Multi-Model Data Assimilation for Structures

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Digital Twins
What constitutes a digital twin?

Does a Response Amplitude Operator (RAO) count?

Does it need to model everything?

From SNAME Ship Structure Committee SSC-459, “RELIABILITY-BASED PERFORMANCE ASSESSMENT OF DAMAGED SHIPS”

How does the Navy envision digital twins?

A digital twin is ultimately a systems of systems model that enables greater insight into the platform.

Digital Twin is a continuous blending of data, physics-based models, and machine learning combined with our best knowledge of the ocean battlespace to forecast platform performance. These insights improve situational awareness and enables a user to readily identify optimum actionable decisions.
Are we overselling the capability?

- We have been talking about this for years

What can digital twins provide?

- Machinery data
- Maintenance Records
- Wind/Wave History
- Structural data

Insights are provided across a range of timescales

- Course and Speed Adjustments
- Ship Routing Decisions
- Expected loads during deployment
Start with Structural Health Monitoring

Aim: Collect sufficient data to inform a digital twin and enable improved Condition Based Maintenance.
Prior Work: Real-time Model Updating
Real-Time Model Updating Through Error Minimization

A frequency-based model updating technique was developed to update an FEA model of the system.

**Experimental**

DROPBEAR experimental testbench
- roller connection
- cantilever beam

**Analytical**

- FEA model
  - Euler-Bernoulli beam
    - $u_1(t)$
    - $u_2(t)$

- sample $n$ unique roller locations

- construct $n$ FEA models

- calculate fundamental frequency ($\nu = \lambda Mv$)
  - $K_1, M_1$
  - $\omega_1$
  - $K_2, M_2$
  - $\omega_2$
  - $\vdots$
  - $K_n, M_n$
  - $\omega_n$

- identify best FEA model
  - select FEA model with lowest error

- probability density function (PDF) of roller locations

- update PDF based on current system state

- current system state
Experimental Results: Model Updating

- FEA nodes = 10
  - iteration time = 0.32 ms
  - MAE = 31.07 mm
  - percent error = 8.9%

- FEA nodes = 20
  - iteration time = 0.86 ms
  - MAE = 13.09 mm
  - percent error = 3.7%

- FEA nodes = 40
  - iteration time = 4.04 ms
  - MAE = 10.05 mm
  - percent error = 2.9%

Cycle time of 5 ms

Average cycle time (ms)
- 0.00
- 0.25
- 0.50
- 0.75
- 1.00
- 1.25
- 1.50
- 1.75
- 2.00
- 2.25
- 2.50
- 2.75
- 3.00
- 3.25
- 3.50
- 3.75
- 4.00
- 4.25
- 4.50
- 4.75
- 5.00

Number of FEA nodes
- 10
- 20
- 30
- 40

Number of particles
Experimental Results: Algorithm Timing

- Code run in parallel on multi-core processors using floating point precision variables.
- The FFT causes a delay in the estimation of the system.
- The length of the FFT is a function of the dynamics in the system.
Experimental Results: Impact and Stochastic Testing

Impact Testing

- The frequency-based model updating algorithms can track the system state through an impact.
- This has benefits for system tracking of fuzes in hard target penetrating systems.

Stochastic Testing

- A random data set was used to investigate the algorithm’s tracking capably.
- The algorithm is shown to accurately track the system state.
Multi-model Data Assimilation
Question: Can we develop a method to update fatigue and impact in a single model?

Research plan:
• Thrust 1: Develop a 1-D test structure.
• Thrust 2: Develop tools for multi-modal data assimilation.
Thrust 1: Develop a 1-D test structure.

- System being designed/developed to simulate a component in a ship hull.
- System will be used to generate Distribution-A data for future publications.

- Similar system developed with the AFRL for real-time model updating of structures experiencing high-rate dynamics.
Thrust 2: Develop tools for multi-modal data assimilation.

- Develop an FEA-based model updating framework that can consider both cracks (e.g., fatigue) and impacts.
FEA Modeling

- Due to lab closures at Carderock and the UofSC, the experimental setup was built as an FEA model.
- Roller movement simulates damage caused by impact.
Background: Modal Analysis

Modal analysis is used to find the mode shapes and frequencies of a structure during free vibration.

Starting with the equation of motion:

\[ M\ddot{x} + C\dot{x} + Kx = 0 \]

the damping coefficient can be ignored as its effect on the natural frequency is less than 0.0005\%, resulting in the expression:

\[ M\ddot{x} + Kx = 0 \]

assuming a temporal solution:

\[ x(t) = \Phi(A_n \cos(\omega_n t) + B_n \sin(\omega_n t)) \]

yields the following expression:

\[ \left(-\Omega_n^2 M\Phi + K\Phi_n\right)q_n(t) = 0 \]

where \( q_n(t) = 0 \) is a trivial solution, therefore the eigenvalues and eigenvectors are solved for using the general eigenvalue problem formulation:

\[ K\Phi_n = \lambda_n M\Phi_n \]

where:

\[ \lambda_n = \Omega_n^2 \]

and:

\[ \omega_n = \sqrt{\lambda_n} \]
The challenge with updating a model for multiple parameters is uniqueness.

- For a given measurable frequency, the structure could be in a number of damage states.
Investigating a variety of parameters can potentially help.

