Modeling of Power and Energy Transduction of Embedded Piezoelectric Wafer Active Sensors for Structural Health Monitoring

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Outline

- PWAS Power and Energy Transduction
  - Transmitter
  - Receiver
  - Pitch-catch

- Methods
  - Wave propagation method
  - Normal mode expansion method

- Conclusions
Transmitter Behavior

- **Electrical response (7-mm transmitter)**
  - **Active power**
    \[ P_{\text{active}} = \frac{1}{2} Y_R \dot{V}^2 \]
  - **Reactive power**
    \[ P_{\text{reactive}} = \frac{1}{2} Y_i \dot{V}^2 \]
  - **Reactive power is dominant**
    - capacitive behavior
Transmitter Mechanical Response

- Active power converts to mechanical power
- Mechanical power at both ends are equal
- Mechanical power converts to wave power
- Waves contain axial and flexural waves

\[
P_{active} = \langle P_A \rangle + \langle P_{A'} \rangle
\]

\[
\langle P_A \rangle = \langle P_{A'} \rangle
\]

\[
\langle P_{A'} \rangle = \langle P_{wave} \rangle
\]

7-mm Transmitter
Transmitter Size Effects

- Reactive power is dominant
- Power rating increases when frequency increases
- Power rating increases when transmitter size increases
- Wave power is relatively small
- Tuning effects
  - Maximum wave power output depends on both frequency and transmitter size
- Increasing transmitter size and frequency requires more input electrical power
- Increasing transmitter size may NOT increase the wave power
Receiver Behavior


Receiver PWAS (Wave Detector) → Oscilloscope

- Only a small portion of wave power converts to receiver output electrical power

Wave Power

Receiver Mechanical Power MB+MB'

Receiver Electrical Power

LAMSS
Laboratory for Active Materials and Smart Structures
Receiver Effects

- Sensing
  - High resistive load (High Z)
- Receiver tuning effects
  - Large receiver size may not output high voltage under constant strain axial wave

- Power harvesting
  - 7-mm receiver
- Load impedance match
  - Maximum power output at 400 kHz and 100 Ω
Pitch-catch Energy Transduction

Transmitter PWAS
INPUT $V_1$


Shear-stress excitation of structure

PWAS-structure interaction

Ultrasonic guided waves from transmitter PWAS

Receiver PWAS
OUTPUT $V_2$


Shear-stress excitation of PWAS

Structure-PWAS interaction

Ultrasonic guided waves arrive at receiver PWAS

Transmitter PWAS
(Wave Exciter) $\nu_i$

Receiver PWAS
(Wave Detector) $\nu_r$

Lamb waves
Pitch-catch Power Transduction

- Power Rating under constant 10-V voltage input
- Electrical Active Power (T)
- Mechanical Power at A,A'
- Receiver Electrical Power Output
- Axial and flexural Wave Power

7-mm transmitter and receiver
PWAS Modeling

Assumptions:
- 1-D axial and flexural wave
- Ideal bonding connection
- Ideal excitation source
- Fully-resistive external load

Axial waves
\[ \rho A \dddot{u}(x,t) - EA\dddot{u}(x,t) = N'_e \]
\[ N_e(x,t) = \left\{ \hat{F} \left[ -H(x - x_0) + H(x - x_0 - l) \right] \right\} e^{-i\omega t} \]

Flexural waves
\[ \rho A \dddot{w}(x,t) + EI\dddot{w}(x,t) = -M''_e(x,t) \]
\[ M_e(x,t) = -\left\{ \frac{h}{2} \hat{F} \left[ -H(x - x_0) + H(x - x_0 - l) \right] \right\} e^{-i\omega t} \]
Wave Propagation Method

- **Transmitter**
  \[ \hat{F}_A = k_{IA}(\Delta \hat{u}_A - u_{ISA}) \]

- **Receiver**
  \[ \hat{F}_B = (1 - \frac{k_{31}^2 Y_0}{Y_e + (1 - k_{31}^2)Y_0})k_{IB} \Delta \hat{u}_B = R(\omega)k_{IB} \Delta \hat{u}_B \]
  \[ R(\omega) = 1 - \frac{k_{31}^2 Y_0}{Y_e + (1 - k_{31}^2)Y_0} = \frac{Y_e + (1 - 2k_{31}^2)Y_0}{Y_e + (1 - k_{31}^2)Y_0} \] (Receiver ratio of external load)

