Experimental determination of Representative Volume Element (RVE) size in woven composites

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- Meso-scale

ABSTRACT

A systematic approach is proposed to estimate the length scales of the representative volume element (RVE) in orthogonal plain woven composites. The approach is based on experimental full-field deformation measurements at mesoscopic scales. Stereovision digital image correlation (DIC) is conducted to determine the full-field strain distribution in on- and off-axis specimens loaded axially in tension. A sensitivity analysis is carried out to optimize the image correlation parameters. Using the optimized set of image correlation parameters, full-field strains are measured and used in conjunction with a simple strain averaging algorithm to identify the length scales at which globally applied and spatially-averaged local strains converge in values. The size of a virtual window containing local strain data, the average of which has the same value as the global strain, is identified as the RVE dimensions for the examined material. The smallest RVE sizes found in this work are shown to be both strain and angle dependent. The largest RVE dimension obtained is reported as a unique, strain and orientation insensitive RVE size for the woven composite examined.

1. Introduction

Macro-scale non-linear mechanical response of woven composites has been documented to be the result of complex fiber-matrix interactions; whereas, the degree of this non-linearity itself is a function of the angle between the loading direction and the principal fiber axes [1–3]. The Complex deformation mechanisms that govern such orientation-dependent nonlinear response can be investigated by studying the deformation response of off-axis specimen at micro and meso-scales [4]. Although micro-scale studies can provide useful information on the response of individual components in a woven composite, deformation characteristics at this scale are basically incapable of providing any evidence on the prevailing fiber-matrix interaction mechanisms. On the other hand, meso-scale analyses have been established to overcome this challenge by allowing a more accurate examination of the deformation mechanisms at yarn scales [5–8]. Data extracted from meso-scale studies on woven composites not only reveals the governing deformation mechanisms, but can also be used to capture the local deformation response, in order to validate micromechanical and finite element simulations, the concepts currently of great interest in the area of composite research [9–12].

Traditionally, verification and validation of numerical approaches, particularly finite element analyses, is generally conducted at only one length scale. However, it has long been realized that the deformation response of fiber composites at smaller scales is clearly different from that of macroscales. Accordingly, researchers commonly attempt to take advantage of homogenization algorithms to determine bulk deformation behavior from the local response. Such homogenization algorithms are required to be conducted over a specifically selected volume of the material which is small enough to capture the local components’ response, while sufficiently large to encompass all individual constituents and represent the material as a whole [13]. Therefore, the concept of representative volume element (RVE) has been introduced and successfully implemented as the underlying concept in homogenization techniques [14,15].

To date, several numerical studies have been carried out attempting to characterize the RVE and its length scales for different material systems and under various loading conditions [15–18]. On the other hand, there are not many experimental works to characterize RVE, particularly in the case of woven composites. Digital image correlation (DIC) has been proven to be a promising technique, enabling accurate deformation measurements at a wide range of length scales, thus allowing for experimental characterization of RVE [19,20]. However, certain challenges exist in the application of DIC, particularly at small scales and for highly heterogeneous deformation patterns, not to mention that both cases are present in meso-scale study of woven composites [21]. Application of a fine speckle pattern on millimeter and sub-millimeter sized fields of interests, as well as selection of...
appropriate image correlation parameters are among the most challenging tasks associated with meso-scale DIC. Speckling methods involving direct deposition of nano and micrometer sized particles have emerged as a solution for the speckling challenge for high resolution small-scale digital image correlation [22]. However, accurate DIC-based measurements not only depend on the speckle pattern, but are also highly sensitive to the strain calculation algorithms in an image correlation process [23–26]. These parameters include but are not limited to subset, step and filter size, all of which have been proven to be capable of remarkably altering the strain measurement accuracy.

With the rapidly growing applications of DIC, great attention has been drawn towards the study of governing image correlation parameters, usually with the purpose of measurement error minimization [27–29]. However, in terms of local deformation characterization of woven composites and the contribution of such local deformation response to the RVE length scales, there still exists a gap in the literature. Accordingly, the present work attempts to provide a systematic experimental-based approach on the characterization of the local deformation response and the length scale of the representative volume element in woven composites subjected to on- and off-axis loading conditions. To this purpose, a concise study is first conducted on the selection of an optimized set of image correlation parameters. Error analyses along with the identification of full-field strain distribution are carried out by varying the three governing image correlation parameters, i.e. subset, step and filter sizes. Upon determination of the optimized image correlation parameters, full-field meso-scale strain distribution is obtained and used to characterize the representative volume element and its length scales.

2. Experimental

2.1. Materials and specimen geometry

The material examined in this work is a three-layered plain woven carbon fiber reinforced composite with 0.8 mm total thickness. This composite is made of 70 vol% carbon fibers of ~7 µm in diameter, processed into towels interwoven to an orthotropic plain weave fabric with 0.8 mm yarn widths, as shown in Fig. 1.