“Z” direction mode shapes for beam:

- Mode shape 2
- Mode shape 4
- Mode shape 5
- Mode shape 7

Change in participation factors:

Modes jump to new frequency.
Modes change with a change in the K or M matrix (i.e. fatigue or roller movement).
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Gradient decent-based model updating approach

- Gradient descent is a simple method for optimization (first-order iterative).
- Provides a simple space to inspect results.
- Provides a simple approach that is mathematically (hopefully) tractable.
- Several options for potential options if we can show the response surface in concave:
  - Particle Swarm
  - Stochastic gradient decent
  - Batch gradient decent

https://medium.com/@DBCerigo/on-why-gradient-descent-is-even-needed-25160197a635
Determine an effective cost function

Initial testing investigated the direct model-based approach (DMBA):

\[ J_{DMBA} = \sum_{i=1}^{n} \left( \frac{\omega_i^{true} - \omega_i^{trial}}{\omega_i^{trial}} \right)^2 + \alpha \sum_{i=1}^{n} \frac{(1 - \sqrt{MAC_i})^2}{MAC_i} \]

where:

\[ MAC_i = \frac{\left| (\phi_i^{true})^T (\phi_i^{trial}) \right|^2}{\left| (\phi_i^{true})^T (\phi_i^{true}) \right| \left| (\phi_i^{trial})^T (\phi_i^{trial}) \right|} \]

The objective function considers \( n \) modes with \( \omega_i^{true} \) and \( \omega_i^{trial} \) being the modal frequencies of a true structure and a trial model, respectively, for the \( i \)th mode. The weighting term, \( \alpha \), allows the objective function to weight the mode shape differences relative to the modal frequencies.

Building on prior work performed in collaboration with NSWC-Carderock

Model-based fatigue crack detection (1/8” cut) in aluminum plate with stiffener.

Results for probability of damage location.

Determine an effective cost function

Furthermore, the **Flexibility-based Approach (FBA)** was also investigated. The FBA method provides an objective function by computing the differences between the truncated flexibility matrices for the true and trial models. This method is less sensitive to higher modes.

The stiffness matrix $K$ and the flexibility $F$ matrix linked through the mode shapes $\Phi$ as follows:

$$ K = M \bar{\Phi} \Omega \bar{\Phi}^T M $$

and

$$ F = \check{\Phi} \Omega^{-1} \check{\Phi}^T = \sum_{i=1}^{n} \frac{1}{\omega_i^2} \check{\phi}_i \check{\phi}_i^T $$

where the bar denotes a mass-normalized quantity. The normalized mode shape ($\check{\phi}_i$) is coupled to the mass-normalized mode via a mass normalization constant $d_i$:

$$ \check{\phi}_i = \phi_i d_i $$

For a select number ($n$) of the lower mode shapes, a truncated flexibly matrix is defined as:

$$ F_{\text{turn}} = \sum_{i=1}^{n} \left( \frac{d_i}{\omega_i} \right)^2 \check{\phi}_i \check{\phi}_i^T $$

Thereafter, a flexibility matrix is constructed for the true and trail FEA case, the difference is defined as:

$$ \Delta F_{\text{turn}} = F_{\text{true}}^{\text{turn}} - F_{\text{trial}}^{\text{turn}} $$

Where $\Delta F_{\text{trun}}$ is a matrix. Lastly, a scalar value for $\Delta F_{\text{trun}}$ can be obtained by computing the Frobenius norm of $\Delta F_{\text{trun}}$, defined as:

$$ \| \Delta F_{\text{turn}} \|_F = \sqrt{\sum_{i=1}^{n} \sum_{i=j}^{m} x_{i,j}^2} $$
• Model tested for a test condition with a 10 mm crack and a roller at 700 mm.
• Modes 2, 4, 5, and 7 are modes that participate in the Z-axis.
• We consider a growing number of modes:
  • Mode 2
  • Modes 2 and 4
  • Modes 2, 4, and 5
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• The use of 4 modes gives the best response surface.
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Discussion on results

• The direct mode-based approach (DMBA) is not on-its-own able to provide a local-min at the correct model parameters.
• With additional information, (i.e. deflection shape, crack length, roller location) the correct parameters could be obtained.
• The Flexibility-based Approach is shown to be a more efficient method for updating the model for various parameters.
• The Flexibility-based Approach requires knowledge of the mass normalization constants of the structure. These can be obtained either experimentally or numerically.
• The affects of noise in the system need to be studied.
• It can be challenging to extract the higher modes from experimental systems.
Key next steps in the research

• Investigate cost functions better suited for tracking multiple models.
• Integrate the participation factor into the cost function.
• Develop Bayesian-based method to obtain probabilities associated with each estimated parameter.
• Investigate the use of different gradient decent solvers.
• Develop experimental test-bench at the UofSC to experimentally validate this work.
Long-term Vision for the Proposed Work
**Dynamic Data Driven Applications System**

- **Static Input Data (Design Data)**
- **On Platform Sensor Systems**
- **Off Platform Sensor Systems**

**Measurement Control**
- resample
- reposition

**Model Selection**
- Data Assimilation
- Prediction

**Ship Model**
- structural
- equipment
- corrosion

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**Informed Response Management**

**Integrated Ship Information System**
- process real-time information
- predictions: intensity, trends
- analysis: risk assessment

**Real-Time Monitoring**
- impact detection
- decision-making

**Long-Term Monitoring**
- route planning
- fatigue life predictions

**Informed Response Management**
- real-time response (close hatches)
- real-time power distribution
- dockyard resource allocation
- cruise planning
Questions?

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