PREDICTIVE SIMULATION OF PIEZOELECTRIC WAFER ACTIVE SENSORS FOR STRUCTURAL HEALTH MONITORING

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Piezoelectric wafer active sensors (PWAS) are lightweight and inexpensive enablers for structural health monitoring (SHM). After a brief review of PWAS physical principles, basic modeling, and PWAS ultrasonic guide-wave methods, the focus of the paper shifts towards the modeling and simulation. The predictive simulation includes both analytical and finite element analyses. The analytical methods have the advantage of expediency and speed; thus, they allow wide parameter studies. However, analytical methods are limited to simple geometries. Finite element methods can model complicated real-world geometries, but they are computationally intensive and require fine meshes/time intervals for accuracy. The relative advantages and limitations of the two approaches are also discussed. The paper ends with conclusions and suggestions for further work.

1. Introduction

Structural health monitoring (SHM) uses a set of permanently attached sensors to obtain on demand information about the structural performance and state of health. The benefits of monitoring the structural state include design feedback, performance enhancement, on-demand condition-based maintenance, and predictive fleet-level prognosis[1]. On-board structural sensing systems have been envisioned for determining the health of a structure by monitoring a set of sensors over time, assessing the remaining useful life from the recorded data and design information, and advising of the need for structural maintenance actions. An onboard SHM system could contain (a) sensors and sensor clusters; (b) electronics; (c) data processing and communications. The sensors can be either passive (strain, temperature, acceleration, impact, acoustic emission, etc.) or active (e.g., ultrasonic transducers that can interrogate the structure to detect damage presence, extent, and intensity). Structural health monitoring (SHM) is a multidisciplinary process involving several disciplines that must be closely coordinated.

Guided-waves techniques for nondestructive evaluation (NDE) and structural health monitoring (SHM) applications are increasingly popular due to their ability to cover large areas with a relatively small number of sensors [2]. Piezoelectric wafers attached directly to structural elements, have gained large popularity due to low cost, simplicity, and versatility [3]. Piezoelectric wafer active sensors (PWAS) have emerged as one of the major SHM technologies because the same sensor installation can be used with a variety of damage detection methods such as: (a) embedded guided-wave ultrasonics,
i.e., pitch-catch, pulse-echo, phased arrays; (b) high-frequency modal sensing, i.e., the electro-mechanical (E/M) impedance method; (c) passive detection, i.e., acoustic emission and impact detection. PWAS transducers couple the electrical and mechanical effects (mechanical strain, $S_{ij}$, mechanical stress, $T_{kl}$, electrical field, $E_k$, and electrical displacement, $D_j$) through the tensorial piezoelectric constitutive equations

$$ S_{ij} = s_{ijkl}^E T_{kl} + d_{klj} E_k $$

$$ D_j = d_{ijkl} T_{kl} + \varepsilon_{jk}^T E_k $$

where, $s_{ijkl}^E$ is the mechanical compliance of the material measured at zero electric field ($E=0$), $\varepsilon_{jk}^T$ is the dielectric permittivity measured at zero mechanical stress ($T=0$), and $d_{klj}$ represents the piezoelectric coupling effect. PWAS utilize the $d_{31}$ coupling between in-plane strains, $S_1, S_2$, and transverse electric field, $E_3$. The main advantage of PWAS over conventional ultrasonic probes is in their small size, light weight, low profile, and inexpensive cost. Extensive experimental data has been accumulated in using PWAS transducers for active structural sensing with guided waves. However, less has been achieved in the modeling and analysis of these active sensing methods; some preliminary modeling and analysis results are given next.

