

\[ z_i = F_i z_{i-1} \]

and the field transfer matrix \( F_i \) can be calculated as

\[ F_i = B(l_i)B^{-1}(0) \]  
(7.18)

For a uniform continuous isotropic beam, the point transfer matrix \( P \) is considered the unit matrix \( I \), i.e.,

\[ P_i = I \]  
(7.19)

With the expressions for the field transfer matrices \( M_i \) and point transfer matrices \( P_i \) we can proceed now to study the structural response of pristine or damaged uniform beams.

### 7.2.1 Free vibrations of a uniform beam

Before developing the analytical model using the transfer matrix method introduced above for a multi-layer damaged beam we will first start with a much simpler case, a straight uniform beam of length \( L \), mass per unit length \( m \), Young’s modulus \( E \), and density \( \rho \). This case was analyzed in Chapter 5 using modal expansion method, and is reanalyzed here for comparison using the transfer matrix method. A schematic of the beam showing the state vectors and the free-body diagram is presented in Figure 30.
Starting from the left-end of the beam the relations between the state vectors are

$$z_1^R = P_1 z_{BC}^L \quad z_2^L = F_1 z_1^R \quad z_{BC}^R = P_2 z_2^L$$

(7.20)

From the above relations, the state vector at the right-end of the beam can be expressed in terms of the state vector at the left-end of the beam as

$$z_{BC}^R = U z_{BC}^L \quad \text{where} \quad U = P_2 F_1 P_1$$

(7.21)

The field transfer matrix $F_1$ is calculated using Eq. (7.18) for $x = 0$ and for $x = L$.

$$F_1(\omega) = B(L, \omega)B^{-1}(0, \omega)$$

(7.22)

To find out the point transformation matrices $P_1$ and $P_2$ we will use the equilibrium equation and the compatibility conditions at node 1 and node 2.
\[ u_i^R = u_{BC}^L \]
\[ w_i^R = w_{BC}^L \]
\[ \varphi_i^R = \varphi_{BC}^L \]
\[ N_i^R = N_{BC}^L \]
\[ M_i^R = M_{BC}^L \]
\[ V_i^R = V_{BC}^L \]

\[ \begin{bmatrix} u^R \\ w \\ \varphi \\ N \\ M \\ V \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^L \\ w \\ \varphi \\ N \\ M \\ V \end{bmatrix}_{BC} \Rightarrow P_1 = I \tag{7.24} \]

@ node #2

\[ u_{BC}^R = u_2^L \]
\[ w_{BC}^R = w_2^L \]
\[ \varphi_{BC}^R = \varphi_2^L \]
\[ N_{BC}^R = N_2^L \]
\[ M_{BC}^R = M_2^L \]
\[ V_{BC}^R = V_2^L \]

\[ \begin{bmatrix} u^R \\ w \\ \varphi \\ N \\ M \\ V \end{bmatrix}_{BC} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^L \\ w \\ \varphi \\ N \\ M \\ V \end{bmatrix}_2 \Rightarrow P_2 = I \tag{7.26} \]

Eq. (7.21) can be rewritten

\[ \begin{bmatrix} d_{BC}^g \\ p_{BC}^g \end{bmatrix} = \begin{bmatrix} U_{dd} & U_{dp} \\ U_{pd} & U_{pp} \end{bmatrix} \begin{bmatrix} d_{BC}^L \\ p_{BC}^L \end{bmatrix} \tag{7.27} \]
where the displacement and internal forces vectors are \( \mathbf{d} = \begin{bmatrix} u \\ w \\ \varphi \end{bmatrix} \) and \( \mathbf{p} = \begin{bmatrix} N \\ M \\ V \end{bmatrix} \).

From Eq. (7.27) we get the following system of equations

\[
\begin{align*}
\mathbf{d}^R_{BC} &= U_{dd} \mathbf{d}^L_{BC} + U_{dp} \mathbf{p}^L_{BC} \\
\mathbf{p}^R_{BC} &= U_{pd} \mathbf{d}^L_{BC} + U_{pp} \mathbf{p}^L_{BC}
\end{align*}
\]  

(7.28)

Applying the free-free boundary conditions

at \( x=0 \) : 
\[
N^L_{bc} = M^L_{bc} = V^L_{bc} = 0
\]  

at \( x=0 \) : 
\[
N^R_{bc} = M^R_{bc} = V^R_{bc} = 0
\]  

(7.29)

Eq. (7.29) can also be written as

\[
\mathbf{p}^L_{BC} = \mathbf{p}^R_{BC} = 0
\]  

(7.30)

The system of equations (7.28) becomes

\[
\begin{align*}
\mathbf{d}^R_{BC} &= U_{dd} \mathbf{d}^L_{BC} \\
0 &= U_{pd} \mathbf{d}^L_{BC} \\
\leftrightarrow \mathbf{U}_{dd} \mathbf{d}^L_{BC} - \mathbf{I} \mathbf{d}^L_{bc} = 0
\end{align*}
\]  

(7.31)

The system of equations (7.31) is a linear homogeneous system and has a unique non-trivial solution if and only if its determinant is non-zero. This is an eigenvalue problem and the solutions of the homogeneous system of equations yields the eigenvalues \( \omega_i \) (natural circular frequencies)

\[
\begin{vmatrix}
U_{dd} & -\mathbf{I} \\
U_{pd} & 0
\end{vmatrix} = 0 \quad \Rightarrow \quad \omega_i
\]  

(7.32)

The eigenvalues are calculated using a frequency search algorithm. For each natural frequency the eigenvectors can be calculated using the singular value decomposition (SVD) method. In this way, the state vector for the left-side of the beam (boundary conditions) is fully determined. i.e.,
Knowing the state vector $z^{BC}_i$, the coefficient vector $a$ can be calculated as

$$z^{BC}_L = \mathbf{B}(0, \omega) \cdot a \quad \Rightarrow \quad a = \mathbf{B}^{-1}(0, \omega) \cdot z^{BC}_L$$  \hspace{1cm} (7.34)

With the coefficients $A_1 \ldots A_6$ known the mode shapes for the axial and flexural vibrations can be computed.

$$u(x) = A_1 g_1(x) + A_2 g_2(x)$$  \hspace{1cm} (7.35)

$$w(x) = A_3 f_1(x) + A_4 f_2(x) + A_5 f_3(x) + A_6 f_4(x)$$  \hspace{1cm} (7.36)

To validate our method to calculate the natural frequencies for free vibrations of a uniform beam, the results obtained using the transfer matrix method were compared against the exact solution of modal analysis, Eq. (6.13) and Eq. (6.38). We considered a steel beam with the Young’s modululs $E = 200$ GPa, density $\rho = 7750$ kg/m$^3$, length $l = 100$ mm, width $b = 8$ mm and thickness $h = 2.6$ mm. The natural frequencies obtained using the two methods are presented in Table 2.

Table 2 Natural frequencies comparison for a uniform beam

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Modal analysis</th>
<th>Transfer matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1357.7</td>
<td>1357.6</td>
</tr>
<tr>
<td>2</td>
<td>3742.5</td>
<td>3742.4</td>
</tr>
<tr>
<td>3</td>
<td>7336.8</td>
<td>7336.8</td>
</tr>
<tr>
<td>4</td>
<td>12128</td>
<td>12128</td>
</tr>
<tr>
<td>5</td>
<td>18117</td>
<td>18117</td>
</tr>
<tr>
<td>6</td>
<td>25304</td>
<td>25304</td>
</tr>
<tr>
<td>7</td>
<td>25400</td>
<td>25400</td>
</tr>
<tr>
<td>8</td>
<td>33689</td>
<td>33689</td>
</tr>
</tbody>
</table>
From the Table 2, we can see a very good agreement between the numerically simulated results obtained using the transfer matrix method and those obtained through classical modal analysis. These results confirm our confidence in being able to predict the natural frequencies of vibrations using the transfer matrix approach, a method that is suitable for modeling more complex structures with damage.

7.2.2 Forced vibration of a uniform beam

The forced vibration analysis of straight uniform beam of length $L$, mass per unit length $m$, Young’s modulus $E$, and density $\rho$ can be performed in as follows. For illustration we will consider the excitation $F$ to be an offset horizontal force applied to the beam at the two ends as shown in Figure 31. The offset excitation force $F$ will create at the neutral axis an axial force $N$ and a bending moment $M$.

![Figure 31 Forced vibration of a uniform beam: (a) schematic of beam elements; (b) free-body diagram](image)

We will start the analysis of forced vibrations of a uniform beam by assuming a value for $\omega$ and finding the field transfer matrix and the point transfer matrices. First the field transfer matrix can be expressed as

$$\mathbf{F}(\omega) = \mathbf{B}(L, \omega) \mathbf{B}^{-1}(0, \omega) \quad (7.37)$$
The next step is to calculate the point transformation matrices $P_1$ and $P_2$. To do this, at each node we will impose the compatibility conditions and apply the equilibrium equations.

@ node #1

Compatibility conditions

$$u_1^R = u_{bc}^L$$
$$w_1^R = w_{bc}^L$$
$$\varphi_1^R = \varphi_{bc}^L$$ \hspace{1cm} (7.38)

Equilibrium equations

$$\sum F_x : \quad N_1^R = N_{bc}^L + N_1^f$$ \hspace{1cm} (7.39)

$$\sum M : \quad M_1^R = M_{bc}^L + M_1^f$$ \hspace{1cm} (7.40)

$$\sum F_y : \quad V_1^R = V_{bc}^L$$ \hspace{1cm} (7.41)

In matrix form Eqs. (7.38) through (7.41) can be written as

$$\begin{bmatrix}
\mathbf{u}^R \\
\mathbf{w} \\
\mathbf{\varphi} \\
\mathbf{N} \\
\mathbf{M} \\
\mathbf{V}_1
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}^L \\
\mathbf{w} \\
\mathbf{\varphi} \\
\mathbf{N} \\
\mathbf{M} \\
\mathbf{V}_{bc}
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}$$ \hspace{1cm} (7.42)

$$\mathbf{z}_1^R = \mathbf{P}_1 \cdot \mathbf{z}_{bc}^L + \mathbf{P}_1^f$$ \hspace{1cm} (7.43)
where $P_1 = I$ and the forcing term $P_1^F$ is $P_1^F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ N_1^F \\ M_1^F \\ 0 \end{bmatrix}$.

In a similar manner

@ node #2

Compatibility conditions

\[ u^R_{BC} = u^L_2 \]
\[ w^R_{BC} = w^L_2 \]
\[ \varphi^R_{BC} = \varphi^L_2 \] (7.44)

Equilibrium equations

\[ \sum F_x : \quad N^R_{BC} = N^L_2 - N^F_2 \] (7.45)

\[ \sum M : \quad M^R_{BC} = M^L_2 - M^F_2 \] (7.46)

\[ \sum F_y : \quad V^R_{BC} = V^L_2 \] (7.47)

In matrix form Eqs. (7.44) through (7.47) can be written as

\[
\begin{bmatrix}
\begin{array}{c}
\mathbf{u}_{BC} \\
\mathbf{w} \\
\mathbf{\varphi} \\
\mathbf{N} \\
\mathbf{M} \\
\mathbf{V}
\end{array}
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}^L \\
\mathbf{w} \\
\mathbf{\varphi} \\
\mathbf{N} \\
\mathbf{M} \\
\mathbf{V}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
N^F_1 \\
-M^F_1 \\
0
\end{bmatrix}
\] (7.48)

\[ z^R_{BC} = P_2 \cdot z^L_2 + P^F_2 \] (7.49)
where \( P_2 = I \) and the forcing term is \( P_2^F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -N_i^F \\ -M_i^F \\ 0 \end{bmatrix} \)

The following relations between state vectors can be written

\[
\begin{align*}
  z_i^R &= P_1 \cdot z^L_{BC} + P_1^F; \\
  z_2^L &= F_1 \cdot z_i^R; \\
  z_i^R &= P_2 \cdot z_2^L + P_2^F
\end{align*}
\] (7.50)

hence, the state vector at the right-end of the beam can be expressed as

\[
\begin{align*}
  z_{BC}^R &= P_2 \cdot z_2^L + P_2^F \\
          &= P_2 \cdot F_1 \cdot z_1^R = P_2 \cdot F_1 \cdot \left( P_1 \cdot z_{BC}^L + P_1^F \right)
\end{align*}
\] (7.51)

\[
\begin{align*}
  z_{BC}^R &= U \cdot z_2^L + T
\end{align*}
\] (7.52)

where \( U = P_2 \cdot F_1 \cdot P_1 \); \( T = P_2 \cdot F_1 \cdot P_1^F + P_2^F \)

Using the displacement and internal forces vectors \( d \) and \( p \), Eq. (7.52) can be expanded as

\[
\begin{bmatrix}
  d_{BC}^R \\
  p_{BC}^R
\end{bmatrix} =
\begin{bmatrix}
  U_{dd} & U_{dp} \\
  U_{pd} & U_{pp}
\end{bmatrix}
\begin{bmatrix}
  d^L_{BC} \\
  p^L_{BC}
\end{bmatrix} +
\begin{bmatrix}
  T_d \\
  T_p
\end{bmatrix}
\] (7.53)

Applying the free-free boundary conditions:

at \( x = 0 \): \( N^L_{BC} = M^L_{BC} = V^L_{BC} = 0 \) \( \Rightarrow \) \( p^{L}_{BC} = 0 \)

at \( x = L \): \( N^R_{BC} = M^R_{BC} = V^R_{BC} = 0 \) \( \Rightarrow \) \( p^{R}_{BC} = 0 \)

Eq. (7.53) can be rewritten as

\[
\begin{bmatrix}
  d_{BC}^R \\
  0
\end{bmatrix} =
\begin{bmatrix}
  U_{dd} & U_{dp} \\
  U_{pd} & U_{pp}
\end{bmatrix}
\begin{bmatrix}
  d_{BC}^L \\
  0
\end{bmatrix} +
\begin{bmatrix}
  U_{dd} & U_{dp} \\
  U_{pd} & U_{pp}
\end{bmatrix}
\begin{bmatrix}
  0 \\
  p_{1}^F + p_2^F
\end{bmatrix}
\] (7.54)

Which will lead to the following non-homogeneous system of equations

\[
\begin{bmatrix}
  d_{bc}^R = U_{dd} \cdot d_{bc}^L + U_{dp} \cdot p_1^F \\
  0 = U_{pd} \cdot d_{bc}^L + U_{pp} \cdot p_1^F + p_2^F
\end{bmatrix}
\] (7.55)
where the unknowns are the two displacement vectors $\mathbf{d}_{BC}^L$ and $\mathbf{d}_{BC}^R$. The system of equations (7.55) can be manipulated and rewritten in matrix form as

$$
\begin{bmatrix}
-U_{dd} & \mathbf{I} \\
-U_{pd} & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{d}_{BC}^L \\
\mathbf{d}_{BC}^R
\end{bmatrix}
= 
\begin{bmatrix}
U_{dp} & 0 \\
U_{pp} & \mathbf{I}
\end{bmatrix}
\begin{bmatrix}
\mathbf{p}_1^F \\
\mathbf{p}_2^F
\end{bmatrix}
$$

or in a more general case

$$
\mathbf{M} \cdot \mathbf{X} = \mathbf{N} \tag{7.57}
$$

The non-homogeneous system of equations (7.57) can be solved for the unknowns $\mathbf{d}_{BC}^L$ and $\mathbf{d}_{BC}^R$. Once they are found, the state vectors $\mathbf{z}_{BC}^L$ and $\mathbf{z}_{BC}^R$ can be calculated as

$$
\mathbf{z}_{BC}^L = \begin{bmatrix} \mathbf{d}_{BC}^L \\ \mathbf{p}_{BC}^L \end{bmatrix}; \quad \mathbf{z}_{BC}^R = \begin{bmatrix} \mathbf{d}_{BC}^R \\ \mathbf{p}_{BC}^R \end{bmatrix} \tag{7.58}
$$

Knowing the state vector $\mathbf{z}_{BC}^L$ the coefficient vector $\mathbf{a}$ can be calculated as

$$
\mathbf{z}_{BC}^L = \mathbf{B}(0, \omega) \cdot \mathbf{a} \quad \Rightarrow \quad \mathbf{a} = \mathbf{B}^{-1}(0, \omega) \cdot \mathbf{z}_{BC}^L = \begin{bmatrix} A_1 \\ \vdots \\ A_6 \end{bmatrix} \tag{7.59}
$$

The process is repeated for the other $\omega$ values over the frequency interval of interest.

### 7.3 Transfer matrix method for multi-layer disbonded beams

In the previous section, we developed a mathematical model to study the force vibration of a uniform beam using the transfer matrix method. Our goal is to further develop the mathematical model to be able to predict the structural response of multi-layer structures (i.e. bonded layers), calculate the frequency response function, and find the electromechanical impedance.
7.3.1 Modeling of multi-layer beams

We considered two uniform beams bonded together using an epoxy adhesive creating a one multi-layer beam under the Euler-Bernoulli assumption (no shear deformations and no rotary inertia terms). On top of the beam, we attached a PWAS transducer which will apply a force $F_P$ on the structure.

For the analysis, we considered ideal bonding between the PWAS and the structure (pin-force model). The first step is to find the location of the elastic center for the multi-layer beam. Once the location of the elastic center is found, all of the multi-layer beam material properties will be expressed with respect to the elastic center. To better understand the problem let’s consider a schematic of a beam element (Figure 33):
Considering no axial force, only bending, we can write

\[ \int_{D} \sigma dA = 0 \]  \hspace{1cm} (7.60)

Using the Hooke’s law

\[ \sigma = E \varepsilon = E \kappa y \]  \hspace{1cm} (7.61)

where the curvature \( \kappa = \frac{1}{r} \)

Eq. (7.60) becomes

\[ \int_{D} E \kappa y dA = \int_{D} E \kappa y b dy = b \kappa \int_{y_{min}}^{y_{max}} E y dy = 0 \]  \hspace{1cm} (7.62)

Since both the width \( b \) and the curvature \( \kappa \) are non-zero, we must have

\[ \int_{y_{min}}^{y_{max}} E y dy = 0 \]  \hspace{1cm} (7.63)

From Figure 33, the arbitrary location \( y \) of the element \( dy \) can be expressed as

\[ y = y^* - \bar{y} \]  \hspace{1cm} (7.64)

where \( \bar{y} \) is the unknown location of the elastic center.