- **In-plane surface displacement**
  \[ u_p(t) = \frac{i\hat{F}_A}{2EA} \begin{cases} 
    \left( -e^{-i\xi_0 x} + e^{-i\xi_0 (x_d + l_A)} \right) \frac{\varepsilon}{\xi_0} e^{i(\xi_0 x - \omega t)} & \\
    +3\left( -e^{-i\xi_F x} + e^{-i\xi_F (x_d + l_A)} \right) \frac{\varepsilon}{\xi_F} e^{i(\xi_F x - \omega t)} \end{cases} \]
  \[ + \frac{i\hat{F}_B}{2EA} \begin{cases} 
    \left( -e^{-i\xi_0 x} + e^{-i\xi_0 (x_d + l_B)} \right) \frac{\varepsilon}{\xi_0} e^{i(\xi_0 x - \omega t)} & \\
    +3\left( -e^{-i\xi_F x} + e^{-i\xi_F (x_d + l_B)} \right) \frac{\varepsilon}{\xi_F} e^{i(\xi_F x - \omega t)} \end{cases} \]

- **PWAS elongation**
  \[ \Delta u_A = (\hat{F}_A C_{AA}(\omega) + \hat{F}_B C_{BA}(\omega))e^{i\omega t} \]
  \[ \Delta u_B = (\hat{F}_A C_{AB}(\omega) + \hat{F}_B C_{BB}(\omega))e^{i\omega t} \]
  - where \( C_{AA}(\omega), C_{AB}(\omega), C_{BA}(\omega), C_{BB}(\omega) \) are frequency dependent structural coefficients

- **Receiver output voltage**
  \[ \hat{V}_B(\omega) = \frac{k_{31}^2 Y_{0B}}{Y_e + (1 - k_{31}^2)Y_{0B}} \frac{k_{IA} C_{AB}(\omega)}{k_{IA} C_{AA}(\omega) + R(\omega)k_{IB} C_{BB}(\omega) - 1} \hat{V}_A(\omega) \]
**Alternative Approach: Normal Mode Expansion Method**

- **Axial vibrations**
  \[ \rho A \dddot{u}(x,t) - EA \ddot{u}(x,t) = N_e'(x,t) \]

- **Flexural vibrations**
  \[ \rho A \dddot{w}(x,t) + EI \ddot{w}(x,t) = -M_e'(x,t) \]

- **Surface displacement of a free-free beam**

  \[
  u_F(x,t) = \frac{\hat{F}}{\rho A} \left[ \sum_{n_u} \Delta U_{n_u}(x_0, l) \frac{\omega_n^2}{\omega_n^2 - \omega_e^2} \right. \\
  + \left( \frac{h}{2} \right)^2 \sum_{n_w} \frac{\Delta W_{n_w}(x_0, l)}{\omega_n^2 - \omega_e^2} \right] e^{-i\omega t} 
  \]

  \[
  U_{n_u}(x) = A_{n_u} \cos(\gamma_{n_u} x), \quad A_{n_u} = \frac{2}{L}, \quad \gamma_{n_u} = \frac{n_u \pi}{L}, \quad \omega_n = \gamma_{n_u} c, \quad c = \sqrt{\frac{E}{\rho}} \quad n_u = 1, 2, ... 
  \]

  \[
  W_{n_w}(x) = A_{n_w} \left[ \cosh \gamma_{n_w} x + \cos \gamma_{n_w} x - \sigma_{n_w} (\sinh \gamma_{n_w} x + \sin \gamma_{n_w} x) \right] 
  \]

  \[
  \omega_n = \gamma_{n_u}^2 a, \quad a = \frac{EL}{\sqrt{\rho A}}, \quad I = \frac{bh^3}{12}, \quad A_{n_w} = \frac{1}{\sqrt{\int_0^L W_{n_w}^2(x) dx}}, \quad n_w = 1, 2, ...
  \]
Non-harmonic Excitation

- **Convolution**

\[ \text{out}(t) = fRf(t) * \text{in}(t) = \mathcal{F}^{-1}
\left[ FRF(\omega) \right] \text{IN}(\omega) \]

- **Frequency response function (FRF)**

\[
FRF(\omega) = \frac{V_B(\omega)}{V_A(\omega)} = \frac{k_{31}^2}{r_y + (1 - k_{31}^2)k_{iA}C_{AA}(\omega) + R(\omega)k_{iB}C_{BB}(\omega) - 1}
\]
Conclusions

- A systematic investigation of power and energy transduction in PWAS attached to structure during SHM process
- Pitch-catch wave propagation between transmitter PWAS and receiver PWAS
- PWAS transmitter:
  - Active power, reactive power, power rating were investigated
  - Reactive power dominates the power supply requirements!
  - Active electrical power converts to mechanical power
  - Mechanical power becomes axial and flexural waves power
- PWAS receiver:
  - Sensing optimization
  - Power harvesting optimization
- Future power and energy analysis:
  - 2-D wave propagation
    - multi-mode Lamb waves
Thank you.