![Figure 1](image1.png) ![Figure 1](image2.png)

**Figure 1** Bond-layer interaction between PWAS and structure: (a) micrograph; (b) modeling

### 2. Shear-Lag Analysis of Structurally-attached PWAS

Giurgiutiu and Santoni-Bottai [5] developed a shear lag solution for the stress/strain transfer between a structurally attached PWAS and the support structure (Figure 1). Earlier studies of this subject [6] assumed axial and flexural vibrations with linear strain distribution across the thickness; this assumption is good only for low values of the frequency-thickness product $f_{ld}$, but would not be appropriate for ultrasonic guided waves (e.g., Lamb waves) because the latter have complicated multi-mode strain distributions across the thickness. To overcome this limitation, we derived a generic shear lag solution that is not limited to the low frequency-thickness values[5]. This generic solution takes into account the exact thickness distribution of displacements and stresses corresponding to the Lamb wave modes existing at a particular ultrasonic frequency-thickness product value. This study [5] showed that essential parameters such as the axial-flexural repartition number, $\alpha$, and the shear lag parameter, $\Gamma$ as well as the tuning curves, depend on the frequency-thickness product. Santoni-Bottai and Giurgiutiu [7][8] extended this work to the case of multiple Lamb wave modes excited in the structure, when the shear stress in the bonding layer depends on the number of modes present in the structure $M$, the PWAS size, $2a$, the modal wavenumbers, $\xi_m$, $m=1,...,M$, and the shear lag parameter $\Gamma$, i.e.,
\[ \tilde{r}(\xi_n) = \frac{2i}{\Gamma^2 + \xi_n^2} \left\{ \Gamma^2 - \sum_{m=1}^{M} \eta_m \xi_m \left[ \frac{2\Gamma e^{-i\xi_n a}}{\Gamma^2 + \xi_m^2} \left( \Gamma \cosh \Gamma a + i\xi_m \sinh \Gamma a \right) + 1 \right] \right\} \]

\[ \times \left( \Gamma \sin \xi_n a \cosh \Gamma a \right) - \xi_n \cos \xi_n a \sinh \Gamma a \right) \left[ \frac{2i\eta_n a}{\Gamma^2 + \xi_n^2} \times \left( \sin(\xi_m - \xi_n) a - \sin(\xi_m + \xi_n) a \right) \right] \]

\[ + \frac{2i\eta_n a}{\Gamma^2 + \xi_n^2} \times \left( \Gamma \sin \xi_n a \sinh \Gamma a - \xi_n \cos \xi_n a \cosh \Gamma a \right) \]

\[ - \frac{\xi_m}{2} \frac{e^{(\Gamma + i\xi_m) a} - e^{-i(\Gamma + i\xi_m) a}}{(\Gamma + i\xi_n)^2} - \frac{\xi_m}{2} \frac{e^{-(\Gamma - i\xi_m) a} - e^{(\Gamma - i\xi_m) a}}{(\Gamma - i\xi_n)^2} \]

\[ (2) \]

The use of the exact solution given by Equation (2) has shown a substantial improvement in the PWAS-Lamb wave tuning curves and an almost perfect match with the experimental measurements [7]. Figure 2 shows a comparison of the experimental and theoretical tuning curves for the first antisymmetric (A0) and symmetric (S0) modes. Blue circles represent experimental S0 data; red crosses represent experimental A0 data; solid lines represent theoretical A0 (red) and S0 (blue) from the simplified ideal-bonding low-frequencies model; dash line represent theoretical A0 (red) and S0 (blue) values intermediate model; Dash dot lines represent theoretical A0 (red) and S0 (blue) exact model of Equation (2).

Figure 2 PWAS tuning curves: (a) \( t_b = 1 \mu m \); (b) \( t_b = 30 \mu m \) [7]

Figure 2a shows results corresponding to the assumption of a thin-bond \( t_b = 1 \mu m \); we notice that the peaks and zeros of the A0 theoretical curves do not overlap with the experimental ones, while those of the S0 are closer to the experimental values. The prediction curves derived with the improved theory of Equation (2) are almost coincidental with the ideal-bonding low-frequencies model. This is so because a thin 1 \( \mu m \) bond is close to the ideal-bonding case. However, this assumption is not realistic and does
not agree with the experimental data (see green circle around the zero at $\sim 200$ kHz. In contrast, Figure 2b, shows predictions for a thicker bond ($t_b = 30 \mu$m); now, the first A0 peak and zero predictive by the improved analysis are almost coincident with the experimental values, while the S0 behavior has not changed significantly. This illustrates the importance of using an exact model.