Hence Eq. (7.63) becomes
\[ \int E\left(y^* - \bar{y}\right)dy = 0 \]  \hspace{1cm} (7.65)

or

\[ \int Ey^*dy^* - \bar{y}\int Edy^* = 0 \]  \hspace{1cm} (7.66)

The location of the elastic center can now be calculated as

\[ \bar{y} = \frac{\sum \int Ey^*dy^*}{\sum \int Edy^*} \]  \hspace{1cm} (7.67)

The flexural rigidity \( EI \) can be expressed with respect to the elastic center. To do this we start from the bending moment formula

\[ M = \int_A y\sigma dA \]  \hspace{1cm} (7.68)

Substituting Eq. (7.61) into Eq. (7.68)

\[ M = b\kappa \int Ey^2 dy \]  \hspace{1cm} (7.69)

The stress can also be expressed as

\[ \sigma = \frac{My}{I}; \quad E\varepsilon = \frac{My}{I}; \quad E\kappa = \frac{My}{I} \Rightarrow M = EI\kappa \]  \hspace{1cm} (7.70)

From Eq. (7.69) and Eq. (7.70)

\[ (EI)_{eq} \kappa = b\kappa \int Ey^2 dy \]  \hspace{1cm} (7.71)

And the expression for the equivalent flexural rigidity for a multi-layer beam becomes

\[ (EI)_{eq} = b\sum \int Ey^2 dy \]  \hspace{1cm} (7.72)

In a similar way the equivalent axial stiffness can be expressed as

\[ (EA)_{eq} = b\sum \int Edy \]  \hspace{1cm} (7.73)

and the equivalent mass is
\[ m_{eq} = \sum \rho A \quad (7.74) \]

The field transfer matrices for each interval (see Figure 32) can be calculated according to Eq. (7.18) as

\[ F_i(\omega) = B_{eq}(L_i, \omega)B_{eq}^{-1}(0, \omega) \quad (7.75) \]

where \( B_{eq} \) is the equivalent dynamic matrix given by Eq. (7.12) using the equivalent properties derived above.

To determine the point transfer matrices we will apply the compatibility conditions and the equilibrium equations to each node.

@ node #1

Compatibility conditions

\[
\begin{align*}
    u_1^R &= u_{BC}^L \\
    w_1^R &= w_{BC}^L \\
    \varphi_1^R &= \varphi_{BC}^L \\
\end{align*}
\quad (7.76)
\]

Equilibrium equations

\[
\begin{align*}
    \sum F_x : & \quad N_1^R = N_{BC}^L \\
    \sum M : & \quad M_1^R = M_{BC}^L \\
    \sum F_y : & \quad V_1^R = V_{BC}^L \\
\end{align*}
\quad (7.77, 7.78, 7.79)
\]

In matrix for Eqs. (7.76) through (7.79) can be written as

\[
\begin{bmatrix}
    u \\
    w \\
    \varphi \\
    N \\
    M \\
    V
\end{bmatrix}^R =
\begin{bmatrix}
    1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    u \\
    w \\
    \varphi \\
    N \\
    M \\
    V
\end{bmatrix}^L
\quad (7.80)
\]
\[ z_i^R = P_1 \cdot z_{bc}^L, \text{ where } P_1 = I \]  

@ node #2

Compatibility conditions

\[ u_2^R = u_2^L \]
\[ w_2^R = w_2^L \]
\[ \varphi_2^R = \varphi_2^L \]  

(7.82)

Equilibrium equations

\[ \sum F_x : \]
\[ N_2^R = N_2^L + N^F \]  

(7.83)

\[ \sum M : \]
\[ M_2^R = M_2^L + M^F \]  

(7.84)

\[ \sum F_y : \]
\[ V_2^R = V_2^L \]  

(7.85)

In matrix form Eqs. (7.82) through (7.85) can be written as

\[
\begin{bmatrix}
  u^R \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}_{2} =
\begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  u^L \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}_{2} +
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\]

(7.86)

\[ z_2^R = P_2 \cdot z_2^L + P_1^F \]  

(7.87)

where \( P_2 = I \) and the forcing term \( P_1^F \) is

\[
P_1^F =
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  N^F \\
  M^F \\
  0
\end{bmatrix}
\]

@ node #3

Compatibility conditions
Equilibrium equations

\[ \sum F_x: \quad N_3^R = N_3^L - N^F \quad (7.89) \]
\[ \sum M: \quad M_3^R = M_3^L - M^F \quad (7.90) \]
\[ \sum F_y: \quad V_3^R = V_3^L \quad (7.91) \]

In matrix for Eqs. (7.88) through (7.91) can be written as

\[
\begin{bmatrix}
  u^R \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}
= \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  u^L \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}
+ \begin{bmatrix}
  0 \\
  0 \\
  -N^F \\
  -M^F \\
  0 \\
  0
\end{bmatrix} \quad (7.92)
\]

\[ z_3^R = P_3 \cdot z_3^L + P_2^F \quad (7.93) \]

where \( P_3 = I \) and the forcing term is \( P_2^F = \)

\[
\begin{bmatrix}
  0 \\
  0 \\
  0 \\
  -N^F \\
  -M^F \\
  0
\end{bmatrix}
\]

@ node #4

Compatibility conditions

\[ u_{BC}^R = u_4^L \]
\[ w_{BC}^R = w_4^L \]
\[ \varphi_{BC}^R = \varphi_4^L \quad (7.94) \]

Equilibrium equations
\[ \sum F_x : \quad N_{BC}^R = N_4^L \quad (7.95) \]
\[ \sum M : \quad M_{BC}^R = M_4^L \quad (7.96) \]
\[ \sum F_y : \quad V_{BC}^R = V_4^L \quad (7.97) \]

In matrix for Eqs. (7.76) through (7.79) can be written as

\[
\begin{bmatrix}
  u^R \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}_{BC}
= \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  u^L \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}_{4}
\]

\[ z_{BC}^R = P_4 z_4^L, \text{ where } P_4 = I \quad (7.99) \]

In order to be able to calculate the frequency response function for the forced vibration of a multi-layer beam we need to determine the unknown state vectors: \( z_{BC}^L, z_1^R, z_2^R, z_3^R \) and \( z_{BC}^R \).

Next we will applying the free-free boundary conditions:

at \( x = 0 \): \( N_{BC}^L = M_{BC}^L = V_{BC}^L = 0 \quad \Rightarrow \quad p_{BC}^L = 0 \quad (7.100) \)

at \( x = L \): \( N_{BC}^R = M_{BC}^R = V_{BC}^R = 0 \quad \Rightarrow \quad p_{BC}^R = 0 \quad (7.101) \)

Considering the free-free boundary conditions, the corresponding unknown displacements and internal forces vectors are: \( d_{BC}^L, d_1^R, p_1^R, d_2^R, p_2^R, d_3^R, p_3^R, d_{BC}^R \). But from Eq. (7.81), \( p_1^R = p_{BC}^L = 0 \) and hence we will have seven unknowns \( d_{BC}^L, d_1^R, d_2^R, p_2^R, d_3^R, p_3^R, d_{BC}^R \) which requires seven equations in order to solve for all the unknowns.
To simplify the mathematical expressions since all the point transfer matrices $P$ are equal with the unity matrix $I$ we will drop them from the future calculations. From the equations (7.81), (7.87), (7.93), and (7.99) we obtain the following system of equations

\[
\begin{align*}
 z_1^R &= z_{BC}^L \\
 z_2^R &= z_2^L + P_1^F \\
 z_3^R &= z_3^L + P_2^F \\
 z_{BC}^R &= z_4^L
\end{align*}
\]  

(7.102)

which can be further expanded into

\[
\begin{align*}
 d_1^R - d_1^F &= 0 \\
 F_{3}^{dd} \cdot d_3^R + F_{3}^{ddp} \cdot p_3^R - d_3^B &= 0 \\
 F_{3}^{dp} \cdot d_3^R + F_{3}^{pp} \cdot p_3^R &= 0 \\
 F_{1}^{dp} \cdot d_1^R - d_2^R &= 0 \\
 F_{1}^{dp} \cdot d_1^R - p_2^- &= -p_1^- \\
 F_{2}^{dp} \cdot d_2^R + F_{2}^{pp} \cdot p_2^R - d_3^- &= 0 \\
 F_{2}^{dp} \cdot d_2^R + F_{2}^{pp} \cdot p_2^R - p_3^- &= -p_2^-
\end{align*}
\]  

(7.103)

The system of equations (7.103) is a non-homogeneous system of equations and can be written in a matrix form as

\[
\mathbf{M} \cdot \mathbf{X} = \mathbf{N}
\]  

(7.104)

where matrices $\mathbf{M}$ and $\mathbf{N}$ are derived in Eq. (A.2) of Appendix A, and the unknown column vector $\mathbf{X}$ is

\[
\mathbf{X} = \begin{bmatrix} d_{BC}^L & d_1^R & d_2^R & p_2^R & d_3^R & p_3^R & d_{BC}^R \end{bmatrix}^T
\]  

(7.105)

From Eq. (7.104), the unknown column vector of displacements and internal forces can be solved

\[
\mathbf{X} = \mathbf{M}^{-1} \cdot \mathbf{N}
\]  

(7.106)
7.3.2 FRF of pristine multi-layer beam under PWAS excitation

With the axial and flexural displacements $u$ and $w$ and the rotation $w'$ calculated, the total displacement between the two ends of a PWAS attached to the beam at the distances $x_1$ and $x_2$ from the left end can be calculated according to Eq. (6.57), i.e.,

$$u_p = u(x_2) - u(x_1) - \frac{h}{2} [w'(x_2) - w'(x_1)]$$

or in terms of the state vectors components

$$u_p = \left( u^L - u^R \right) - \frac{h}{2} \left( \varphi^L - \varphi^R \right)$$  \hspace{1cm} (7.107)

Dividing Eq. (7.107) by the excitation force $F$, the frequency response function $FRF(\omega)$ can be expressed as

$$\frac{u_p}{F} = \left( \frac{u^L - u^R}{F} \right) - \frac{h}{2} \left( \frac{\varphi^L - \varphi^R}{F} \right)$$  \hspace{1cm} (7.108)

$$FRF(\omega) = FRF_u(\omega) - FRF_w(\omega)$$  \hspace{1cm} (7.109)

where

$$FRF_u(\omega) = \left( \frac{u^L - u^R}{F} \right); \text{ and } FRF_w(\omega) = \frac{h}{2} \left( \frac{\varphi^L - \varphi^R}{F} \right)$$

The dynamic structural stiffness of the multi-layer beam can be calculated as

$$k_{str}(\omega) = \frac{1}{FRF(\omega)}$$  \hspace{1cm} (7.110)

Knowing the material properties and the dimensions of the PWAS the electromechanical impedance can be expressed as shown in Eq. (6.62) through Eq. (6.64), i.e.,
\[ Z = \frac{1}{Y} = \frac{1}{i\omega C} \left[ 1 - \kappa_{31}^2 \left( 1 - \frac{1}{\varphi \cot \varphi + r} \right) \right]^{-1} \]  

(7.111)

where \( \varphi = \frac{2\gamma_p E_p}{l} \), \( \gamma_p = \omega \rho_p s_{11}^E \)

Since there is no classical vibration analytical expression to predict the vibration response of a multi-layer uniform beam we will compare the results for a uniform beam under PWAS excitation against the transfer matrix method for a multi-layer beam under PWAS excitation when the material properties for each layer are the same.

We will consider the same uniform beam as in Section 6.2.1. with the following material properties: Young’s modulus \( E = 200 \) GPa, density \( \rho = 7750 \) kg/m\(^3\), length \( l = 100 \) mm, width \( b = 8 \) mm and thickness \( h = 2.6 \) mm. The location of the sensor from the left end of the beam is 40 mm. The material properties for the PWAS (APC-850) are presented in Table 3.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compliance, in plane</td>
<td>( s_{11}^E )</td>
<td>( 15.30 \cdot 10^{-12} ) Pa(^{-1})</td>
</tr>
<tr>
<td>Compliance, thickness wise</td>
<td>( s_{33}^E )</td>
<td>( 17.30 \cdot 10^{-12} ) Pa(^{-1})</td>
</tr>
<tr>
<td>Dielectric constant</td>
<td>( \varepsilon_{13}^T )</td>
<td>( 1750 \cdot \varepsilon_0 ); ( \varepsilon_0 = 8.85 \cdot 10^{-12} ) F/m</td>
</tr>
<tr>
<td>Thickness wise induced strain coefficient</td>
<td>( d_{33} )</td>
<td>( 400 \cdot 10^{-12} ) m/V</td>
</tr>
<tr>
<td>In-plane induced strain coefficient</td>
<td>( d_{31} )</td>
<td>( -175 \cdot 10^{-12} ) m/V</td>
</tr>
<tr>
<td>Coupling factor, parallel to electric field</td>
<td>( k_{33} )</td>
<td>0.72</td>
</tr>
<tr>
<td>Coupling factor, transverse to electric field</td>
<td>( k_{31} )</td>
<td>0.36</td>
</tr>
<tr>
<td>Poisson ratio</td>
<td>( \nu )</td>
<td>0.35</td>
</tr>
<tr>
<td>Density</td>
<td>( \rho )</td>
<td>7700 kg/m(^3)</td>
</tr>
</tbody>
</table>
The excitation frequency range was 1 kHz to 30 kHz. A hysteretic structural damping $g = 1\%$ was considered. The results between the transfer matrix method and the classical modal analysis are shown in Figure 34. It can be observed a very good match for both frequency response function as well as the real part of the electromechanical impedance spectrum. It can also be seen from the plots that the resonant frequencies calculated using the transfer matrix method for multi-layer beams when the material properties are the same for each layer are also in agreement with the theoretical values for uniform beams (see Table 2).
Figure 34  Comparison between the transfer matrix method and modal analysis: (top) frequency response function; (bottom) real part of electromechanical impedance
In view of these results the advantages of the transfer matrix method over the modal analysis method are very clear. It was proven that with a much simpler method requiring less mathematical calculations it is possible to get accurate predictions about the structural response of complicated and complex structures.

7.3.3 Modeling of multi-layer cracked beams

The methodology to model a damaged multi-layer beam is almost the same as previously demonstrated with the difference that we have to divide the beam in several strategic segments. In addition, the disbond damage will create a discontinuity in the material which will split the beam in two branches as illustrated in Figure 35.
Going forward with the calculations we will use the following notations: the top part of the beam will be denoted with a \( t \) as a subscript and the bottom part with a \( b \) as a subscript (i.e. \( z_{3t}^{L} \) and \( z_{3b}^{L} \)).

For each of the beam segments, the location of the elastic center and the equivalent material properties need to be calculated as presented in previous section. Once they are calculated, for a given frequency range, the matrix \( B_{eq} \) can be determined for each beam segment using Eq. (7.12). The field transfer matrices \( F \) can now be calculated using

\[
F(\omega) = B_{eq}(L, \omega)B_{eq}^{-1}(0, \omega)
\]

The next step is to determine the point transfer matrices \( P \) at each node of the beam. To do this we will use the compatibility conditions and the equilibrium equations at each node along the beam.

@ node #1

After applying the compatibility conditions and the equilibrium equations the relation between the state vector at the right of the node #1 and the state vector at the left of the node (the boundary conditions) can be expressed in matrix form as

\[
\begin{bmatrix}
  u \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}_{R} =
\begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  u \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}_{L}
\]

(7.112)

\[
\begin{bmatrix}
  u \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}_{L} = P_{i} \cdot \begin{bmatrix}
  u \\
  w \\
  \varphi \\
  N \\
  M \\
  V
\end{bmatrix}_{BC}
\]

(7.113)

@ node #2
Compatibility conditions

top

\[
\begin{align*}
u_{2t}^R &= u_2^L - r_1 \phi_2^L \\
w_{2t}^g &= w_2^L ; \quad d_{2t}^g = D_{2t} \cdot d_2^L \quad \text{where} \quad D_{2t} = \begin{bmatrix} 1 & 0 & -r_1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\phi_{2t}^g &= \phi_2^L
\end{align*}
\] (7.114)

bottom

\[
\begin{align*}
u_{2b}^g &= u_2^L + r_2 \phi_2^L \\
w_{2b}^g &= w_2^L ; \quad d_{2b}^g = D_{2b} \cdot d_2^L \quad \text{where} \quad D_{2b} = \begin{bmatrix} 1 & 0 & r_2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\phi_{2b}^g &= \phi_2^L
\end{align*}
\] (7.115)

Equilibrium equations

\[
\begin{align*}
\sum F_x : & \quad N_2^L = N_{2t}^R + N_{2b}^R \\
\sum M : & \quad M_2^L = -r_1 N_{2t}^R + M_{2t}^R + r_2 N_{2b}^R + M_{2b}^R \\
\sum F_y : & \quad V_2^L = V_{2t}^R + V_{2b}^R
\end{align*}
\] (7.116-7.118)

Or in matrix form

\[
p_2^L = p_{2t}^R + p_{2b}^R ; \quad \text{where} \quad p_{2t} = \begin{bmatrix} 1 & 0 & 0 \\ -r_1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad p_{2b} = \begin{bmatrix} 1 & 0 & 0 \\ r_2 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \]
\] (7.119)

@ node #3

For the top segment we also need to take into account the force applied by the PWAS onto the structure. After applying the compatibility conditions and the equilibrium equations the following matrix equation can be written
\[
\begin{bmatrix}
\mathbf{u}^R \\
\mathbf{w} \\
\mathbf{\varphi} \\
\mathbf{N} \\
\mathbf{M} \\
\mathbf{V}
\end{bmatrix}_{3t} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}^L \\
\mathbf{w} \\
\mathbf{\varphi} \\
\mathbf{N} \\
\mathbf{M} \\
\mathbf{V}
\end{bmatrix}_{3t} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]  

(7.120)

\[
\mathbf{z}^R_{3t} = \mathbf{P}_{3t} \cdot \mathbf{z}^L_{3t} + \mathbf{P}^F_1
\]  

(7.121)

where \( \mathbf{P}_{3t} = \mathbf{I} \) and the forcing term \( \mathbf{P}^F_1 \) is \( \mathbf{P}^F_1 = \)

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
N^F \\
M^F \\
0
\end{bmatrix}
\]

For the bottom segment the PWAS has no influence and the state vector to the right of the node is the same with the state vector at the left of the node as expressed by the following equation

\[
\mathbf{z}^R_{3b} = \mathbf{P}_{3b} \cdot \mathbf{z}^L_{3b} ; \quad \text{where} \quad \mathbf{P}_{3b} = \mathbf{I}
\]  

(7.122)

@ node #4

Similar to node # 3 we can write the compatibility and equilibrium equations for the top part of the beam as

\[
\begin{bmatrix}
\mathbf{u}^R \\
\mathbf{w} \\
\mathbf{\varphi} \\
\mathbf{N} \\
\mathbf{M} \\
\mathbf{V}
\end{bmatrix}_{4t} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}^L \\
\mathbf{w} \\
\mathbf{\varphi} \\
\mathbf{N} \\
\mathbf{M} \\
\mathbf{V}
\end{bmatrix}_{4t} +
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-N^F \\
M^F \\
0
\end{bmatrix}
\]  

(7.123)

\[
\mathbf{z}^R_{4t} = \mathbf{P}_{4t} \cdot \mathbf{z}^L_{4t} + \mathbf{P}^F_2
\]  