3. Transfer Matrix Method Approach to Modeling the PWAS Detection of Disbonds
Transfer matrix method (TMM) is an efficient analytical approach for solving 1-D vibration and wave propagation problems. TMM starts with the exact closed-form solutions describing the vibration and wave propagation in simple uniform segments and then uses a state-vector formulation and boundary matching to connect segments with different properties. The state vector at the right end of the beam is expressed in terms of the state vector at the left end of the beam using exact beam functions. Boundary conditions are matched at the left and right beam ends.

$$z^R_{BC} = P_2 \cdot z^L_{BC} + P^F_1 = P_2 \cdot F_1 \cdot z^R_1 = P_2 \cdot F_1 \cdot (z^L_{BC} + P^F_1)$$  \hspace{1cm} (3)

Hence, one writes

$$z^R_{BC} = U \cdot z^L_{BC} + T$$  \hspace{1cm} (4)

$$U = P_2 \cdot F_1 \cdot P_1 \quad \text{and} \quad T = P_2 \cdot F_1 \cdot P^F_1 + P^F_2$$  \hspace{1cm} (5)

![Figure 3](image)

Figure 3  TMM calculation of E/M impedance spectrum of a PWAS on bonded metallic coupon [9]

The TMM approach permits the modeling of branched structures, such as in the case of a disbond or split in an adhesively bonded structure. Cuc and Giurgiutiu [9] used the TMM approach to simulate analytically the detection of disbonds in adhesive joints using PWAS transducers [9]. TMM was used for modeling a cracked multi-layer adhesively bonded beam with a PWAS attached to the top surface.
The model starts with a single segment in the good-bond region. At point 2, the model splits into two branches representing the structure above and below the disbond. The two branches reunite when the disbond ends, and the rest of the beam is modeled with just one segment. The PWAS transducer is accommodated by making a separate segment for the structure above which the PWAS sits. Figure 3 shows the E/M impedance spectrum curves predicted for a small adhesively bonded coupon having three PWAS transducers mounted on the upper surface. This analytical study (which was backed up by experimental measurements) was aimed at determining two things:

(a) the changes that occur in the spectrum when disbands appear in the adhesive joint

(b) the sensitivity of the spectrum to changes in the PWAS location

The spectrum shown in Figure 3 corresponds to PWAS #2, which is placed directly above the disbond. Similar changes, though of lesser amplitude, were observed for PWAS #1 and #3, which are not placed on top of the disband but in its vicinity. For the pristine specimen, the E/M impedance spectrum of PWAS #2 place exactly in the middle of the beam, shows three peaks in the $30 - 60$ kHz range, i.e., at $\sim 31$ kHz, $\sim 43$ kHz, and $\sim 56$ kHz. When disbond damage was applied, these peaks shifted to $\sim 38$ kHz, $\sim 42.5$ kHz, and $\sim 52$ kHz. The changes in the first and third peak are major, and easily detectable; they are indicative of disband damage presence. This answers point (a) by indicating that significant changes take place in the spectrum when disband damage appears. To address point (b), we introduced a very small (1-mm) shift in the PWAS location. This 1-mm shift generated new small peaks at $\sim 36$ kHz, $\sim 49$ kHz, but left the three major peaks virtually unchanged. This answers point (b), i.e., it shows that the spectrum is also sensitive to changes in PWAS location, but changes are generally small. They do not impede our damage detection capability, because the changes due to disbond are of much larger amplitude. In addition, during the SHM process, the PWAS location is fixed, and hence the differences between the pristine baseline and the damaged spectra will only be due to damage. The effect of modeling the exact PWAS location is nonetheless important when trying to compare theoretical predictions and experimental results.