(7.124)
where $P_{4l} = I$ and the forcing term $P_{2}^{F}$ is $P_{2}^{F} =  \begin{bmatrix} 0 \\ 0 \\ 0 \\ -N^{F} \\ M^{F} \\ 0 \end{bmatrix}$

For the bottom segment we have

$$z_{4b}^{R} = P_{4b} \cdot z_{4b}^{L}; \quad \text{where} \quad P_{4b} = I$$  \hspace{1cm} (7.125)

@ node #5

Compatibility conditions

**top**

$$\begin{align*}
  u_{5t}^{L} &= u_{5t}^{R} - r_{1} \varphi_{5t}^{R} \\
  w_{5t}^{L} &= w_{5t}^{R} \\
  \varphi_{5t}^{L} &= \varphi_{5t}^{R}
\end{align*}$$

where $D_{5t} = \begin{bmatrix} 1 & 0 & -r_{1} \\ 0 & 1 & 0 \end{bmatrix}$ \hspace{1cm} (7.126)

**bottom**

$$\begin{align*}
  u_{5b}^{L} &= u_{5b}^{R} + r_{2} \varphi_{5b}^{R} \\
  w_{5b}^{L} &= w_{5b}^{R} \\
  \varphi_{5b}^{L} &= \varphi_{5b}^{R}
\end{align*}$$

where $D_{5b} = \begin{bmatrix} 1 & 0 & r_{2} \\ 0 & 1 & 0 \end{bmatrix}$ \hspace{1cm} (7.127)

Equilibrium equations

$$\begin{align*}
  \sum F_{x} & : & N_{5}^{R} &= N_{5t}^{L} + N_{5b}^{L} \\
  \sum M & : & M_{5}^{R} &= -r_{1} N_{5t}^{L} + M_{5t}^{L} + r_{2} N_{5b}^{L} + M_{5b}^{L} \\
  \sum F_{y} & : & V_{5}^{R} &= V_{5t}^{L} + V_{5b}^{L}
\end{align*}$$

Or in matrix form

$$P_{5}^{R} = P_{5t} \cdot P_{5t}^{L} + P_{5b} \cdot P_{5b}^{L};$$  \hspace{1cm} (7.131)
where \( P_{5_l} = \begin{bmatrix} 1 & 0 & 0 \\ -r_1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \); \( P_{5_b} = \begin{bmatrix} 1 & 0 & 0 \\ r_2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \)

@ node #6

For the last node after applying the compatibility conditions and the equilibrium equations the relation between the state vector at the right of the node #6 (the boundary conditions) and the state vector at the left of the node can be expressed in matrix form as

\[
\begin{bmatrix} u^R \\ w \\ \phi \\ N \\ M \\ V \end{bmatrix}_{BC} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u^L \\ w \\ \phi \\ N \\ M \\ V \end{bmatrix}_6
\]

(7.132)

\[
z^R_{BC} = P_6 \cdot z^L_6 \text{, where } P_6 = I
\]

(7.133)

Now that all of our point transfer matrices \( P \) have been determined we will consider the following simplification, all the point transfer matrices \( P = I \) will be ignored.

The next step is to determine all the unknown state vectors: \( z^L_{BC} \), \( z^R_{2b} \), \( z^R_{3b} \), \( z^R_{3b} \), \( z^R_{4r} \), \( z^R_{5b} \), \( z^R_{2b} \) and \( z^R_{BC} \). To relate the state vectors at the two ends of a segment we will use Eq. (7.2)

\[
z^L_i = F_i z^R_{i-1}
\]

Applying the free-free boundary conditions:

at \( x = 0: \)

\[
N^L_{BC} = M^L_{BC} = V^L_{BC} = 0 \implies P^L_{BC} = 0
\]

(7.134)

at \( x = L: \)

\[
N^R_{BC} = M^R_{BC} = V^R_{BC} = 0 \implies P^R_{BC} = 0
\]

(7.135)
Considering the free-free boundary conditions, the corresponding unknown displacements and internal forces vectors are: \( \mathbf{d}_{BC}^L, \mathbf{d}_2^R, \mathbf{p}_2^R, \mathbf{d}_2^B, \mathbf{p}_2^B, \mathbf{d}_3^R, \mathbf{p}_3^R, \mathbf{d}_3^B, \mathbf{p}_3^B, \mathbf{d}_4^R, \mathbf{p}_4^R, \mathbf{d}_4^B, \mathbf{p}_4^B, \mathbf{d}_5^R, \mathbf{p}_5^R, \mathbf{d}_{BC}^R \). To solve for the unknowns we need 16 equations. These equations result from the compatibility and the equilibrium equations conditions; they will form a non-homogeneous system of equations with 16 variables (displacements and internal forces) which can be written in matrix form

\[
\mathbf{M} \cdot \mathbf{X} = \mathbf{N} \quad (7.136)
\]

where matrices \( \mathbf{M} \) and \( \mathbf{N} \) are derived in Eq. (B.2) and Eq. (B.3) of Appendix B. The unknown column vector \( \mathbf{X} \) is

\[
\mathbf{X} = \begin{bmatrix} \mathbf{d}_{BC}^L & \mathbf{d}_2^R & \mathbf{p}_2^R & \mathbf{d}_2^B & \mathbf{p}_2^B & \mathbf{d}_3^R & \mathbf{p}_3^R & \mathbf{d}_3^B & \mathbf{p}_3^B & \mathbf{d}_4^R & \mathbf{p}_4^R & \mathbf{d}_4^B & \mathbf{p}_4^B & \mathbf{d}_5^R & \mathbf{p}_5^R & \mathbf{d}_{BC}^R \end{bmatrix}^T
\]

From Eq. (7.136), the unknown column vector of displacements and internal forces can be solved

\[
\mathbf{X} = \mathbf{M}^{-1} \cdot \mathbf{N} \quad (7.137)
\]

### 7.3.4 FRF of cracked multi-layer beam under PWAS excitation

With the axial and flexural displacements \( u \) and \( w \) and the rotation \( w' \) calculated the total displacement between the two ends of a PWAS transducer attached to the beam at the distances \( x_1 \) and \( x_2 \) from the left end can be calculated as

\[
u_p = u(x_2) - u(x_1) - \frac{h}{2} [w'(x_2) - w'(x_1)]
\]

or, in terms of the state vectors components,

\[
u_p = (u_{si}^L - u_{si}^R) - \frac{h}{2} (\varphi_{si}^L - \varphi_{si}^R) \quad (7.138)
\]
Dividing Eq. (7.138) by the excitation force \( F \), the frequency response function \( FRF(\omega) \) can be expressed as

\[
\frac{u_p}{F} = \left( \frac{u'_{41} - u'_{31}}{F} \right) - \frac{h}{2} \left( \varphi'_{u} - \varphi'_{31} \right)
\]

\( (7.139) \)

\[
FRF(\omega) = FRF_u(\omega) - FRF_w(\omega)
\]

\( (7.140) \)

where

\[
FRF_u(\omega) = \left( \frac{u'_{41} - u'_{31}}{F} \right); \quad \text{and} \quad FRF_w(\omega) = \frac{h(\varphi'_{41} - \varphi'_{31})}{2F}
\]

The dynamic structural stiffness of the multi-layer beam can be calculated as

\[
k_{str}(\omega) = \frac{1}{FRF(\omega)}
\]

\( (7.141) \)

Knowing the material properties and the dimensions of the PWAS the electromechanical impedance can be expressed as (see Eq. (6.62) through Eq. (6.64))

\[
Z = \frac{1}{Y} = \frac{1}{i\omega C} \left[ 1 - \kappa_{31}^2 \left( 1 - \frac{1}{\varphi \cot \varphi + r} \right) \right]^{-1}
\]

\( (7.142) \)

where \( \varphi = \frac{\gamma_p}{2}, \quad \gamma_p = \omega \rho_p \sigma_{ii}^E \)

In conclusion, in this chapter a new method to calculate the structural response to a damage using the transfer matrix method was developed. The electromechanical impedance measured by a small PWAS transducer attached to a beam was reproduced analytically by using the transfer matrix approach. The advantages of such a simple but yet robust method are: short computing time, average PC hardware requirements, possibility of modeling complex structures with cracks.
Chapter 8
Validation of the mathematical model and experimental impedance method

In this chapter, the validations of the results for the analytical model as well as the experiments are studied. In order to make comparison between the results obtained for the pristine and damaged specimens with the analytical and experimental methods, it is important to validate the readings for the sensors bonded on the specimens. The location of the sensor as well as the adhesive layer will affect the impedance readings. In this chapter we will show what role the sensor location plays in the impedance spectrum. The capacitance of each sensor was measured. The impedance spectrum for each sensor was recorded and analyzed.

8.1 Validation of the analytical method on a small beam specimen

To validate the analytical model we considered two small beam specimens, one pristine and one damaged. Two aluminum strips 178 mm x 37 mm x 1.55 mm were bonded together (Figure 36). The damage consisted of a 25 mm disbond. The material properties for the aluminum and the adhesive Hysol® EA 9309.3NA are: $E_{\text{Al}} = 70 \text{ GPa}$,
$\rho_{Al} = 2780 \text{ kg/m}^3$, $E_{ad} = 2.23 \text{ GPa}$, and $\rho_{ad} = 1150 \text{ kg/m}^3$. On each specimen, three locations were considered where sensors were placed (PWAS #1, PWAS #2, and PWAS #3).

Two cases will be considered in our analysis: (a) the symmetric case when the sensors are symmetric with respect to the two ends of the specimens and (b) the asymmetric case where one sensor (PWAS #3) is shifted 1 mm towards one end.

8.1.1 Symmetric sensor placement

First, we considered perfect symmetry between the sensors meaning that PWAS #1 and PWAS #3 were located at exact the same distance from the left and right end of the specimen and that PWAS #2 was located exactly in the middle of the specimen (the location of the sensors is presented in Table 4).

Table 4 Location of the sensors from the left edge of the beam for the analytical method assuming perfect symmetry.

<table>
<thead>
<tr>
<th></th>
<th>Pristine specimen</th>
<th>Damaged specimen</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWAS #1</td>
<td>35 mm</td>
<td>35 mm</td>
</tr>
<tr>
<td>PWAS #2</td>
<td>85.5 mm</td>
<td>85.5 mm</td>
</tr>
<tr>
<td>PWAS #3</td>
<td>136 mm</td>
<td>136 mm</td>
</tr>
</tbody>
</table>

Figure 36 Pristine and damaged specimens considered in the analytical model.
The same layout was used for the, pristine and damaged cases. The impedance response in the range of 30-60 kHz was simulated for each sensor.

For the pristine specimen, the impedance spectrum (Figure 37) shows perfect match between the PWAS #1 and PWAS #3 which is what we expected since the sensors are placed symmetric to the left and right ends of the specimen.
For the damaged specimen, we also see a perfect match between the readings from PWAS #1 and PWAS #3 (Figure 38). These results validate our analytical model; we are now confident that we can predict with high accuracy the impedance response from PWAS permanently attached to the structure. Next we will look at what effect the adhesive layer has and how it will change the resonant frequencies.
Figure 39 Effects of the disbond on the analytical EMI spectrum for symmetric case: (top) PWAS #1 on pristine vs. damaged specimen; (bottom) PWAS #3 on pristine vs. damaged specimen
When we compared the impedance spectrum for the sensors placed in the same location but on different specimens (pristine and damaged, Figure 39) we could see the effects of a 25 mm disbond damage. The impedance plots in Figure 39 are the same for PWAS #1 (top) and PWAS #3 (bottom) because the two transducers are placed symmetrically. This proves again that the analytical model is predicting consisting results between the sensors on the left and right end of the specimens. Looking at the frequency plots for PWAS #1 pristine and damaged cases, we observe the change due to the presence of the disbond which is shifting of resonant frequencies and new peaks.

The next step is to look at the changes in the resonant frequencies for sensor PWAS #2 located in the middle of the specimen. On the damaged specimen, the sensor is centered above the disbond damage (Figure 40). A clear change in the impedance spectrum is visible. The disbond presence is changing the mechanical impedance of the structure and this results into new peaks with large amplitude in the electromechanical impedance spectrum. The new features are an obvious sign of the disbond existing in the structure underneath the sensor. The theoretical model gives a strong prediction of the damage; in later chapters, we will show that this is also in good correlation with the experimental results.
Figure 40 Analytical electro-mechanical impedance spectrum, symmetric case for PWAS #2 located on the pristine and damaged specimens. New features associated with the presence of the disbond.

8.1.2 Effect of asymmetric sensor placement

To determine the effect of an asymmetrical placement of the sensors, the location of PWAS #1 was slightly shifted by 1 mm closer to the left end of the specimen. All the other sensors were kept in the same location.
By moving PWAS #1 by just 1 mm, we can already see changes in the impedance spectrum due to the small asymmetry (Figure 41). New resonant frequencies as well as an increase in amplitude for some frequencies are the new features associated with the small change in the sensor location.
The same changes in the electro-mechanical impedance can be observed for the signals collected from the damaged specimen if one sensor is shifted by 1 mm (Figure 42). In this case PWAS #1 was placed 1 mm closer to the left end. We can see a consistency in the effect that asymmetric sensor placement has on the impedance spectrum. While there is an overall good overlap of the signals, in both cases we see an increase in the amplitude of some resonant frequencies.
Next, PWAS #2 on the pristine specimen was moved 1 mm to the right and the electro-mechanical impedance spectrum was compared with the case when the sensor is placed in the center of the specimen (Figure 43). For a small 1 mm change, the mechanical impedance of the structure has changed and as expected more resonant peaks are present in the same frequency domain.
The last situation we analyzed for the asymmetric case was that of shifting PWAS #2 located on the damaged specimen on top of disbonds by 1 mm and then compare the impedance spectrum with the data collected when the sensor is not shifted (Figure 44). By looking at the electro-mechanical impedance we see that the frequency spectrum for the shifted sensor has changed (red line) showing new resonant frequencies and increased amplitude of some frequencies.
One question still needs to be answered for an asymmetric placement of the sensors: if the sensor on the pristine specimen is shifted from the location of the same sensor on the damaged specimen, is it still possible to detect the damage? The answer to this question is given in Figure 45. Even though the impedance spectrum of the shifted sensor (PWAS #2 on the pristine specimen) has changed, it is clearly shown in Figure 45 that the presence of the damage will significantly change the impedance spectrum. A small change in the location of one sensor will not diminish the damage detection capabilities.
In conclusion, from the symmetric and asymmetric analytical simulations we were able to show that the model of the impedance response gives consistent results for both pristine and damaged bonded specimens. The plots show a perfect left-right overlap for the symmetric case when the sensors are symmetrically located with respect to the beam center. We could also see how the damage will change the impedance spectrum: shifting of resonant frequencies, new resonant peaks.

The same simulation was then conducted for the asymmetric case when one of the sensor (PWAS #1) was shifted by 1 mm. The electro-mechanical impedance spectrum changed due to the small shift for both the pristine and damaged specimens.

From this simulation we can conclude that a small change in the location of the sensors will affect the impedance response. To be able to reproduce experimental results by analytical modeling it is important to know the exact location of the sensors as well as information about the damage location and damage size. However, experimental damage detection is still possible even if the sensors are shifted, because the differences between “pristine” and “damage” cases are still very clear.

### 8.2 Validation of the experimental method on a small beam specimen

To validate the experimental method, two aluminum specimens were designed and fabricated, one pristine and one with an artificial disbond. Both specimens were made out of two aluminum strips 7075-T6, 178 mm x 37 mm x 1.55 mm bonded together using an epoxy paste adhesive, Hysol® EA 9309.3NA. A disbond in one of the specimens was created as a discontinuity of the epoxy adhesive in the middle of the
specimen. This was done using a strip of teflon tape inserted between the two aluminum strips. After cure, the teflon tape was removed. Each specimen was instrumented with three PWAS transducers (Figure 46); the location of the sensors is shown in Table 5.

Table 5 Location of the sensors from the left edge of the beam

<table>
<thead>
<tr>
<th></th>
<th>Pristine specimen</th>
<th>Damaged specimen</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[mm]</td>
<td>[mm]</td>
</tr>
<tr>
<td>PWAS #1</td>
<td>35.2</td>
<td>34.8</td>
</tr>
<tr>
<td>PWAS #2</td>
<td>84.5</td>
<td>84.3</td>
</tr>
<tr>
<td>PWAS #3</td>
<td>134.5</td>
<td>134.3</td>
</tr>
</tbody>
</table>

Using an HP4194A impedance analyzer, the electromechanical impedance was measured at several locations to check for consistency and validation of the experimental data. The sensors were excited with a signal in the frequency range 30-60 kHz. Figure 47 shows the impedance spectrum for two sensors, PWAS #1 and PWS #3 located on the pristine specimen. We can observe a very good match between the frequency responses of the two sensors.
The same impedance spectrum was taken for sensors PWAS #1 and PWAS #3 on the damaged specimen (Figure 48). It can be observed a fair match between the two measurements. When looking at the impedance spectrum for the sensors in the same location but on different specimens (pristine and damaged), interesting changes could be observed (Figure 47).
Figure 48  Experimental electro-mechanical impedance spectrum for PWAS #1 and PWAS #3 on damaged specimen.

Figure 49 (top), associated with PWAS #1, shows that the two spectra are somehow close, but we start seeing some of the effects of the disbond, shifting of the resonant frequencies as well as an increase in the amplitude of a few resonant frequencies. Figure 49 (bottom) shows similar changes associated with the damage on PWAS #3: shifting in resonance frequencies, new resonant peaks as well as increase in amplitude.
Figure 49  Experimental electro-mechanical impedance spectrum: (top) PWAS # 1 on the pristine and damaged specimens; (bottom) PWAS # 3 on the pristine and damaged specimens
If we compare now the impedance spectrum for the sensors placed in the central location (PWAS #2) but one on a pristine specimen and the other one on damaged specimen with disbond, we see the frequency spectrum of Figure 50. There is almost no similarity between the pristine and damaged signals, and we can see some very significant changes due to the disbond: new peaks with large amplitude.

Figure 50 Experimental electro-mechanical impedance spectrum for PWAS #2 on the pristine and damaged specimens
8.3 Analytical model vs. experimental results

In this section we will compare the analytical results from Section 7.1 with the results of our experiments given in Section 7.2. Both the pristine and damaged specimens were modeled using exactly the dimensions measured on the test specimens. A digital caliper and a digital micrometer were used to measure as accurate as possible the thickness, length, width and the location of the sensors for each specimen. These dimensions are listed in Table 6.

Table 6 Measured specimen dimensions and location of the sensors

<table>
<thead>
<tr>
<th></th>
<th>Pristine specimen [mm]</th>
<th>Damaged specimen [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>178.4</td>
<td>178.4</td>
</tr>
<tr>
<td>Width</td>
<td>36.8</td>
<td>36.7</td>
</tr>
<tr>
<td>Aluminum thickness</td>
<td>1.55</td>
<td>1.55</td>
</tr>
<tr>
<td>Total thickness of specimen</td>
<td>3.22</td>
<td>3.22</td>
</tr>
<tr>
<td>PWAS #1 location from left edge</td>
<td>35.2</td>
<td>34.9</td>
</tr>
<tr>
<td>PWAS #2 location from left edge</td>
<td>83.9</td>
<td>84.3</td>
</tr>
<tr>
<td>PWAS #3 location from left edge</td>
<td>134.5</td>
<td>134.3</td>
</tr>
</tbody>
</table>

An HP impedance analyzer was used to apply a voltage on each PWAS with the frequency varying from 10 kHz to 60 kHz and the electromechanical impedance spectrum was recorded. The numerical results from the Matlab program for the mathematical model were then compared with the experimental results for various sensors as discussed next.
Figure 51  Comparison between the analytical and experimental electro-mechanical impedance spectrum for PWAS #1: (top) pristine specimen; (bottom) damaged specimen
Figure 52  Comparison between the analytical and experimental electro-mechanical impedance spectrum for PWAS #3: (top) pristine specimen; (bottom) damaged specimen
In Figure 51 and Figure 52, the impedance spectra are shown for two sensors, PWAS #1 and PWAS #3. The analytical and experimental results are superposed for the pristine and damaged specimens. For each case, we can see an overall fair correlation between the two impedance spectra with some frequencies being almost coincidental. However, the experimental electromechanical impedance spectrum shows more resonant peaks than the analytical model and this is attributed to the fact that the 1-D analytical model is not taking into account any width vibrations of the actual specimen. One can also observe other consistency aspects: (a) between the theoretical and the experimental results for the same sensors; (b) between the pristine and damaged specimens; and (c) between PWAS #1 and PWAS #3 for both pristine and damaged specimens.

The impedance spectra for PWAS #2 located in the middle of the beam for pristine and damage specimens are shown in Figure 53. For each case, the analytical results are compared with the experimental results. For the damaged specimen (25 mm disbond), the sensor (PWAS#2) is placed on the aluminum strip on top of the damage. By looking at each graph (top and bottom) of Figure 53, we see a fair to good correlation between the theory and experiments. If we now compare the top graph (pristine) with the one on the bottom (damaged-disbond), we see significant changes in the impedance spectrum. A significant increase in the amplitude of the resonant frequencies as well as new peaks are indicators of the damage, which changed the mechanical impedance of the structures and thus in terms changed the real part of the electrical impedance of the sensor. The theoretical model predicts very well the damaged response of the structure.
Figure 53  Comparison between the analytical and experimental electro-mechanical impedance spectrum for PWAS #2: (top) pristine specimen; (bottom) damaged specimen
From the results presented in this chapter, we can conclude that the matrix transfer method to model the structure response due to a damage (disbond) in an adhesively bonded specimen using the electromechanical impedance gives good results. The analytical numerical results are in accordance with the experimental results. The transfer matrix method has the advantage of a significant reduction in the computational complexity and also a very fast simulation time compared to other methods of analysis.

It is also important to notice the sensitivity of the method to the general dimensions of the specimen, the location of the sensors and also the damage characteristics. It was shown that a small change in the location of one sensor will produce a significant change in the impedance response. When comparing predictive analytical results against experimental results, special attention needs to be paid to modeling exactly the location of the sensors attached to the structure. However, in real world applications, the PWAS remains in the same location on the structure while damage is initiated and progresses.

The last important comment is about the good prediction of the changes in the impedance spectrum due to the presence of the damage. However, the analysis of these changes has only been qualitative. In later chapters, we will show that the use of a damage index can qualitatively and quantitatively predict the damage.
Chapter 9
Finite element analysis of multi-layer beam

In previous chapters, we studied analytically the response of structures to PWAS excitation and calculated numerically the electromechanical impedance response for pristine and damaged structures. Another method to predict the response to PWAS excitation under various damage conditions is the finite element analysis (FEA) method. The FEA method is used in many engineering applications; however, for complicated structures, it may present the disadvantage of increased computational time, large storage capabilities, and memory limitations.

In this chapter, we will use the FEA approach to calculate the electromechanical impedance and to compare with our analytical model. In order to do this we will use two FEA techniques: (a) the conventional FEA where a harmonic force is applied onto the structure and the respective axial and flexural displacement amplitudes are calculated; (b) the coupled-field FEA where the electromechanical impedance is calculated directly from the coupling between the structure’s stress field and the PWAS electrical field. In both cases, the FEA results will be compared with the analytical and experimental results.
9.1 Conventional FEA of simple uniform beam

Before addressing the more complicated case of a bonded lap-joint structure with disbond damage, we start with a simpler case in which we consider a simple uniform beam with a PWAS attached to its surface (Figure 54).

![Figure 54](image)

Figure 54 Modeling of forces and moments acting on the beam due to PWAS excitation using conventional FEA

For the 1-D situation shown in Figure 54 the sensor and the structure interact through axial forces and bending moments. We will consider perfect bonding between the PWAS and the beam; hence, the interaction between the PWAS and structure will take place only at the two ends of the PWAS (this is also known as the pin-force model). For the FEA analysis, we considered a steel beam with the Young’s modululs $E = 200$ GPa, density $\rho = 7750$ kg/m$^3$, length $l = 100$ mm, width $b = 8$ mm and thickness $h = 2.6$ mm. The PWAS transducer is located at $x = 40$ mm from the left end of the beam and the length of the sensor is $L_p = 7$ mm. The material properties of the PWAS are listed in Table 3.

The commercial FEA software ANSYS was used to model the beam and perform the analysis. The beam was meshed using PLANE42 elements element with two degrees of freedom at each node. A harmonic force $F$ of unit amplitude was applied at the nodes corresponding to the locations of the two ends of the PWAS transducer (Figure 55).
A harmonic analysis was conducted in the frequency range of 1 kHz to 30 kHz; the axial displacements in the X-direction at the two ends of the PWAS as well as the rotations about the Z-axis at the two ends were calculated. The FEA analysis parameters are shown in Table 7.

Table 7 Analysis parameters for conventional FEA

<table>
<thead>
<tr>
<th>FEA parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element type</td>
<td>PLANE42</td>
</tr>
<tr>
<td>Element edge length</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Force</td>
<td></td>
</tr>
<tr>
<td>@ node 338</td>
<td>-1</td>
</tr>
<tr>
<td>@ node 324</td>
<td>1</td>
</tr>
<tr>
<td>Harmonic frequency range</td>
<td>1 kHz – 30 kHz</td>
</tr>
<tr>
<td>Number of substeps</td>
<td>401</td>
</tr>
<tr>
<td>Constant damping ratio</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Using Eqs. (6.57) and (6.61) the dynamic structural stiffness $k_{str}(\omega)$ was calculated. Following the method described in Section 5.5, the electromechanical impedance at the PWAS terminals was calculated. These conventional FEA results, analytical and experimental results are shown in Figure 56.
Figure 56  Comparison of the conventional FEA vs. analytical and experimental results for uniform beam

From Figure 56, it can be seen that both methods, conventional FEA and analytical transfer matrix method, predict well the natural resonant frequencies. The analytical model does not predict well the double spike in the natural frequencies around 26 kHz but the conventional FEA does. Although not perfect, the overlap of the results with the analytical method is promising.

9.2 Coupled-field FEA of simple uniform beam

In order to include the piezoelectric properties of the PWAS in the finite element analysis, the coupled-field FEA is used. In this case, the mechanical stress field of the structure is coupled with the electrical field of the PWAS and a change in one field will
generate a change in the other field based on the piezoelectric principles. The PWAS transducer was meshed and a voltage was applied to the two PWAS electrodes. The voltage applied was of unity amplitude. Then, a harmonic analysis was performed and the change with frequency of the electrical charge on the two PWAS electrodes was recorded. In order to be able to use the voltage DOF, the PWAS was modeled using a PLANE13 four nodes element with up to four DOF per node. To select the VOLT degree of freedom in the element type in ANSYS, the KEYOPT(7) was used. The FEA parameters for the coupled-field analysis are presented in Table 8.

Table 8 Analysis parameters for coupled-field FEA

<table>
<thead>
<tr>
<th>FEA parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element type</td>
<td></td>
</tr>
<tr>
<td>PWAS</td>
<td>PLANE13</td>
</tr>
<tr>
<td>Beam</td>
<td>PLANE42</td>
</tr>
<tr>
<td>Element edge length</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Voltage applied on the top electrode</td>
<td>1 V</td>
</tr>
<tr>
<td>Harmonic frequency range</td>
<td>1 kHz – 30 kHz</td>
</tr>
<tr>
<td>Number of substeps</td>
<td>401</td>
</tr>
<tr>
<td>Constant damping ratio</td>
<td>0.005</td>
</tr>
</tbody>
</table>

The beam was modeled using a four node PLANE42 element with two degrees of freedom at each node (Figure 57). The harmonic analysis was carried out in the frequency range of 1 kHz to 30 kHz. A constant damping factor of 0.005 was applied during the simulation.
From the electrical charge on the PWAS electrodes, the electrical current was calculated and then the electromechanical impedance at the PWAS location. For each frequency in the spectrum, the electromechanically impedance is given by

\[ Z = \frac{V}{I} \]  

(9.1)

where \( V \) is the voltage and \( I \) is the current. Knowing the electric charge \( Q \), the current can be expressed as

\[ I = \omega Q = i 2 \pi f \cdot [\text{Re}(Q) + i \text{Im}(Q)] \]  

(9.2)

For a unity voltage applied on the electrodes, the real part of the electromechanical impedance becomes

\[ Z = \text{Re}\{i 2 \pi f \cdot [\text{Re}(Q) + i \text{Im}(Q)]\} \]  

(9.3)
Figure 58 Comparison of the coupled-field FEA (10^3 multiplication factor) vs. analytical and experimental results for uniform beam.

The results of these calculations are presented in Figure 58. From the comparison between the coupled-field FEA, analytical and experimental results we can see that both the coupled-field FEA and the analytical results using the transformation matrix method predict well the experimental electromechanical impedance method. As in the conventional FEA, the coupled-field FEA method also predicts the double resonant peak at around 25 kHz.

We observe that there are differences in the amplitudes of the off-resonant frequencies. These differences are consistent between the three methods (analytical, conventional FEA and coupled-field FEA) and the experimental results. They can be
attributed to the fact that in our analytical investigation as well in the FEA simulation we consider a 1-D model whereas the experimental results are based on a 3-D specimen.

9.3 FEA analysis of multi-layer damaged beam

We will consider now the more complicated case of a multi-layer beam in which two strips of aluminum were bonded together using an epoxy adhesive (see Table 6 for dimension information).

The two specimens, pristine and damaged, were modeled in ANSYS, both conventional FEA and couple-field FEA were performed. The results were compared against the analytical results (transfer matrix method) and the experimental results. For the FEA analysis, just like in the analytical model, we considered the problem to be 1-D. The bond between the PWAS and the beam was considered ideal. The parameters used in the finite element analysis are listed in Table 9.

<table>
<thead>
<tr>
<th>FEA parameter</th>
<th>Conventional FEA</th>
<th>Coupled-field FEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PWAS</td>
<td>-</td>
<td>PLANE13</td>
</tr>
<tr>
<td>Beam</td>
<td>PLANE42</td>
<td>PLANE42</td>
</tr>
<tr>
<td>Element edge length</td>
<td>0.5 mm</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Force</td>
<td></td>
<td></td>
</tr>
<tr>
<td>@ node 2335</td>
<td>-1 N</td>
<td>-</td>
</tr>
<tr>
<td>@ node 2321</td>
<td>1 N</td>
<td>-</td>
</tr>
<tr>
<td>Voltage applied on the top electrode</td>
<td>-</td>
<td>1 V</td>
</tr>
<tr>
<td>Harmonic frequency range</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pristine model</td>
<td>1 kHz – 30 kHz</td>
<td></td>
</tr>
<tr>
<td>Damage model</td>
<td>30 kHz – 60 kHz</td>
<td></td>
</tr>
<tr>
<td>Number of substeps</td>
<td></td>
<td>401</td>
</tr>
</tbody>
</table>

Table 9 FEA parameters for conventional and coupled-field analysis
For the conventional FEA the multi-layer beam was modeled using a four node PLANE42 element. A unit axial force was applied at the location of the two ends of the PWAS. For the coupled-field analysis we used the couple-field four node PLANE13 element which allows the VOLT degree of freedom to be used. The multi-layer beam was modeled using a four node PLANE42 element. The mesh of the pristine and damaged models for the conventional FEA is shown in Figure 59.

![Figure 59 Conventional FEA: mesh and axial force applied at the two ends of the PWAS](image)

Similarly, the mesh and the voltage applied on top and bottom electrodes of the PWAS transducers for the pristine and damaged multi-layer model coupled-field FEA is shown in Figure 60.
Figure 60  Couple-field FEA: mesh and voltage load applied on the top and bottom electrodes of the PWAS

The harmonic FEA analysis for the pristine model was performed in the frequency range of 1 kHz to 30 kHz. However, the FEA analysis of the damaged model was performed in the frequency range of 30 kHz to 60 kHz, because the experimental results (confirmed also by analytical simulations) indicated that the damage will affect more the frequency range of 30 kHz to 60 kHz. The finite element analysis was conducted for PWAS #2 on both models. On the damaged specimen, PWAS #2 is placed directly on top of the disbond (Figure 61).

Figure 61  Location of sensors on the pristine and damage models for FEA analysis

A comparison between the finite element results with both conventional and coupled-field analysis, analytical results, and experimental results is shown in Figure 62 for the pristine case.
From Figure 63 we can see that all three methods predict well the frequency peaks of the measured electromechanical impedance spectrum for the frequency range up to 15 kHz. For frequencies greater than 15 kHz some frequency shifting is noticed between the conventional FEA, coupled-field FEA and the analytical results. We associated the shifting to a coarser mesh used for the FEA analysis; this shift will decrease as the element length becomes smaller and smaller. However, a finer mesh will affect the time needed to run the simulation and the needed memory allocation. In addition, we wanted to mention that there is not a perfect match between calculated results and experimental results because the analytical simulation is 1-D whereas the experimental results are for a 3-D case.
Figure 63 Comparison between FEA, analytical, and experimental results for multi-layer disbonded beam ($10^3$ multiplication factor for CF FEA)

For the damaged case shown in Figure 63, the FEA simulation predicts a few resonant frequencies in the electromechanical impedance spectrum but it is not as close to the experimental results as the analytical model. More work needs to be done in this area to find the relationship between the type of structure, type of damage and how to “fine tune” the FEA properties in order to get simulation results much closer to the experimental and analytical results.

In summary, the goal of this chapter was to compare the results obtained from the analytical model with the predictions from the finite element analysis and to validate them against the experimental results. For the simple case of a uniform beam, the FEA results show good correlation between the resonant peaks of the electromechanical
impedance. We can say with confidence that the analytical model using the transformation matrix method is reliable and is predicting results very close to the experimental ones.

For the pristine multi-layer adhesively bonded beam both, the conventional and coupled-field FEA results when compared with the analytical and experimental results show a good overlap between the resonant peaks for lower frequencies but a shift occurs at higher resonant frequencies. The shift as well as the difference in the amplitude of the off-resonance frequencies are attributed to the difference in the models we used (1-D for analytical and FEA simulations and 3-D for experiments).

For the more complex case of a multi-layer beam with a damage (partial disbond between two aluminum strips), the FEA analysis becomes much more complex. The 1-D model is not sufficient to accurately predict the electromechanical impedance spectra.
Chapter 10
Variation of damage indices with disbond size for structural health monitoring

In order to be able to make quantitative analysis of the changes in the structural response of complex structures due to the presence of various damages, we need to make use of damage indexes or damage metrics. A damage index (DI) is a scalar number and is the result of a comparison process between two sets of data, “pristine” and “damaged”. In most cases one set of data is the baseline or the pristine case and the other one is the current situation or the damaged case. The DI should reveal the differences between the two sets of data and it should be sensitive to the changes in the spectral features affected by the damage presence and should not be sensitive to the common cause variations (i.e. temperature, humidity, etc.)

In this chapter we will introduce several most common damage indices to compare impedance spectra and to assess the presence of damage: root mean square deviation (RMSD), mean absolute percentage deviation (MAPD), and correlation coefficient deviation (CCD). Each DI will be presented and analytical results for various damage sizes will be discussed and analyzed. The results for various DI types will be compared to determine which ones react better to certain damage growth.
10.1 Damage indices for structural health monitoring

Three simple damage indices are commonly used in structural health monitoring: root mean square deviation (RMSD), mean absolute percentage deviation (MAPD), and correlation coefficient deviation (CCD).

10.1.1 Root mean square deviation (RMSD)

In general the root mean square deviation (RMSD) is used as a measure of the differences between the values predicted analytically by a model and the values obtained from experiments. Each individual difference is called a residual and the root mean square deviation will combine all the residuals into a single values. The mathematical expression for the RMSD is given by

$$ RMSD = \sqrt{\frac{\sum_{i=1}^{N} [\text{Re}(Z^d_i) - \text{Re}(Z^p_i)]^2}{\sum_{i=1}^{N} [\text{Re}(Z^p_i)]^2}} $$

(10.1)

where \( N \) is the number of frequencies in the electromechanical impedance spectrum, \( \text{Re}(Z^p_i) \) is the real part of the electromechanical impedance for the pristine case, \( \text{Re}(Z^d_i) \) is the real part of the electromechanical impedance for the damage case, and \( i = 1, \ldots, N \) is the index of the frequency point in the spectrum.

10.1.2 Mean absolute percentage deviation (MAPD)

In statistical analysis, the absolute deviation is defined as the absolute difference between an element and a given point. The point from which the deviation is measured can be the mean value or the median value of a set of data. The mean absolute percentage
deviation is the mean absolute deviation scaled with the absolute value of the original point.

In our case, we have calculated the deviation between the pristine case and the damage case. The mathematical expression for the mean absolute percentage deviation, MAPD, is given by the expression

$$\text{MAPD} = \frac{1}{N} \sqrt{\sum_{i} \left(\frac{\text{Re}(Z_i^d) - \text{Re}(Z_i^p)}{\text{Re}(Z_i^p)}\right)^2}$$  \hspace{1cm} (10.2)

10.1.3 Correlation coefficient deviation (CCD)

The correlation coefficient can indicate a predictive relationship between two sets of data. In our case, the correlation coefficient deviation can be used to characterize the relation between the damage size and the change in the electromechanical impedance spectrum.

The mathematical expression for the CCD index is given by the following equation

$$\text{CCD} = 1 - \frac{1}{(N-1)\sigma_{Z^d} \sigma_{Z^p}} \sum_{i} \left[\text{Re}(Z_i^d) - \text{Re}(Z_i^p)\right] \left[\text{Re}(Z_i^d) - \text{Re}(Z_i^p)\right]$$ \hspace{1cm} (10.3)

where $N$ is the number of frequencies in the electromechanical impedance spectrum, $\sigma_{Z^d}$ and $\sigma_{Z^p}$ are the standard deviations for the pristine and damage data sets.