4. Power and Energy Transduction between PWAS and Structure

An analytical investigation of power and energy transduction between PWAS and structure during the structural health monitoring process was recently performed by Lin and Giurgiuƫiu [10]. This preliminary work uses an analytical approach applied to the simple model depicted in Figure 4. The study used a 1-D analytical model to capture the power and energy flow from the electrical source energizing the transmitter PWAS through various stages of transduction up to the signal captured by an instrument connected to the receiver PWAS. The model consists of a transmitter PWAS (A) and a receiver PWAS (B) bonded to a metallic beam. The following energy conversion stages were considered:

(a) piezoelectric transduction between the electric source and the transmitter PWAS

(b) mechanical transmission of shear stresses from the PWAS to the structure

(c) excitation of ultrasonic waves traveling through the structure from the transmitter to the receiver

(d) capturing of ultrasonic waves arriving at the receiver PWAS

(e) mechanical conversion of structural waves into shear stresses acting from the structure onto the receiver PWAS

(f) piezoelectric conversion at the receiver PWAS and measurement of the resulting signal by the electrical instrument.
We have developed two analytical approaches, one based on standing waves (vibration), the other based on traveling waves. The standing-waves model is appropriate for a finite-dimensional specimens; when excited harmonically, such a specimen will enter a state of vibration caused by the ultrasonic guided waves bouncing back and forth between the specimen boundaries in a standing-wave pattern. The traveling-waves model is appropriate for the study of large specimens in which the boundary effects can be neglected or for the study of wave-propagation events that happen before the waves bounce back from the reflecting boundaries. In order to account for the electronic effects, we considered a voltage source of voltage $V_A$, source impedance $Z_A$ and maximum current $I_{A\text{max}}$ and measuring instrument characterized by instrument admittance $Y_e$.

The standing-waves model was based on normal modes expansion; in the simplified case of only axial (extensional) and flexural (bending) vibrations, the voltage $V_B$ at PWAS B is found in terms of the voltage $V_A$ at PWAS A in the following form

$$
\dot{V}_B(\omega) = \frac{k_{31}^2 Y_{0B}}{Y_e + (1-k_{31}^2)Y_{0B}} R(\omega)k_{iA}k_{ib} \left[ C_{AB}^2(\omega) - C_{AA}(\omega)C_{BB}(\omega) \right] + k_{iA}C_{AA}(\omega) + R(\omega)k_{ib}C_{BB}(\omega) - 1 \dot{V}_A(\omega) \tag{6}
$$

where $Y_{0B}$ is the admittance of PWAS B, $k_{iA}$ and $k_{ib}$ are the internal stiffnesses of PWAS A and B, $k_{31}$ is the piezoelectric-transduction coupling factor of the PWAS material. The expressions $R(\omega)$, $C_{AA}(\omega)$, $C_{AB}(\omega)$, $C_{BB}(\omega)$ are defined in ref. [10].

The propagating-waves model assumes that axial and flexural propagating waves generated at PWAS A are felt at PWAS B and transduced into an electrical voltage which, in turn, will transduce into a reflected ultrasonic wave that will be felt back at A and will influence its ultrasonic output. Hence, the voltage $V_B$ at B is found in terms of the voltage $V_A$ at A in the following form

$$
\dot{V}_B(\omega) = \frac{k_{31}^2 Y_{0B}}{Y_e + (1-k_{31}^2)Y_{0B}} R(\omega)k_{iA}k_{ib}C_{AB}(\omega)C_{BB}(\omega) - (k_{iA}C_{AA}(\omega) - 1)(R(\omega)k_{ib}C_{BB}(\omega) - 1) \dot{V}_A(\omega) \tag{7}
$$

The coefficients $C_{AA}(\omega)$, $C_{AB}(\omega)$, $C_{AB}(\omega)$, $C_{BB}(\omega)$ are expressed in terms of propagating waves and are different from those of Equation (6) (see ref. [10] for details). The model was used to predict the
frequency response functions for voltage, current, complex power, active power, etc. At the input side, it was found that the reactive electric power is dominant and hence defines the size of the energizing power supply/amplifier (Figure 5a). At the PWAS structure interface, it was found that only the active electrical power gets converted into mechanical power, which is transmitted across the PWAS-structure interface and energizes the axial and flexural waves propagating into the structure. A parametric study was conducted w.r.t. the transmitter PWAS size: it was found that proper size and excitation frequency selection facilitates ultrasonic wave excitation through tuning effects. Figure 5b displays peaks and valleys that depend on frequency and PWAS size. It is apparent that a larger PWAS does not necessarily ensure more power transmission -- careful frequency-size tuning is necessary! Similar tuning effects were also found at the receiver PWAS where a parametric study of the receiver size, receiver impedance and external electrical load provides useful design guidelines for PWAS-based sensing and/or energy harvesting. This preliminary analysis shown here needs to be validated and verified through experiments and then extended to cover multi-modal Lamb waves, various structural situations (structural variability, structural joints, flaws/damage, nonlinear friction in joints and cracks, adhesive bonding/delamination, etc.), and more complicated excitation and detection electronic circuitry.