10.1.4 DI comparison for disbond detection

The RMSD, MAPD and the CCD damage indices have been calculated to account for two situations: (a) the length of the damage is changing; and (b) the opening of the damage is changing. We used the transfer matrix method, developed in this dissertation,
to calculate the real part of the electromechanical impedance Re(Z) as detailed in Section 6.2.

We considered the case of a beam modeled as a joint between two aluminum strips bonded together using an epoxy adhesive (Figure 64).

![Figure 64](image.png)

Figure 64  PWAS location on pristine and damage models; variation of disbond length ΔL and disbond opening Δh

Table 10  Measured specimen dimensions and location of the sensors

<table>
<thead>
<tr>
<th></th>
<th>Pristine specimen [mm]</th>
<th>Damaged specimen [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>178.4</td>
<td>178.4</td>
</tr>
<tr>
<td>Width</td>
<td>36.8</td>
<td>36.7</td>
</tr>
<tr>
<td>Aluminum thickness</td>
<td>1.55</td>
<td>1.55</td>
</tr>
<tr>
<td>Total thickness of specimen</td>
<td>3.22</td>
<td>3.22</td>
</tr>
<tr>
<td>PWAS #1 location from left edge</td>
<td>35.2</td>
<td>34.9</td>
</tr>
<tr>
<td>PWAS #2 location from left edge</td>
<td>83.9</td>
<td>84.3</td>
</tr>
<tr>
<td>PWAS #3 location from left edge</td>
<td>134.5</td>
<td>134.3</td>
</tr>
</tbody>
</table>

The dimensions of the two models, pristine and damage are listed in Table 10. The frequency range for our analysis was chosen between 1 kHz and 60 kHz. The small dimension differences between the two specimens as well as difference in the PWAS
locations are due to the manufacturing process. Although great care was used when the specimens were manufactured, some variation was hard to avoid with the equipment in our machine shop.

### 10.2 Variation of damage indices for PWAS #2

#### 10.2.1 Variation of damage indices with the length of disbond damage

To verify the sensitivity of the transfer matrix method to detect disbond damage with different sizes, the length of the damage located under PWAS #2 (disbond) was increased from 0.5 mm to 20 mm. The location of the start of the disbond was maintained constant at $x = 83$ mm from the left end of the. The damage indices RMSD, MAPD, CCD, calculated for each disbond value are listed in Table 11.

<table>
<thead>
<tr>
<th>Damage length [mm]</th>
<th>RMSD</th>
<th>MAPD</th>
<th>CCD</th>
<th>RMSD</th>
<th>MAPD</th>
<th>CCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.018</td>
<td>0.018</td>
<td>0.000</td>
<td>0.011</td>
<td>0.045</td>
<td>0.000</td>
</tr>
<tr>
<td>1</td>
<td>0.018</td>
<td>0.018</td>
<td>0.000</td>
<td>0.012</td>
<td>0.045</td>
<td>0.000</td>
</tr>
<tr>
<td>2</td>
<td>0.023</td>
<td>0.020</td>
<td>0.000</td>
<td>0.014</td>
<td>0.051</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>0.062</td>
<td>0.047</td>
<td>0.003</td>
<td>0.039</td>
<td>0.119</td>
<td>0.005</td>
</tr>
<tr>
<td>6</td>
<td>0.104</td>
<td>0.094</td>
<td>0.008</td>
<td>0.065</td>
<td>0.237</td>
<td>0.013</td>
</tr>
<tr>
<td>8</td>
<td>0.673</td>
<td>0.150</td>
<td>0.122</td>
<td>0.419</td>
<td>0.379</td>
<td>0.214</td>
</tr>
<tr>
<td>10</td>
<td>0.648</td>
<td>0.210</td>
<td>0.115</td>
<td>0.403</td>
<td>0.528</td>
<td>0.202</td>
</tr>
<tr>
<td>12</td>
<td>1.210</td>
<td>0.299</td>
<td>0.310</td>
<td>0.753</td>
<td>0.753</td>
<td>0.546</td>
</tr>
<tr>
<td>14</td>
<td>1.258</td>
<td>0.325</td>
<td>0.319</td>
<td>0.783</td>
<td>0.818</td>
<td>0.562</td>
</tr>
<tr>
<td>16</td>
<td>1.158</td>
<td>0.344</td>
<td>0.308</td>
<td>0.721</td>
<td>0.866</td>
<td>0.542</td>
</tr>
<tr>
<td>18</td>
<td>0.919</td>
<td>0.366</td>
<td>0.317</td>
<td>0.572</td>
<td>0.922</td>
<td>0.559</td>
</tr>
<tr>
<td>20</td>
<td>1.607</td>
<td>0.397</td>
<td>0.568</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
The effects of an increased damage length on PWAS #2 and the numerical results for the three damage indices, RMSD, MAPD, CCD, were carried out analytically in MATLAB. The goal was to see if, by the help of the damage indices we could detect the damage presence and length. We also wanted to compare how the various damage indices change as the length of disbond damage changes. The analytical values of the damage indices (DI) were normalized to their maximum value, such that an easy comparison of the three damage indices can be done.

Figure 66 shows the electromechanical impedance spectra for each damage length. This figure allows us to visually correlate the changes in DI’s given in Table 11.

Changes of DI values with the damage length increase with the changes in the impedance spectra.
Figure 65 Changes in the damage indices and the real part of the electromechanical impedance for different damage lengths for PWAS #2 (dotted line is the damaged case; solid line is the pristine case).
When we plotted the analytical electromechanical impedance results (Figure 65) a narrower frequency interval of 30 kHz to 60 kHz was chosen to better show the changes in the impedance spectra due to increased damage.

The electromechanical impedance spectra in Figure 65 show increased frequency response changes as the damage increases. The following changes were observed due to damage growth:

(a) peak shifts, usually to the left, indicative of increased structural stiffness

(b) peak splitting or coalescence

(c) increase in peak heights

(d) appearance of new peaks

The values of the damage indices are also changing with the change in the damage size. A comparison of all three DI’s is presented in Figure 66.

![Figure 66 DI comparison for different damage lengths for PWAS #2](image)
From Figure 66, it can be seen that all three damage indices modify with the increase in damage length. However, each of the DI curves has a different shape. The only DI that shows monotonic increase with the damage length is the mean absolute percentage deviation (MAPD). The other two, RMSD and CCD, are not monotonic with damage length; this behavior is attributed to the fact that they are not as sensitive to the small continuous changes in the electromechanical impedance spectrum but rather to the more abrupt ones. This behavior is also shown in Figure 66 for the damage length of 10, 12, and 16 mm respectively. The change in the RMSD and CCD values from 10 mm to 12 mm are much bigger than the change in values from 12 mm to 16 mm.

10.2.2 Variation of damage indices with opening of the disbond damage

Next, we look at the damage indices changes when the disbond damage opens (grows) from 0% to 100 %, where 100% is considered to be fully disbonded. We will consider the same adhesively bonded beam as presented in Section 9.2. In this case, the length of the disbond is maintained constant at $a = 20$ mm located at $x = 83$ mm from the left end of the beam. The disbond was then simulated to open up from 0% (“kissing bond”) to 100% disbond (fully disbonded, the adhesive layer fully removed locally between the two aluminum strips). The damage indices RMSD, MAPD and CCD were calculated for each disbond opening and the results are listed in Table 12.
<table>
<thead>
<tr>
<th>Damage growth [%]</th>
<th>RMSD</th>
<th>MAPD</th>
<th>CCD</th>
<th>RMSD</th>
<th>MAPD</th>
<th>CCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0.010</td>
<td>0.002</td>
<td>0.000</td>
<td>0.041</td>
<td>0.037</td>
<td>0.002</td>
</tr>
<tr>
<td>10</td>
<td>0.022</td>
<td>0.005</td>
<td>0.000</td>
<td>0.093</td>
<td>0.084</td>
<td>0.008</td>
</tr>
<tr>
<td>20</td>
<td>0.048</td>
<td>0.011</td>
<td>0.002</td>
<td>0.200</td>
<td>0.179</td>
<td>0.038</td>
</tr>
<tr>
<td>30</td>
<td>0.073</td>
<td>0.017</td>
<td>0.004</td>
<td>0.307</td>
<td>0.275</td>
<td>0.090</td>
</tr>
<tr>
<td>40</td>
<td>0.098</td>
<td>0.024</td>
<td>0.007</td>
<td>0.411</td>
<td>0.372</td>
<td>0.162</td>
</tr>
<tr>
<td>50</td>
<td>0.122</td>
<td>0.030</td>
<td>0.011</td>
<td>0.511</td>
<td>0.471</td>
<td>0.252</td>
</tr>
<tr>
<td>60</td>
<td>0.145</td>
<td>0.036</td>
<td>0.015</td>
<td>0.608</td>
<td>0.569</td>
<td>0.358</td>
</tr>
<tr>
<td>70</td>
<td>0.169</td>
<td>0.042</td>
<td>0.020</td>
<td>0.705</td>
<td>0.670</td>
<td>0.486</td>
</tr>
<tr>
<td>80</td>
<td>0.192</td>
<td>0.049</td>
<td>0.027</td>
<td>0.805</td>
<td>0.776</td>
<td>0.638</td>
</tr>
<tr>
<td>90</td>
<td>0.216</td>
<td>0.056</td>
<td>0.034</td>
<td>0.905</td>
<td>0.883</td>
<td>0.812</td>
</tr>
<tr>
<td>100</td>
<td>0.239</td>
<td>0.063</td>
<td>0.042</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The analytical results in Table 12 show a good correlation between the damage growth (disbond opening) and the changes in the values for all the damage indices: root mean square deviation (RMSD), mean absolute percentage deviation (MAPD), and the correlation coefficient deviation (CCD). Figure 67 compares the real part of the electromechanical impedance spectrum for two cases: (a) very small damage opening (1% disbond) and (b) almost complete disbond (90% damage opening).
Figure 67  Comparison of the damage indices and the real part of the electromechanical impedance for 1% opening of the disbond and 90% opening.

- **1% opening**
  - RMSD = 0
  - MAPD = 0
  - CCD = 0

- **90% opening**
  - RMSD = 0.905
  - MAPD = 0.883
  - CCD = 0.812
A plot of the damage index values vs. disbond opening is given in Figure 67. From Figure 67 it can be seen that all three damage indices, RMSD, MAPD and CCD predict the damage opening in a similar fashion. However two of them, RMSD and MAPD have a more linear correlation with disbond damage opening. Of this two, it is apparent that the mean absolute percentage deviation (MAPD) is slightly better than root mean square deviation (RMSD). Hence, it is recommended that MAPD is used as metric to characterize the disbond damage because it has a monotonic response to spectral changes due to the two types of damage growth, both disbond length and disbond opening.

![Figure 68: DI comparison for different damage opening for PWAS #2](image-url)

Figure 68  DI comparison for different damage opening for PWAS #2
10.3 Variation of damage indices for PWAS #1

10.3.1 Variation of damage indices with the length of disbond damage

In the previous section we addressed the sensitivity of the DI to detect changes in the disbond located under PWAS #2. In this part we want to study what effect the damage growth will have on the adjacent PWAS transducers. We will consider the damage located under PWAS #2 (disbond), its length was increased from 0.5 mm to 20 mm. The electromechanical impedance was calculated at PWAS #1, located to the left of the disbond (Figure 64). The location of the start of the disbond was maintained constant at $x = 83$ mm from the left end of the. The damage indices RMSD, MAPD, CCD, calculated for each disbond value are listed in Table 15.

Table 13 Changes of DI values with the damage length increase for PWAS #1

<table>
<thead>
<tr>
<th>Damage length [mm]</th>
<th>Calculated</th>
<th>Normalized</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSD</td>
<td>MAPD</td>
</tr>
<tr>
<td>0.5</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>1</td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>2</td>
<td>0.008</td>
<td>0.005</td>
</tr>
<tr>
<td>4</td>
<td>0.048</td>
<td>0.026</td>
</tr>
<tr>
<td>6</td>
<td>0.133</td>
<td>0.075</td>
</tr>
<tr>
<td>8</td>
<td>0.266</td>
<td>0.135</td>
</tr>
<tr>
<td>10</td>
<td>0.343</td>
<td>0.173</td>
</tr>
<tr>
<td>12</td>
<td>0.665</td>
<td>0.193</td>
</tr>
<tr>
<td>14</td>
<td>0.931</td>
<td>0.220</td>
</tr>
<tr>
<td>16</td>
<td>0.513</td>
<td>0.243</td>
</tr>
<tr>
<td>18</td>
<td>0.301</td>
<td>0.251</td>
</tr>
<tr>
<td>20</td>
<td>0.414</td>
<td>0.252</td>
</tr>
</tbody>
</table>
Figure 69 Changes in the damage indices and the real part of the electromechanical impedance for different damage lengths for PWAS #1 (dotted line is the damaged case; solid line is the pristine case).
The effects of an increased damage length on PWAS #1 and the numerical results for the three damage indices, RMSD, MAPD, CCD, were carried out analytically in MATLAB. The analytical values of the damage indices (DI) were normalized to their maximum value, such that an easy comparison of the three damage indices can be done.

Figure 69 shows the electromechanical impedance spectra for each damage length. This figure allows us to visually correlate the changes in DI’s given in Table 13 with the changes in the impedance spectra.

![DI comparison for different damage lengths for PWAS #1](image)

A comparison of all three DI’s is presented in Figure 70. From Figure 70, it can be seen that all three damage indices modify with the increase in damage length. However, each of the DI curves has a different shape. The only DI that shows monotonic increase with the damage length is the mean absolute percentage deviation (MAPD). The other two, RMSD and CCD, are not monotonic with damage length; this behavior is
attributed to the fact that they are not as sensitive to the small continuous changes in the electromechanical impedance spectrum but rather to the more abrupt ones.

10.3.2 Variation of damage indices with opening of the disbond damage

The effect of the disbond growth located under PWAS #2 on the damage indices for PWAS #1 is investigated. The disbond damage opens (grows) from 0% to 100 %, where 100% is considered to be fully disbonded. We will consider the same adhesively bonded beam as presented in Section 9.2. In this case, the length of the disbond is maintained constant at $a = 20$ mm located at $x = 83$ mm from the left end of the beam. The disbond was then simulated to open up from 0% (“kissing bond”) to 100% disbond (fully disbonded, the adhesive layer fully removed locally between the two aluminum strips). The damage indices for PWAS #1, RMSD, MAPD and CCD were calculated for each disbond opening and the results are listed in Table 14.

<table>
<thead>
<tr>
<th>Damage growth [%]</th>
<th>RMSD</th>
<th>MAPD</th>
<th>CCD</th>
<th>RMSD</th>
<th>MAPD</th>
<th>CCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
<td>0.044</td>
<td>0.040</td>
<td>0.002</td>
</tr>
<tr>
<td>10</td>
<td>0.003</td>
<td>0.001</td>
<td>0.000</td>
<td>0.098</td>
<td>0.090</td>
<td>0.010</td>
</tr>
<tr>
<td>20</td>
<td>0.007</td>
<td>0.003</td>
<td>0.000</td>
<td>0.205</td>
<td>0.189</td>
<td>0.042</td>
</tr>
<tr>
<td>30</td>
<td>0.010</td>
<td>0.004</td>
<td>0.000</td>
<td>0.310</td>
<td>0.288</td>
<td>0.096</td>
</tr>
<tr>
<td>40</td>
<td>0.014</td>
<td>0.006</td>
<td>0.000</td>
<td>0.414</td>
<td>0.387</td>
<td>0.172</td>
</tr>
<tr>
<td>50</td>
<td>0.017</td>
<td>0.008</td>
<td>0.000</td>
<td>0.517</td>
<td>0.489</td>
<td>0.267</td>
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<tr>
<td>60</td>
<td>0.020</td>
<td>0.009</td>
<td>0.000</td>
<td>0.618</td>
<td>0.591</td>
<td>0.382</td>
</tr>
<tr>
<td>70</td>
<td>0.024</td>
<td>0.011</td>
<td>0.000</td>
<td>0.717</td>
<td>0.694</td>
<td>0.514</td>
</tr>
<tr>
<td>80</td>
<td>0.027</td>
<td>0.012</td>
<td>0.001</td>
<td>0.813</td>
<td>0.796</td>
<td>0.661</td>
</tr>
<tr>
<td>90</td>
<td>0.030</td>
<td>0.014</td>
<td>0.001</td>
<td>0.908</td>
<td>0.898</td>
<td>0.824</td>
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<tr>
<td>100</td>
<td>0.033</td>
<td>0.016</td>
<td>0.001</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Figure 71 compares the real part of the electromechanical impedance spectrum for two cases: (a) very small damage opening (1% disbond) and (b) almost complete disbond (90% damage opening).

Figure 71 Comparison of the damage indices and the real part of the electromechanical impedance for 1% opening of the disbond and 90% opening

1% opening
RMSD = 0
MAPD = 0
CCD = 0

90% opening
RMSD = 0.030
MAPD = 0.014
CCD = 0.001

Figure 71  Comparison of the damage indices and the real part of the electromechanical impedance for 1% opening of the disbond and 90% opening
A plot of the damage index values vs. disbond opening is given in Figure 67. From Figure 67 it can be seen that all three damage indices, RMSD, MAPD and CCD predict the damage opening in a similar fashion. However two of them, RMSD and MAPD have a more linear correlation with disbond damage opening. Of this two, it is apparent that the mean absolute percentage deviation (MAPD) is slightly better than root mean square deviation (RMSD). Hence, it is recommended that MAPD is used as metric to characterize the disbond damage because it has a monotonic response to spectral changes due to the two types of damage growth, both disbond length and disbond opening.

Figure 72 DI comparison for different damage opening for PWAS #1
10.4 DI comparison between PWAS #1 and PWAS #2

In this section a comparison between the three damages indices, RMSD, MAPD and CCD, for each of the PWAS transducers (PWAS #1 and PWAS #2) is presented.

10.4.1 Variation of damage indices with the length of disbond damage

We consider first variations in the disbond length and we look at the corresponding changes in the values of the damage indices. For each of the DI, a comparison between the values calculated from the electromechanical impedance response for PWAS #1 and PWAS #2 is given. The results for the three DI’s (RMSD, MAPD and CCD) are shown in Figure 73, Figure 74, and Figure 75.

![RMSD DI vs. damage length](image)

Figure 73 Comparison between DI’s for PWAS #1 and PWAS #2: RMSD DI vs. damage length
Figure 74  Comparison between DI’s for PWAS #1 and PWAS #2: MAPD DI vs. damage length

Figure 75  Comparison between DI’s for PWAS #1 and PWAS #2: CCD DI vs. damage length
From Figure 73, Figure 74, and Figure 75, it can be seen that each of the damage indices increases as the disbond length increases. It is noticeable that the values of the DI’s for PWAS #1, located at around 45 mm from the tip of the disbond, are still changing with the disbond length increase. In other words, PWAS #1 is still sensitive to the presence of the disbond being 45 mm away from the disbond. For small disbond lengths (< 2 mm) the changes in the electromechanical impedance for PWAS #1 are negligible.

10.4.2 Variation of damage indices with opening of the disbond damage

The variation in the DI’s values with the disbond opening for PWAS #1 and PWAS #2 is considered next. The results for the three DI’s (RMSD, MAPD and CCD) are shown in Figure 76, Figure 77, and Figure 78.

![RMSD Diagram](image)

**Figure 76** Comparison between DI’s for PWAS #1 and PWAS #2: RMSD DI vs. disbond opening
Figure 77  Comparison between DI's for PWAS #1 and PWAS #2: MAPD DI vs. disbond opening

Figure 78  Comparison between DI's for PWAS #1 and PWAS #2: CCD DI vs. disbond opening
From Figure 76, Figure 77, and Figure 78 it can be seen that the values of the three DI’s (RMSD, MAPD and CCD) for PWAS #1 are much smaller compared with those for PWAS #2. This is expectable since the opening of the disbond is very small (< 0.12 mm) and PWAS #1 is located 45 mm to the left of the disbond tip, whereas PWAS #2 is directly on top of the disbond. However, the data show that the electromechanical impedance for PWAS #1 is still affected by the disbond location and opening growth.

10.5 Conclusions

From the results presented in this chapter we can draw several conclusions. A positive damage detection for different types of damages, locations and damage growths using only one damage index may be very difficult. Different damage indices respond in different ways to changes in the impedance spectrum. Some are more sensitive than others, to more abrupt changes in the spectra like new peaks and some do take into account small continuous changes like, increase in amplitude and frequencies shifts.

Another factor worth to be mentioned is the frequency range to be analyzed. A frequency range which does not have sufficient resonant frequencies might give erroneous results due to a lack of statistically relevant data set. For this reason it is recommended to apply these damage metrics for frequencies ranges with a high density of resonant peaks.

Two disbond damage situations were studied in this chapter: (a) changes in the disbond length from 0.5 mm to 20 mm; and (b) increase in disbond opening from 1% to 100% disbond. In both cases, we were successful in showing that the damage indices can predict the damage growth. It seems that the mean absolute percentage deviation
(MAPD) in the frequency range of 30 kHz to 60 kHz is the best metric to predict disbond damage growth. The use of such damage metrics can be applied directly to the measured experimental data and does not need any pre-processing of data.

PART III

Experiments on representative specimens

Chapter 11
Experiments on a bonded lap-joint specimen

Experimental work was conducted to validate the assumption that small PWAS can excite Lamb waves in bonded specimens and that this waves will propagate and be received by other PWAS acting as receivers. A long aluminum lap-joint specimen was fabricated using two aluminum 2024T3 stripes as shown in Figure 79 and Figure 81.
Figure 79 Lap-joint specimen consisting of two aluminum stripes bonded together. Artificially disbonds were introduced in the specimen.

The two aluminum stripes are 80mm x 1220mm, and 75mm x 1220mm with a thickness of 1mm. They were bonded together using an epoxy paste adhesive, Loctite Hysol® EA 9309.3NA. The bonded overlap of the two aluminum stripes is 20mm. Two disbonds were artificially created using Mylar® polyester film that was introduced between the two aluminum stripes, removed after the adhesive cured. Thus a discontinuity of the adhesive layer was produced. One disbond was 25 mm wide located at 290 mm from the left edge and the second disbond was 51 mm wide located at 765 mm from the left edge.

Next, the specimen was instrumented with an array of PWAS sensors as presented in Figure 80.

Figure 80 Location of the PWAS on the lap-joint specimen

The sensor array has 11 columns and three rows (A, B, and C). One row of sensors (A) is located on the top aluminum plate, the second row on the bond line (B),
and the third row is located on the second aluminum plate (C). The spacing between each column of sensors is 100 mm and the distance between PWAS on row A and PWAS on row B is 30mm. The distance between PWAS on row B and PWAS on row C is 28mm. PWAS #3B is located above the first disbond and PWAS #8B is located above the second disbond.

To prove successful transmission and reception of Lamb waves, the pitch-catch method was used. In this case, one PWAS was used as a transmitter and another as a receiver. The instrumentation set-up is shown in Figure 81. An HP 33210 signal generator was used to produce a 3-count tone burst with a frequency of 390 kHz. The wave traveled through the specimen and was capture using a Tektronix TDS210 digital oscilloscope.
To study the propagation of Lamb waves, the dispersion phenomenon needs to be addressed. Wave dispersion may occur when the wave speed varies with frequency. Lamb waves are dispersive because the wave speed depends on the product between the wave frequency and the thickness of the plate in which the Lamb waves are traveling. The physical phenomenon can be explained considering a wave packet, or a tone burst, as shown in Figure 82. The tone burst consists of a carrier frequency (tone) and has a short
duration in time (burst). In the frequency domain, there is a dominant frequency, $f_c$, and side frequencies.

**Figure 82**  (a) 5-count sine tone burst; (b) frequency spectrum of the tone burst signal

For non-dispersive wave propagation, the shape of the wave is preserved and the wave speed is constant. All frequency components in the wave packet travel with the same speed and the packet keeps its shape. For dispersive wave propagation, each frequency component travels with a different speed (wave speed is a function of frequency $c = c(f)$) hence the wave packet spreads out, i.e. it disperses. The degree of dispersion depends on the spectrum bandwidth. Typical Lamb waves dispersion curves propagating in plates are shown in Figure 83.
At lower frequency-thickness products, only two Lamb wave modes exist: $S_0$, which is a symmetrical Lamb wave resembling the longitudinal wave; and $A_0$ which is an antisymmetrical Lamb wave resembling the flexural waves. The disadvantage of Lamb waves is that they are highly dispersive because the wave speed varies with frequency.
However, the $S_0$ mode shows less dispersion than the $A_0$ mode at low frequency-thickness values.

### 11.1 Wave propagation damage detection

Lamb waves traveling in bonded specimens encounter complex phenomena at the bond interface. Part of the energy of the incident wave will be transmitted through the bond layer from one side to the other. At the same time, mode conversion and diffraction of the Lamb wave take place. The propagation of the Lamb wave will become highly complicated and several modes may be propagating at the same time. In addition, the viscoelastic behavior of the adhesive layer will cause damping, attenuation of the Lamb wave energy, and frequency shifts. In this section, we will address the possibility to excite Lamb waves in such bonded specimens using small, unobtrusive PWAS transducers.

The propagation of the Lamb waves in the specimen of Figure 79 using pitch-catch method is shown in Figure 84. The results shown in Figure 84 clearly demonstrate the capability of our PWAS to send and receive Lamb waves in the aluminum plate itself and along the bond line between the two plates. It can be observed that for the waves traveling in the aluminum plate alone (Figure 84a) there is attenuation of the signal as the distance increases but the dispersion is very little. However, the Lamb waves traveling along the bond line show a strong dispersion even for the $S_0$ mode, as shown in Figure 84b. This is to be expected because of the damping effect of the adhesive layer and the coupling between the top and bottom plates in the adhesive joint.
Figure 84  (a) S\textsubscript{0} mode Lamb waves traveling in the top aluminum plate; (b) S\textsubscript{0} mode Lamb waves traveling along the bond line
Figure 85 presents the attenuation of the Lamb waves traveling in the aluminum plate outside the bond line and along the bond line.

![S0 Lamb mode attenuation graph](image)

The energy of the signal traveling along the bond line is less than the energy of the signal traveling outside the bond line: the signal is weaker because the adhesive layer is absorbing part of the energy of the transmitted wave.

For a better understanding of the complexity of wave propagation in the bonded structure and the multiple reflections that will arrive at the same time or very close to each other, a simulation of the reflections arriving at each sensor was done. For the sensors located on the bond line, the results are shown in Figure 86.
Figure 86 Simulation of reflections from edges and disbonds arriving at different sensors located on the bond line.

From Figure 86, it can be seen that multiple reflections will arrive at the same time and they will overlap making the identification process difficult. For PWAS #4B located to the right of the first disbond, we see between 20 µsec and 40 µsec that there are three reflections that will overlap: (i) reflection from the bottom edge of the specimen; (ii) reflection from the top edge of the specimen and; (iii) the reflection from the disbond. In order to eliminate some of the reflections from the edges of the specimen, a non-drying modeling clay was used on the periphery of the specimen (Figure 87).
The modeling clay absorbed the energy of the wave arriving at the edge and the reflected wave does not have enough energy to reach back the sensor; thus, the only reflection arriving at the sensor is the one from the disbond.

### 11.1.1 Pitch-catch method for damage detection

For the pitch-catch method we use two PWAS transducers, one acting as a transmitter and another acting as a receiver. A tone burst was sent from one PWAS and received at the other PWAS. As the tone burst travels over the length of the structure between the transmitter and the receiver, its amplitude, frequency and wave mode may change. These features will be greatly affected by any damage that could be present in the structure.

The pitch-catch method was used to detect the 51 mm disbond situated at 775 mm from the left end. The location of the sensors used to identify the damage is presented in Figure 88.
A 3-count tone burst at 130 kHz was sent from each of the sensors placed on the bottom aluminum strip (PWAS #6C, PWAS #7C, and PWAS #8C) and received by the sensors placed on top of the aluminum strip (PWAS #6A, PWAS #7A, and PWAS #8A). As the Lamb wave travels through the bottom aluminum strip, into the adhesive bond and then into the top aluminum strip, we anticipate that the wave sent from PWAS #8C and received at PWAS #8A will be greatly attenuated.
The experimental results are shown in Figure 89. From Figure 89, we see that the waves travelling over a good bonded joint have the same amplitude and the same reflections arriving at the same time (good symmetry of the sensor placement). However, the waves travelling in a bad bonded joint where a disbond is present are much attenuated (PWAS #8A $\rightarrow$ PWAS #8A). The energy of the incident wave cannot leak into the bond layer and then back into the other aluminum plate but goes around the disbond and part of the energy arrives at the receiver PWAS.

Based on the experimental results, it was shown that the pitch-catch method can be successfully used across the bond layer to detect disbonds in the adhesive layer. Damage indices can be developed to predict the presence of damage; based on the attenuation of the signal, it is also possible to quantify the size of the damage. For successful and positive damage identification, the pitch-catch method should be used in conjunction with other damage detection methods (i.e. pulse-echo, electromechanical impedance, phased arrays, etc.)

### 11.1.2 Pulse-echo method for damage detection

For the pulse-echo method, we use only one sensor that acts as both transmitter and receiver. A guided wave is excited and propagated in the structure. If the wave front encounters any damage (i.e. disbonds, cracks, delaminations, etc.) part of the wave will be reflected back to the sensor and it will show in the time domain spectrum as a reflection. The goal of our work is to use the pulse-echo method and detect a disbond 51 mm long in the adhesive joint between the two aluminum strips. To accomplish our goal, we use two sensors located on the bond line, PWAS #6B (the baseline) and PWAS #7B located on the bond line close to the disbond.
In order to properly identify changes in the wave spectrum, we need to know the wave speed in the host structure. For the aluminum plate this might not be so complicated, since we know the material the wave speed can be found from the published literature (or from the dispersion curves Figure 83). For the bond layer however, this is not so immediate. To calculate the wave speed, we measure the time of flight (TOF) between two known locations of the sensors and then calculate the wave speed.

We considered the sensors PWAS #5A and PWAS #6A located on the top aluminum strip and sensors PWAS #5B and PWAS #6B located on the bond line. The measured distance between the sensors was 100 mm. The time of flight measured with the oscilloscope was

\[ \Delta t_{\text{Al}} = 21 \mu \text{sec} \ \Rightarrow \ \nu_{\text{Al}} = 4762 \text{ m/s} \]

\[ \Delta t_{\text{bond}} = 25 \mu \text{sec} \ \Rightarrow \ \nu_{\text{bond}} = 4000 \text{ m/s} \]

Figure 90 Pulse-echo method: PWAS location used to detect the disbond between the two bonded aluminum strips: (a) 3-D view; (b) cross-section in the bond region
Our goal is to use the pulse-echo method to detect the second disbond on the fabricated specimen. The sensors used for disbond detection were sensors PWAS #6B and PWAS #7B located on the bond line (Figure 90). A 3-count tone burst with the frequency of 130 kHz was sent from both PWAS #6B and PWAS #7B. The location of PWAS #6B is far enough from the disbond (175 mm) such that no reflection will be received in the 0 -60 µsec span. Note that we used the modeling clay to reduce the reflections from the specimen edges such that the reflection from the disbond can be more easily identified. The experimental results are shown in Figure 91.

From Figure 91, it can be seen that there are no reflection in the time range of 60 µsec for PWAS #6B, in accordance with our simulation for the arriving reflections shown in Figure 86. The time domain data for PWAS #7B located close to the disbond (67 mm)
shows a reflection packet at around 38 µsec which is the reflection from the disbond, in accordance with the reflections simulation shown in Figure 86.

These experimental results showed that permanently attached PWAS sensors can be used to excite Lamb waves in the bonded structure and to detect a disbond between two aluminum strips. Future work needs to be done to address the signal processing part of the detection method, the wave tuning process, and the development of the damage indices to indicate the presence of the damage.

11.2 Standing wave damage detection – electromechanical impedance method

The impedance method is a damage detection technique complementary to the wave propagation method. The conventional mechanical impedance method consists of exciting vibrations of bonded plates using a specialized transducer that simultaneously measures the applied normal force and the induced velocity. The electro-mechanical (E/M) impedance method is an emerging technology that offers distinctive advantage over the conventional mechanical impedance method. While the mechanical impedance method uses normal force excitation, the electromechanical impedance method uses in-plane strain. The mechanical impedance transducer measures mechanical quantities (force and velocity/acceleration) to indirectly calculate the mechanical impedance, while the electromechanical impedance active sensor measures the E/M impedance directly as an electrical quantity.

The effect of a PWAS transducer affixed to the structure is to apply a local strain parallel to the surface; this creates stationary elastic waves in the structure. Through the
mechanical coupling between the PWAS and the host structure, on one hand, and through the electro-mechanical transduction inside the PWAS, on the other hand, the drive-point structural impedance is directly reflected into the effective electrical impedance as seen at the PWAS terminals.

11.2.1 Validation of the experimental E/M impedance readings

The same specimen presented in Figure 80 was used to validate experimentally the E/M impedance method. An HP4194A impedance analyzer was used to measure the electromechanical impedance signature of the PWAS attached to the structure. Based on initial exploratory tests, the frequency range 650 kHz to 2 MHz was selected. Measurements of the real part of the electromechanical impedance for several sensors were taken. Repeated sampling of the data indicated a stable and reproducible pattern of the impedance spectrum. The results for measurements of PWAS #6A&B and PWAS #7A&B are presented in Figure 92 and Figure 93.
Figure 92 Real part or the electromechanical impedance spectrum for PWAS #6A and PWAS #6B located on the aluminum plate and the bond line respectively.

Figure 93 Real part or the electromechanical impedance spectrum for PWAS #7A and PWAS #7B located on the aluminum plate and the bond line respectively.
As seen from Figure 92 and Figure 93, the resonant spectra for the sensors placed on the aluminum plate (PWAS #6A and PWAS #7A) are consistent from one sensor to the other. In addition, they are different from the resonant spectrum for the sensors placed on the bond line (PWAS #6B and PWAS #7B). These observations clearly reveal the fact that, as anticipated, the adhesive layer has an important impact on the standing Lamb waves pattern and pointwise impedance. The adhesive layer acts like a damper dissipating part of the wave energy, and hence the spectrum for sensors #6B and #7B is much more depressed than for #6A and #7A.

**11.2.2 Damage detection using EMI method**

The electromechanical impedance method was used to detect the two disbonds in the manufactured specimen. The sensors PWAS #6A, 6B, 6C (located in a good bond region) are used as the baseline when compared with the sensors PWAS #3A, 3B, 3C and PWAS #8 A, 8B, 8C which are located next to disbonds (Figure 94).

![Figure 94 E/M impedance: PWAS location used to detect the disbonds between the two bonded aluminum stripss](image)

Each sensor was excited in the frequency range of 650 kHz to 2 MHz using the HP4194A impedance analyzer and the respective electromechanical impedance spectra were collected. The electromechanical impedance spectra for the sensors located on a good bond are shown in Figure 95.
From Figure 95 it can be seen that the sensors locate on the aluminum plates (PWAS #6A and PWAS #6C) have a strong resonant peak at 950 kHz; their electromechanical impedance spectra are similar. The amplitudes of the two resonant frequencies are slightly different because the two aluminum strips have different dimensions; the one on top plate is 80 mm wide while the bottom plate is 75 mm wide.

The sensor located on the bond line, PWAS #6B, has a totally different electromechanical impedance spectrum in the frequency range of 650 kHz to 2 MHz. The thickness of the bonded material (two plates plus adhesive) underneath the sensor is more than twice as the one for just one aluminum plate; in addition the bond layer through its viscoelastic properties will also affect the resonant amplitudes. All these factors are reflected in the impedance spectrum for PWAS #6B shown in Figure 95 which is clearly different from #6A and #6C.
Figure 96 Electromechanical impedance spectra for PWAS #3A, 3B, 3C – 25 mm disbond

Figure 97 Electromechanical impedance spectra for PWAS #8A, 8B, 8C – 51 mm disbond
The electromechanical impedance for sensors PWAS #3A, 3B, 3C and PWAS #8A, 8B, 8C located on the two disbonds are shown in Figure 96 and Figure 97. The impedance spectra for sensors PWAS #3A and PWAS #8A located on the top aluminum layer are very similar with the impedance spectra for PWAS #6A showing the same main resonant peak at 950 kHz. Same results are also for PWAS #3C and PWAS #8C located on the bottom aluminum layer. The main difference in the electromechanical impedance spectra is for PWAS #3B and PWAS #8B. When compared with the impedance spectrum for PWAS #6B they show a strong resonant frequency around the same 950 kHz frequency. The resonant peaks are very close to the ones from the sensors located on the aluminum layers and this is a strong indication of the presence of the disbonds.

11.2.3 Damage indices used for disbond detection

Two of the damage indices (DI) developed in Chapter 9 were used to identify changes due to the presence of the two disbonds. The two DI were: the root mean square deviation (RMSD) and the mean absolute percentage deviation (MAPD). The electromechanical impedance spectra collected from the sensors locate on the bond line were used.