Figure 5  PWAS transmitter under constant 10-V excitation (a) power rating; (b) wave power[10]

5. Multi-Physics Finite Element (MP-FEM) Simulations
Gresil and Giurgiutiu [12] have explored the use of multi-physics finite element method (MP-FEM) to model the generation of elastic waves from an applied electric field applied to a surface-mounted
PWAS transmitter (T-PWAS) and the reception of the elastic wave as electric signal recorded at a PWAS receiver (R-PWAS). In these preliminary studies, we used the ANSYS and ABAQUS commercially available codes and explored the comparative behavior of the available elements such as brick, plate, and shell. We found that different elements and mesh sizes might give quite different wave propagation results. Subsequently, we performed a study on modeling the guided wave generation and reception in a rectangular metallic plate containing a through-hole defect. This benchmark problem has been examined by many investigators and is quite well documented in literature. We modeled 7-mm PWAS transducers bonded to the top of the plate on both sides of the hole. The PWAS transducers operated in pitch-catch mode. A 3-count smoothed voltage tone burst with \( f = 141 \text{ kHz} \) was applied to the T-PWAS and received by the R-PWAS. The presence of the hole in the plate modified the transmitted signal through wave scatter and mode conversion.

![Figure 6 Multi-physics finite element method (MP-FEM) simulation of guided waves generate by a 7-mm PWAS transmitter and scatter from a 12-mm through hole.](image)
Figure 6 shows image snapshots of the guided wave pattern in the plate taken at 10-μs intervals. At the excitation frequency of $f = 141$ kHz, two guided wave modes are present, S0 and A0. The A0 mode is considerably slower than the S0 mode. The A0 mode is also much more dispersive than the S0 mode. At $t = 10 \mu s$, one sees the waves just starting from the T-PWAS. Wave scatter from the hole becomes apparent at $t = 20 \mu s$, with mode conversion very clear at $t = 30 \mu s$. The interaction of the waves with the R-PWAS and the boundaries start to be observable from $t = 40 \mu s$ onwards. By $t = 80 \mu s$, most of the wave power has dissipated into the boundaries. In the future, we will simulate the sensor signals that would be measured on a flawed/damaged realistic structure in comparison to signals that would be measured on a pristine structure. The main difficulty in addressing realistic specimens representative of actual structures is one of scale of complexity.

We have also modeled and developed electronic hardware for multichannel PWAS SHM systems [13], as well as a compact DSP-based electromechanical impedance analyzer[14][15] and other instrumentation and signal processing algorithms for compact active SHM systems.

4. SUMMARY AND CONCLUSION

This paper addressed the need for developing a predictive modeling methodology for structural sensing in active structural health monitoring (SHM) with piezoelectric wafer active sensors (PWAS). A few examples of preliminary work done towards such a predictive methodology have been presented. The exact modeling of the shear-lag interaction between PWAS and multi-modal guided waves in the structure was able to match experimental results much better that previous two-mode theories. The analytical modeling of the power and energy flow between a transmitter PWAS and a receiver PWAS showed very interesting tuning opportunities. The E/M impedance spectrum for a partially disbonded adhesive joint was modeled with the transfer matrix method (TMM) allowed important parameter studies. Multi-physics finite element method (MP-FEM) was used to simulate the traveling of guided waves from a transmitter PWAS to a receiver PWAS while being scattered by a hole defect.

However, the results presented here are just preliminary. Future work should attempt to combine the efficiency of analytical methods with the detailing capability of the FEM approach such as to developed a hybrid method for modeling realistic structures with sufficient computational efficiency as to permit parameter studies. In this way, we will be able to advance from an empirical approach into an analytical rational development of structural health monitoring systems and maintenance strategies.

6. References