The data set for PWAS #6B was considered as the baseline and the damage indices were calculated for each sensor using Eq. (10.1) and Eq. (10.2). The impedance spectra for all the sensors are shown in Figure 98.
The calculated values for the RMDS and MAPD damage indices are listed in Table 15.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>RMSD</th>
<th>MAPD</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWAS #2B</td>
<td>0.326</td>
<td>0.180</td>
</tr>
<tr>
<td>PWAS #3B</td>
<td>0.430</td>
<td>0.240</td>
</tr>
<tr>
<td>PWAS #4B</td>
<td>0.370</td>
<td>0.213</td>
</tr>
<tr>
<td>PWAS #6B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PWAS #7B</td>
<td>0.539</td>
<td>0.470</td>
</tr>
<tr>
<td>PWAS #8B</td>
<td>0.712</td>
<td>0.561</td>
</tr>
<tr>
<td>PWAS #9B</td>
<td>0.526</td>
<td>0.423</td>
</tr>
</tbody>
</table>
For a better understanding of the results and to compare the two damage indices, the DI values were also plotted (Figure 99). PWAS #3B and PWAS #8B are the sensors located directly above the two disbonds.

![Figure 99](image)

**Figure 99** Variation of the two damage indices (RMSD and MAPD) with the location of PWAS

From Figure 99 it can be seen that both damage indices predicted the disbonds located under PWAS #3B and PWAS #8B. Also, as we get closer to the disbond the DI increases (PWAS #2B and PWAS #7B) and as we go away from the disbond the DI decreases (PWAS #4B and PWAS #9B).

In conclusion, we can summarize that successful experimental damage detection was possible using small PWAS transducers permanently attached to the structure. Wave propagation methods (i.e. pitch-catch and pulse-echo) as well as standing waves methods
(electromechanical impedance) were used to detect disbonds in a fabricated specimen with one 25 mm disbond and one 51 mm disbond. We could positively identify damage with each of the methods described above; however, if possible one method should always be used in conjunction with the other method in order to validate the results and to minimize false positive and negative results.
Chapter 12
Experiments on a spacecraft-like panel specimen

Ultrasonic guided waves inspection using Lamb waves is suitable for damage detection in metallic structures. In this chapter we will present experimental results obtained using guided Lamb waves to detect flaws in aluminum specimens with design features applicable to space applications. Two aluminum panels were fabricated from a variable-thickness aluminum top plate, with two bolted I-beams edge stiffeners and four bonded angle stiffeners. Artificial damages were introduced in the two panels: cracks, corrosions, and disbonds. The proposed investigation methods used embedded piezoelectric wafer active sensors (PWAS) to excite and receive Lamb waves. Three wave propagation methods were used: pitch-catch, pulse-echo, and the embedded ultrasonic structural radar (EUSR). In addition, we also used a standing-wave damage detection technique, the electro-mechanical impedance method. Where appropriate, comparison between different methods in detecting the same damage will be performed. The results have demonstrated the ability of piezoelectric wafer active sensors working in conjunction with guided Lamb waves to detect various types of damages present in complex geometry structures typical of space applications.
Two aluminum test panels were fabricated by NextGen Aeronautics, Inc. The panels consist of the skin (Al 7075, 24x23.5x0.125 in) with a 3 in diameter hole in the center, two spars (Al 6061 I-beams, 3x2.5x0.250 in and 24 in length), four stiffeners (Al 6063, 1x1x0.125 in and 18.5 in length) and fasteners installed from the skin side (Figure 100).

![Figure 100 Structural panel design](image)

The stiffeners were bonded to the aluminum skin using a structural adhesive, Hysol EA 9394. Damages were artificially introduced in the two specimens including cracks (CK), corrosions (CR), disbonds (DB), and cracks under bolts (CB). A summary of the type and size of the damages is provided in Table 16.
### Table 16 Induced panel damages

<table>
<thead>
<tr>
<th>Panel 1</th>
<th>Panel 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ID</strong></td>
<td><strong>Size (inches)</strong></td>
</tr>
<tr>
<td>CK1</td>
<td>0.75 x 0.015 x thru thickness</td>
</tr>
<tr>
<td>CK2</td>
<td>0.50 x 0.015 x thru thickness</td>
</tr>
<tr>
<td>CK3</td>
<td>0.50 x 0.015 x thru thickness</td>
</tr>
<tr>
<td>CK4</td>
<td>0.50 x 0.015 x thru thickness</td>
</tr>
<tr>
<td>CR1</td>
<td>1.65 x 0.38 x 0.025</td>
</tr>
<tr>
<td>CR2</td>
<td>0.50 x 0.38 x 0.025</td>
</tr>
<tr>
<td>DB1</td>
<td>2.00 x 0.50</td>
</tr>
<tr>
<td>DB2</td>
<td>1.00 x 1.00</td>
</tr>
<tr>
<td>DB3</td>
<td>2.00 x 0.50</td>
</tr>
<tr>
<td>DB4</td>
<td>1.00 x 1.00</td>
</tr>
</tbody>
</table>

![Figure 101 Schematic of the location and the type of the damage on the Panel 1 specimen (top view)
A schematic of the aluminum Panel 1 specimen showing the location of the damage is presented in Figure 101. Panel 1 contains disbonds, cracks and corrosions. The disbonds are located between the stiffeners and the skin. They are of two types: partial disbonds DB1 and DB3, and through disbonds DB2 and DB4. The corrosions are simulated as machined areas were part of material was removed. The four cracks presented are in the shape of a slit and are through cracks located on the skin of the panel.

The schematic of Panel 2 is presented in Figure 102. Panel 2 contains only corrosions and cracks. The two corrosions are hidden between the skin and the spars and are simulated in the similar way by material removal. Cracks are located under the bolts and are through the thickness cracks.
The panels were instrumented with piezoelectric wafer active sensors (PWAS) as shown in Figure 101 and Figure 102. The PWAS are used for both sensing and receiving Lamb waves. The experimental set-up is presented in Figure 103.

Figure 103 Instrumentation set-up: (a) wave propagation; (b) electromechanical impedance
The instrumentation set-up for the wave propagation method is shown in Figure 103a. An HP 33120 signal generator was used to excite the PWAS with a 3-count sinusoidal burst signal at a frequency of 330 kHz. The received signal was displayed on a Tektronix TDS 210 digital oscilloscope and transferred to a laptop. For the electromechanical impedance method the instrumentation set-up is shown in Figure 103b. The real part of the EM impedance spectrum was recorded using an HP 4194A impedance analyzer, and the frequency range of the excitation was 150 kHz to 1.5 MHz.

A significant number of sensors were initially used to ensure a good signal response near known damage locations. The optimum number of sensors/methods is to be determined such that the signal response will give an acceptable probability of detection for certain types of damage. The detection capabilities used were:

- Wave propagation
  - Pitch-catch: disbond detection
  - Pulse-echo: disbond and crack detection
- Standing wave
  - Electromechanical Impedance (EMI): disbond, crack and corrosion detection
The detection methods used in this experiment are summarized in Table 17. This table provides a quick correlation between the damage type and the detection method. A sample of the test matrix for data collection from the PWAS is presented in
<table>
<thead>
<tr>
<th>Method</th>
<th>Transmitter</th>
<th>Receiver</th>
<th>Excitation freq (kHz)</th>
<th>Excitation ampl (Vpp)</th>
<th>Mode</th>
<th>Damage</th>
</tr>
</thead>
<tbody>
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<td>Pitch-catch</td>
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<td>a3</td>
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<td>DB1</td>
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<tr>
<td></td>
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<td>a11</td>
<td>110</td>
<td>10</td>
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<td>DB2</td>
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<tr>
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<td>a19</td>
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<td>10</td>
<td>S0</td>
<td>DB3</td>
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<td>DB4</td>
</tr>
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</table>

12.1 PWAS calibration

Two PWAS were attached to a testing sample skin to calculate the wave speed for the A0 and S0 Lamb modes. The pitch-catch method was used and a 3-count sine smoothed burst signal was sent from PWAS #1 (the transmitter) to PWAS #2 (the receiver). The location of the two sensors is presented in Figure 105. The optimum
excitation frequency for the A0 mode is 60 kHz, and for the S0 mode is 330 kHz. The signal sent and received is shown in Figure 104.

![Diagram showing time of flight for the wave transmitted from PWAS #1 to arrive at PWAS #2 when the A0 Lamb mode was excited; (b) time of flight for the wave transmitted from PWAS #1 to arrive at PWAS #2 when the S0 Lamb mode was excited.]

Figure 104  (a) time of flight for the wave transmitted from PWAS #1 to arrive at PWAS #2 when the A0 Lamb mode was excited; (b) time of flight for the wave transmitted from PWAS #1 to arrive at PWAS #2 when the S0 Lamb mode was excited.
Figure 105  PWAS location used to calculate the A0 and S0 Lamb modes wave speed

Using the information extracted from Figure 104 the wave speed was calculated as:

\[ \Delta t_{A0} = 84.8 \ \mu \text{sec}, \ \Delta t_{S0} = 28.6 \ \mu \text{sec}, \ L = 143 \ \text{mm} \]

\[ c_{A0} = \frac{L}{\Delta t_{A0}} = 1.7 \ \text{mm/}\mu\text{s}, \ c_{S0} = \frac{L}{\Delta t_{S0}} = 5 \ \text{mm/}\mu\text{s} \]

where \( c_{A0} \) is the phase velocity of the A0 mode, \( c_{S0} \) is the phase velocity of the S0 mode, \( \Delta t \) is the time of flight, and \( L \) is the distance between the two sensors.

To validate the transmission and reception of Lamb waves and to positively identify the reflections in the received signal, the pulse-echo method was used. Knowing the location of the damage (middle circle) and the relative position of the edges of the plate from the two PWAS as well as the group velocity the reflections can be labeled in the time domain response (Figure 106 and Figure 107).
Figure 106 Pulse-echo method: received signal at PWAS #1

Figure 107 Pulse-echo method: received signal at PWAS #2
12.2 Pitch-catch method

The PWAS used for the pitch-catch method and the actual location of the sensors used on Panel 1 is presented in Figure 108.

Figure 108  PWAS location on Panel 1 for pitch-catch tests: (a) top face of Panel 1; (b) bottom face of Panel 1
The locations of the disbonds are presented in Figure 108a: DB1 located between PWAS a1 and PWAS a3, DB2 located between PWAS a9 and PWAS a11, and DB3 between PWAS a17 and PWAS a19. Disbonds DB1 and DB3 are 2.00 in x 0.50 in and represent partial disbond of the stiffener, and DB2 is 1.00 in x 1.00 in and represents total disbond of the stiffener. The results for the disbonds DB1, DB2, and DB3 are presented in Figure 109 and Figure 110. They show the signals sent and received from sensors located on a stiffener with a disbond and the signal received by a sensor located on a stiffener without a disbond.

![Figure 109 Pitch-catch method: the signal traveling over the disbond DB1 is compared with the signal traveling the same distance over a bonded area.](image_url)
Figure 110  Pitch-catch method: the signals traveling over the disbond DB2 and DB3 are compared with the signal traveling the same distance over a bonded area.

It can be seen from Figure 109 that there is a strong difference between the wave traveling over a disbond and the same wave traveling over a good bond. For a better quantification of the damage present in the received signals, a damage index was developed. The damage index (DI) will take into consideration the energy of the wave traveling over a pristine area and the energy of the wave traveling over a disbonded area. The damage index plotted in Figure 111 was calculated with the formula:

\[
DI = 1 - \frac{\text{Energy}_{\text{pristine}}}{\text{Energy}_{\text{damage}}} = \begin{cases} 
0, & \text{no damage} \\
1, & 100\% \text{ damage} 
\end{cases}
\]  

(12.1)

where energy was calculated as:

\[
\text{Energy} = \sum \frac{x^2}{n}
\]

(12.2)
Figure 111 Damage Index of the three signals showing an increase of the DI with the severity of the damage.

Figure 111 shows that the DI increases going from an undamaged area to different types of disbonds and this is in accordance with an increase in the severity of the damage.

12.3 Pulse-echo method

The PWAS used for the pulse-echo method and the actual location of the sensors used on Panel 1 is presented in Figure 112.
For disbond detection using the pulse-echo method, a 3-count smoothed sine burst at 330 kHz was used to excite S0 mode Lamb waves into the testing specimen. The results shown in Figure 113 and Figure 114 refer to the disbond DB2. For this case the PWAS a7, a8, a20, and a21 were used. PWAS a7 is located close to the disbond DB2.
whereas PWAS, a8, a20, and a21 are located in a pristine area where no damage is present. The comparison of the signal received from the damage with the signals where there is no damage is presented in Figure 114.

![Figure 113 Pulse-echo method: pristine data from PWAS a8, a20, & a21 showing a good consistent response](image-url)
Figure 114 Pulse-echo method: initial bang removed, received signal at PWAS a7 from the disbond DB2. Pristine signals from PWAS a8 & a20. Additional reflections due to the presence of damage.

Figure 114 shows clear changes in the received signal (additional reflections) close to the damage. The presence of additional reflections is associated with echoes from the disbonded area.

### 12.4 Electromechanical impedance method

The PWAS used for the EMI method and the actual location of the sensors used on Panel 1 is presented in Figure 115. The electromechanical impedance method was used to detect disbonds, cracks and corrosions. The results are presented in Figure 116 through Figure 118. The impedance spectrum from PWAS a1, a2, and a3 is presented in Figure 116. It can be seen that the resonant spectrums of the signals from PWAS a1 and a3 located on an area with good bond are almost identical. The resonant spectrum from
PWAS a2 located on the disbond DB1 is very different showing new strong resonant peaks associated with the presence of the disbond.

Figure 115  PWAS location on Panel 1 for the EM impedance tests: (a) top face of Panel 1; (b) bottom face of Panel 1
Figure 116  EM Impedance method: resonant frequencies spectrum showing increased amplitude for the signal received at the sensor located on the top of disbond DB1 (PWAS a2)

Figure 117  EM Impedance method: resonant frequencies showing shifted peaks for corroded area CR1 (PWAS b30) vs. undamaged area (PWAS b31)
The electromechanical impedance method also showed good results in detecting corrosion. In Figure 117 the resonant spectrum for the sensors PWAS b30 (corroded area) and PWAS b31 (undamaged area) are presented. The shift in the resonant frequency peak for the PWAS b30 is very clear which is an indication of a structural change due to the corrosion.

![Resonant frequencies of sensors](image)

**Figure 118** EM Impedance method: resonant frequencies of the sensor close to the crack CK1 (PWAS a30) and the sensors in a pristine area (PWAS b29 and b34)

The same phenomena of shifted and prominent resonant frequencies is noticeable in the crack CK1 detection shown in Figure 118. PWAS a29 and PWAS a31 show a consistent resonant spectrum and PWAS a30 presents the features mentioned above.

The experimental results presented in this chapter indicate that PWAS-based structural health monitoring can be successfully applied to the damage detection in metallic specimens typical of current spacecraft structures. The comparison of various damage types and various detection methods indicates that no single method is capable to
universally detect all the damage types that can appear in a typical spacecraft structure. To assist this interpretation, Table 19 gives a summary of how various damage detection methods fared in relation with the various damage types considered in our tests. From Table 19, one sees that some methods are more appropriate for certain damage types and less appropriate for other damage types. The results are based on near-field and medium-field damage detection for a specific damage size. The use of far field damage detection was not practical in our studies due to relatively small size of our specimens.

Table 19 Summary of PWAS detection methods

<table>
<thead>
<tr>
<th>Damage Type</th>
<th>Wave propagation</th>
<th>Standing wave</th>
<th>E/M impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pitch-catch</td>
<td>Pulse-echo</td>
<td></td>
</tr>
<tr>
<td>Disbond</td>
<td>Fair</td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>Cracks</td>
<td>-</td>
<td>Fair</td>
<td>Fair</td>
</tr>
<tr>
<td>Corrosion</td>
<td>-</td>
<td>-</td>
<td>Good</td>
</tr>
<tr>
<td>Crack under bolt</td>
<td>Good</td>
<td>Fair</td>
<td>Fair</td>
</tr>
</tbody>
</table>

Hence, the main conclusion is that a multi-method approach is the advisable course of action for detecting a multitude of damage types as considered in our study. They allow us to conclude that successful damage detection can be achieved by using in combination both traveling waves methods (pitch-catch, pulse-echo, and phased arrays) as well as standing waves methods (electromechanical impedance method). For most cases, most of these damage detection approaches can be achieved with the same installation of PWAS transducers into the structure, which illustrates the versatility of PWAS-based structural health monitoring strategies.
Chapter 13
Experiments on a full scale helicopter blade

Helicopters are continuously subjected to periodic loads and vibration environments that initiate and propagate fatigue damage in many components. Current helicopter maintenance practice requires a large number of parts to be monitored and replaced at fixed intervals. This is a time consuming and also an expensive procedure. Previous chapters have shown that PWAS transducers can be successfully used to detect damage in simple metallic specimens. This chapter will address the possibility of using small PWAS transducers permanently attached to the structure to send and receive guided waves in a full scale adhesively bonded helicopter blade.

13.1 Motivation

One type of helicopter rotor blade is constructed by adhesively bonding titanium sheet metal C-sections together (Figure 119a). This bond is susceptible to extreme temperature conditions, particularly in cold weather in a climate where the relative humidity is high. The rotor blade has a built-up construction consisting of sheet-metal members adhesively bonded with high-performance structural adhesive. Composite substructures are also incorporated at the blade tip. The blade trailing consists of a non-
metallic honeycomb construction. In-service blades have shown disbonds appearing between the structural elements due to in-flight vibrations (Figure 119a).

Figure 119 (a) Helicopter blade cut-out section showing adhesively bonded C-section and disbond location; (b) Disbond inspection using putty knife

The problem that occurs is that the blade sections, held together with the adhesive, can be separated when moisture is trapped between sections and freezes. This separates the C-sections of the blade and forms a crack in the outer skin of the blade. Currently the only way to detect these flaws is to visually inspect the blade for the appearance of cracks, and if one is detected, insert a putty scraper to determine if the epoxy has separated (Figure 119b). This rudimentary and time-consuming task is both inefficient and unable to detect a separation until the skin of the blade cracks. If the detected separation is less than 6-12”, new epoxy can be injected using a hypodermic needle, and the sections are then clamped down until the adhesive takes hold. However, if the separation is larger than 6-12”, it is considered un-reparable, and must be sent for a depot repair. It is possible to prevent such costly repairs with early detection. Piezoelectric wafer active sensors (PWAS) may present a solution to this early detection need. PWAS can both transmit and collect data on ultrasonic vibrations within the structure to which
they are attached. Studying the variations in the transmitted signal and the received signal, one can determine if there is a discontinuity in the material.

### 13.2 Instrumentation setup

The tip section of a helicopter main rotor blade was instrumented with an array of PWAS sensors as shown in Figure 120. The array consists of 15 sensors disposed in five columns and three rows. The sensors on the first and the third row are mounted along the bond line (Figure 120b) while the sensors on the second row are mounted on the skin of the blade. The capability of successfully sending Lamb waves from one sensor and receiving the signal on the other sensors was investigated. A schematic of the instrumentation setup is presented in Figure 120b along with the location of the PWAS sensors on the main rotor blade.
The instrumentation setup consists of an HP 33120 signal generator to create the excitation signal, a Tektronix TDS210 digital oscilloscope to collect the signal from the PWAS, and a computer to store and analyze the signal.
Figure 121  Surface-guided Lamb waves received at (a) sensors B2 & C2 (100 mm apart) when excited from sensor A2 along the bond line (b) received at sensors B1, B2 & B3 when

In our preliminary investigation we sent and received signals using an array of 15 PWAS, some used as transmitters, other used as receivers. Using the signal generator, a 3-count tone burst signal at a frequency of 330 kHz was sent from sensor A1 and received at sensors B1 and C1 along the bond line, as shown in Figure 121a. Also the
same signal was sent from sensor A1 to sensor B1, B2 and B3 across the bond line as shown in Figure 121b.

This shows clearly the possibility of sending and receiving surface Lamb waves along and across the bond line of a helicopter blade. It should be noticed the attenuation of the amplitude of the signal received at sensor B3 due to the fact that it is traveling across two bond lines (Figure 121b). The same attenuation is observed also for Lamb waves traveling along the bond line, the further the sensor from the transmitter the smaller is the amplitude of the received signal (Figure 121a).

13.3 Tuning, attenuation and dispersion of guided waves traveling in a real helicopter blade

Figure 122 represents the signal received at locations B1, C1, D1, and E1 when the PWAS at A1 was excited in the frequency band 1-500 kHz. These signals traveled over the bond area, consisting of the two titanium layers plus the adhesive in between. However it is remarkable that signals of good quality and strength are observed in spite the small size of our PWAS devices. Also noticed in Figure 122 is how frequency affects the amplitude of the signal transmission.

This aspect is important for designing PWAS installation that is tuned to certain Lamb modes. Figure 122 shows that, at relatively low frequency, good signals are received. However, as indicated by the Lamb wave theory, these signals are of the flexural type (A₀) and thus highly dispersive. Of considerable interest are the signals of axial type (S₀), which have a lower dispersion rate, and hence are better suited to ultrasonic NDE. In the bonded region, we observe that the S₀ Lamb waves are
preferentially excited in the 300-450 kHz range. This fact is very encouraging because the S\textsubscript{0} modes have very little dispersion at 300-450 kHz and hence could be use in the pulse-echo mode.

![Graph](image_url)

Figure 122  Lamb waves traveling (a) along the bond line; (b) outside the bond line

The attenuation of the signals when traveling along the bond line is presented in Figure 123. From the two graphs presented in Figure 123 it can be seen that the attenuation of the A\textsubscript{0} mode is much less than the attenuation of the S\textsubscript{0} mode. Also, we observe that the adhesive layer has a higher influence on the S\textsubscript{0} mode. Looking at the
Figure 123a vs. Figure 123b (i.e., on bond line vs. outside the bond line), we noticed that the amplitude of the $S_0$ mode is much smaller for the Lamb waves traveling in the bond line. This is expected since the adhesive layer absorbs part of the energy and the signal that arrives at the sensing PWAS is much weaker than the original excitation signal.

![Diagram of A0 Lamb wave mode](image)

- **Outside bond line**
  - $y = 39.037x^{-0.9245}$
  - $y = 19.849x^{-0.9595}$

- **Bond line**
  - $y = 19.849x^{-0.9595}$

![Diagram of S0 Lamb wave mode](image)

- **Outside bond line**
  - $y = 40.3x^{-0.9863}$
  - $y = 7.8539x^{-1.5562}$

- **Bond line**
  - $y = 7.8539x^{-1.5562}$

Figure 123  Attenuation of the (a) $A_0$ mode Lamb waves traveling along the bond line; (b) $S_0$ mode Lamb waves traveling along the bond line

Considering now the wave signals transmitted from the PWAS at A2 and received at PWAS B2,…, E2 (outside the bond line), we observed that the optimal excitation frequency to excite the $S_0$ Lamb mode appeared in the 250-400 kHz frequency band (Figure 124).
This is explainable through the Lamb wave theory because in the area between two bonding lines the material thickness is much less than in the bonded area. To illustrate this, Figure 124 presents the raw signals collected on sensor B2 and C2 when sensor A2 was excited with a 300 kHz sinusoidal tone burst. The amplitude of the signal collected by the sensor B2 is the highest, since sensor B2 is closest to the transmitter, and the amplitude of the signal is smaller and smaller as we get further away from the transmitter sensor A2.

In this chapter we demonstrated that embedded PWAS can be successfully used for sending and receiving Lamb waves across the bond line. We have also demonstrated that PWAS can be used to send Lamb waves along the bond line. We noticed that the attenuation along the bond line is stronger than outside the bond line. This strong attenuation was especially observed for the S0 Lamb wave mode. For the A0 Lamb wave mode, the attenuation along the bond line was less pronounced. These observations are
useful for developing a disbond detection method using wave leakage and acusto-ultrasonic techniques. However, further work needs to be done in order to understand and determine how the Lamb waves interact with the bond layer, how dispersion and attenuation of the excitation signal will affect the measurements.

Chapter 14
Conclusions and future work

This dissertation has studied the feasibility of using small, unobtrusive, permanently attached, low-cost piezoelectric wafer active sensors (PWAS) for structural health monitoring (SHM) of bonded structures and detection of disbond damage. A review of the vibration and wave propagation principles as well as the state of the art for Lamb waves damage detection methods was presented. The major focus of this research was directed towards the electromechanical impedance method for disbond detection. The electromechanical impedance (EMI) method was modeled extensively with analytical and finite element method; the theoretical predictions were compared with carefully conducted experiments on geometrically tractable specimens. Other damage detection methods, i.e. pitch-catch and pulse-echo, were also introduced but used only experimentally. Other damage detection methods, i.e. pitch-catch and pulse-echo, were also introduced and experimental results were presented.
In most real applications, the structure under investigation is far more complicated than the simple uniform beam used in laboratories. The need for an analytical model to predict the behavior of a multi-layer disbonded structure determined us to push our efforts towards developing an analytical model, which would be simple and robust and can accurately predict the vibration response under PWAS excitation.

Significant work was done by many researchers to address the vibration analysis of simple uniform beam under axial and flexural loading using the modal analysis. The high-frequency vibration modal analysis method using PWAS transducers (Giurgiutiu, 2008) for calculating the electromechanical impedance was used as a starting point in developing a new model. The attention was focused towards the transfer matrix method introduced by Pestel and Leckie (1963) for studying the vibration of mechanical systems. The simplicity of the method which uses transfer matrices and state vectors was very appealing. The first step was to develop the analytical model using the transfer matrix method for the simpler case of uniform beams using the Euler-Bernoulli beam theory (shear deformation and rotary inertia were neglected). Based on the encouraging initial results, we expanded the method to address the multi-layer adhesively bonded beams. The length of the beam was divided in smaller segments, and for each segment the state vectors (displacements and internal forces) were calculated using the field transfer matrices and the point transfer matrices. If damage (disbond) is present in the structure the model becomes more complicated and the beam is divided into branches. For each branch, the mathematical formulation for the equivalent material properties, field and point transfer matrices were developed. Finally, the state vectors, hence the frequency response function at any location on the beam was calculated. The mechanical frequency
response function was used to calculate the electromechanical impedance of the attached PWAS.

A finite element analysis (FEA) was also performed for comparison with the analytical results. A conventional FEA analysis in which a harmonic force was applied at the nodes corresponding to the PWAS two ends was performed first, using the commercial code ANSYS. Then, a coupled-field FEA was performed using the coupled-field option of PLANE13 element in ANSYS. A voltage was applied to the top and bottom electrodes of the PWAS and the electric charged was measured. The electrical charge accumulated on the surface electrodes of the PWAS is directly correlated with the changes in the mechanical stiffness of the structure. The electrical charge was then used to calculate the current and then the electromechanical impedance was calculated as the ratio between the applied voltage and the current.

To validate the analytical model, two sets of experiments were conducted: (i) uniform beam specimen with one PWAS transducer attached; (ii) pristine and damaged (disbonded) specimens. For the second set three PWAS transducers were installed on each specimen and the electromechanical impedance was recorded using an HP4194A impedance analyzer. The experimental electromechanical impedance spectra for the pristine and damage cases were compared against the analytical and FEA results. For the two experimental sets the following results were observed:
(a) Very good overlap of the resonant peaks between the experimental results and the analytical and FEA simulations;
(b) For the multi-layer pristine case the agreement between the experimental results and the analytical and FEA simulation is good for lower resonant frequencies. For the
damaged case, the analytical model predicts more closely the experiments. However, caution need to be used when comparing the results between the three methods since the analytical model and the FEA analysis are based on 1-D models whereas the experimental results are from a real 3-D specimen.

The last part of this research addresses the possibility of using PWAS transducers for damage detection on real world specimens. Experiments were conducted on (1) a fabricated large scale adhesively bonded lap-joint, (2) a spacecraft simulated panel specimen, and (3) on a full scale helicopter blade. The experiments successfully proved the possibility to excite and receive guided Lamb waves in real application specimens despite the attenuation and dispersion encountered while travelling in the structures. Also, the experiments showed promising results in detecting several types of damages: cracks, simulated disbonds, and simulated corrosion using several detection methods: (a) electromechanical impedance; (b) pitch-catch; and (c) pulse-echo. However, if possible one method should always be used in conjunction with the other method in order to validate the results and to minimize false positive and negative results.

14.1 Key contributions

The major contributions resulted from this work are:

1. Development of an analytical method, using the transfer matrix approach, to study the vibration response of complex structures built up from adhesively bonded materials with structural damages in the form of disbonds or cracks. From the vibration response, i.e., from the total displacement at the PWAS ends, the electromechanical impedance was calculated. The novelty of this approach lies in
how the field transfer matrix was derived, namely the functions used in the axial and flexural general solutions.

2. Validation of the analytical results with experimental data. A set of pristine and damaged specimens were fabricated. The disbond damage was artificially simulated by inserting a piece of Mylar between the two bonded aluminum plates. The specimens were each instrumented with three PWAS transducers and the electromechanical impedance was recorded. The comparison between the analytical predictions and the experimental results revealed a very good match of the impedance response for both pristine and damaged cases.

3. Investigation of the effects of the location of the PWAS transducers attached to the beam as well as the effects of the length and height of the disbond were investigated. It was shown that small changes in the location of the PWAS will change the frequency response and hence the electromechanical impedance spectrum. It is important to take this into consideration when the predictions from the analytical model are compared with the experimental results. The changes in the size of the damage also affected the electromechanical impedance spectrum and they were successfully reproduced by our model.

4. Development of a set of damage indices to detect the presence of disbond damage in an adhesively bonded structure. From the results, it was shown that by using only one damage index it may be difficult to have a positive damage detection. Different damage indices respond in different ways to changes in the impedance spectrum. The frequency range is also a key factor. A frequency range which does not have sufficient resonant frequencies might give erroneous results due to a lack
of statistically relevant data set. For this reason it is recommended to apply these damage metrics for frequencies ranges with a high density of resonant peaks.

5. Validation of the concept of embedded SHM method using PWAS transducers through experiments designed and conducted on real world specimens (fabricated adhesively lap-joint, spacecraft simulated panel, full scale real helicopter blade) using wave propagating methods (i.e. pitch-catch and pulse-echo) and standing waves methods (electromechanical impedance. The results showed that we could positively identify damage with each of the methods described above. However, if possible one method should always be used in conjunction with the other method in order to validate the results and to minimize false positive and negative results.

14.2 Future work

The work presented in this dissertation has opened the way towards the understanding and development of a predictive model for damage detection in adhesively bonded structures using PWAS transducers. This is necessary for the development and implementation of an embedded SHM system capable of detecting and assessing the severity of various damages that could initiate and grow to unacceptable levels. The following directions are recommended for future work:

1. Using the Timoshenko beam theory extend the transfer matrix method developed in this work for Euler-Bernoulli beams to cover the transverse shear effects, which for wide beams can not be neglected.

2. Extend the analytical method to plates and shells using the plate theory with shear corrections.
3. Extend the transfer matrix method to wave propagation theory in order to predict the wave propagation in layered structures such as anisotropic adhesive joints and composite materials.

4. Optimize and automate the process of dividing the structure in an adequate number of segments and branches when modeling complicated structures.

5. Develop a graphical user interface (GUI) in MATLAB for an easier and friendlier way of inputting analysis parameters.

6. Perform additional experimental test with an increased number of specimens and with various types and sizes of damages.

7. Further develop appropriate damage indices (DI) for better detection capabilities using the modal features of the EMI spectrum.

8. Perform additional FEA to validate analytical results, to compare them with experimental data, and apply to disbond damage detection in real world structures.
References


GE Inspection Technologies – Phased Array

[cited May 22, 2006]


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Lord Rayleigh (1947) *Theory of Sound* Dover, 1947


APPENDIX A

Coefficient matrices for non-homogeneous system of equation for forced vibrations of multi-layer beam

\[ M \cdot X = N \]  \hspace{1cm} (A.1)

where

\[
M = \begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & F_{1}^{dd} & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & F_{1}^{pd} & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & F_{2}^{dd} & F_{2}^{dp} & -1 & 0 & 0 & 0 \\
0 & 0 & F_{2}^{pd} & F_{2}^{pp} & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & F_{3}^{dd} & F_{3}^{dp} & -1 & 0 \\
0 & 0 & 0 & 0 & F_{3}^{pd} & F_{3}^{pp} & 0 & 0 \\
\end{bmatrix}
\]

\[ N = \begin{bmatrix}
0 \\
-\mathbf{p}_{1}^{f} \\
0 \\
-\mathbf{p}_{2}^{f} \\
0 \\
0 \\
\end{bmatrix} \]  \hspace{1cm} (A.2)
APPENDIX B

In this appendix the coefficient matrices for the force vibration of a damaged multi-layer Euler-Bernoulli beam are expressed.

From Eq. (7.113) through Eq. (7.133) we can write the following system of equations:

\[
\begin{align*}
\mathbf{d}^{R}_{2t} &= \mathbf{D}_{2t} \cdot \mathbf{d}^{dd}_{1} \cdot \mathbf{d}^{R}_{t} + \mathbf{D}_{2t} \cdot \mathbf{F}^{dp}_{1} \cdot \mathbf{p}^{R}_{1} \\
\mathbf{d}^{R}_{2b} &= \mathbf{D}_{2b} \cdot \mathbf{F}^{dd}_{1} \cdot \mathbf{d}^{R}_{t} + \mathbf{D}_{2b} \cdot \mathbf{F}^{dp}_{1} \cdot \mathbf{p}^{R}_{1} \\
\mathbf{F}^{dd}_{1} \cdot \mathbf{d}^{R}_{1} + \mathbf{F}^{pp}_{1} \cdot \mathbf{p}^{R}_{1} &= \mathbf{P}_{2t} \cdot \mathbf{p}^{R}_{2t} + \mathbf{P}_{2b} \cdot \mathbf{p}^{R}_{2b} \\
\mathbf{d}^{R}_{3t} &= \mathbf{F}^{dd}_{2t} \cdot \mathbf{d}^{R}_{3t} + \mathbf{F}^{dp}_{2t} \cdot \mathbf{p}^{R}_{2t} \\
\mathbf{p}^{R}_{3t} &= \mathbf{F}^{pp}_{2t} \cdot \mathbf{p}^{R}_{2t} + \mathbf{p}^{F}_{1} \\
\mathbf{d}^{R}_{3b} &= \mathbf{F}^{dd}_{2b} \cdot \mathbf{d}^{R}_{3b} + \mathbf{F}^{dp}_{2b} \cdot \mathbf{p}^{R}_{2b} \\
\mathbf{p}^{R}_{3b} &= \mathbf{F}^{pp}_{2b} \cdot \mathbf{p}^{R}_{2b} \\
\mathbf{d}^{R}_{4t} &= \mathbf{F}^{dd}_{3t} \cdot \mathbf{d}^{R}_{4t} + \mathbf{F}^{dp}_{3t} \cdot \mathbf{p}^{R}_{3t} \\
\mathbf{p}^{R}_{4t} &= \mathbf{F}^{pp}_{3t} \cdot \mathbf{p}^{R}_{3t} + \mathbf{P}_{2} \\
\mathbf{d}^{R}_{4b} &= \mathbf{F}^{dd}_{3b} \cdot \mathbf{d}^{R}_{4b} + \mathbf{F}^{dp}_{3b} \cdot \mathbf{p}^{R}_{3b} \\
\mathbf{p}^{R}_{4b} &= \mathbf{F}^{pp}_{3b} \cdot \mathbf{p}^{R}_{3b} \\
\mathbf{F}^{dd}_{4t} \cdot \mathbf{d}^{R}_{4t} + \mathbf{F}^{dp}_{4t} \cdot \mathbf{p}^{R}_{4t} &= \mathbf{D}_{5t} \cdot \mathbf{d}^{R}_{5} \\
\mathbf{F}^{dd}_{4b} \cdot \mathbf{d}^{R}_{4b} + \mathbf{F}^{dp}_{4b} \cdot \mathbf{p}^{R}_{4b} &= \mathbf{D}_{5b} \cdot \mathbf{d}^{R}_{5} \\
\mathbf{p}^{R}_{5} &= \mathbf{P}_{5t} \cdot \left( \mathbf{F}^{dd}_{4t} \cdot \mathbf{d}^{R}_{4t} + \mathbf{F}^{dp}_{4t} \cdot \mathbf{p}^{R}_{4t} \right) + \mathbf{P}_{5b} \cdot \left( \mathbf{F}^{dd}_{4b} \cdot \mathbf{d}^{R}_{4b} + \mathbf{F}^{dp}_{4b} \cdot \mathbf{p}^{R}_{4b} \right) \\
\mathbf{d}^{R}_{BC} &= \mathbf{F}^{dd}_{5} \cdot \mathbf{d}^{R}_{5} + \mathbf{F}^{dp}_{5} \cdot \mathbf{p}^{R}_{5} \\
0 &= \mathbf{F}^{pd}_{5} \cdot \mathbf{d}^{R}_{5} + \mathbf{F}^{pp}_{5} \cdot \mathbf{p}^{R}_{5}
\end{align*}
\]
where

\[
T_1 = D_{2t} \cdot F_{1}^{dd} ; \quad T_2 = D_{2t} \cdot F_{1}^{dp} ; \quad T_3 = D_{2b} \cdot F_{1}^{dd} ; \quad T_4 = D_{2b} \cdot F_{1}^{dp} ; \\
T_5 = P_{5t} \cdot F_{4t}^{pd} ; \quad T_6 = P_{5t} \cdot F_{4t}^{pp} ; \quad T_7 = P_{5b} \cdot F_{4b}^{pd} ; \quad T_8 = P_{5b} \cdot F_{4b}^{pp} .
\]