2.2. Tensile testing

Rectangular tensile specimens of 175×25×0.8 mm³ dimensions are extracted for both on-axis (β = 0°) and off-axis (β = 45°) tensile tests, as shown schematically in Fig. 2. At least three specimens are prepared and tested at each angle to ensure the reproducibility of the obtained results. The grip length of 25 mm is marked at either ends of the tensile composite. Load and cross-head displacement data are recorded at a rate of 1 Hz. Global mechanical parameters of the specimens are listed in Table 1. It must be noted here that, there is an one order of magnitude difference between the global failure strain (εf) of on-axis and off-axis specimens.

3. Full-field measurements

3.1. Stereo-imaging

Full-field displacement distribution of a deforming material is measured in digital image correlation by tracking the deformation of a speckle pattern through a series of digital images captured from a region of interest. In case of no or negligible out-of-plane motion, in-plane surface displacement can be obtained using a single camera (2D DIC). However, in applications where considerable out-of-plane displacements are present, the application of stereovision digital image correlation (3D DIC) can significantly enhance the measurements accuracy. The use of stereovision DIC reduces the sensitivity of the measurement system to the out-of-plane displacement, such that the out-of-plane motion will have minimal contribution to the in-plane strain measurements [30]. In case of off-axis woven composites subjected to in-plane tension, it was recently documented that the occurrence of fiber trellising can promote the protrusion of the matrix material on the specimen; thus giving rise to a considerable out-of-plane displacement on the surface [31]. Accordingly, to minimize the measurement errors associated with this out-of-plane displacement, 3D DIC is utilized in the present work. The experimental setup is shown schematically in Fig. 3a. The stereovision system is calibrated using custom made 6.5 mm calibration plates [32] as shown in Fig. 3b, with the obtained calibration details listed in Table 2. It should be noted that the use of larger calibration targets cover a larger portion of the field-of-view is generally favorable and results in a higher level of deformation measurement accuracy in stereovision image correlation. The application of a calibration plate smaller than the entire field-of-view was mainly due to the availability. However, the accuracy of 3D DIC results obtained with a calibration plate that covers at least 50% of the field of view has been confirmed in our study. The DIC software used in this work is Vic-3D [32]. Image correlation parameters are detailed and discussed in the following.

In DIC, the correlation algorithm works by locating a subset in the undeformed image and comparing it with its corresponding subset in the image taken from the deformed state. Mathematical details of this correlation algorithm can be found in [33]. Since each subset is uniquely identified by its gray-scale intensity, it is required for the specimens to be properly speckled with a high-contrast, random and dense speckle pattern. The speckle pattern is used to provide fingerprints to search and track the subsets during the deformation process [34]. Ideally, the average size of a single speckle in the applied pattern must be roughly 5 times larger than the physical size of a single pixel. Owing to the fact that the physical size of a pixel is a function of the resolution of the camera, the optimal speckle pattern is thus an indirect function of the camera resolution, as well. The stereovision camera system utilized in this work consists of a pair of 5 MP CCD Point Gray cameras, each equipped with a 100 mm macro lens. 2448×2048 pixel² images are acquired from a 10×10 mm² speckled area of interest located on the center of the specimen (see Fig. 4a). The magnification factor is measured as 7.21 µm/pixel. Using the above mentioned rule of thumb, speckle patterns consisting of 20–40 µm white particles are applied on the specimen surface. The white speckle pattern is directly applied on the black surface of the specimen, using an airbrush and a
diluted flat white paint (see Fig. 4b). The gray-scale intensity of the original speckle pattern is also illustrated in Fig. 4c, indicating a bell shape distribution, suitable for image correlation purposes [33].

3.2. 3D Image correlation

There are several parameters which govern the accuracy of an image correlation process. The first is the subset size, which governs the spatial resolution of the displacement measurement. A proper image correlation process requires that each subset in the deformed state contain unique gray scale intensity information. This helps in distinguishing the subset from its neighboring subsets. Therefore, the lower bound on the dimensions of the subset size can be set based on the degree of correlation in the deformed images [23]. The degree of correlation is a function of step size, as well. Step size is the value by which the subsets are overlapped and tracked during the correlation process. Therefore, the step size indicates the number and the spacing of data points within the area of interest. Note that a substantially small step size results in the overlapping subsets not to be completely independent of each other, thus increasing the susceptibility of obtaining dependent and repetitive results. Conversely, large step sizes may lead to a loss in spatial resolution of the measured displacement and strain, as discussed in detail in the forthcoming sections. In practice, the step size ($h_{\text{step}}$) must be smaller than $h_{\text{sub}}/2$, with $h_{\text{sub}}$ denoting the subset dimensions, to allow for subset overlapping. Variation of the degree of correlation as a function of the utilized subset and step sizes are studied in this work as the starting point. To this purpose, the degree of correlation is determined for several image correlation cases with different subset and step dimensions. Note that other than subset and step dimensions, all other image correlation parameters are kept constant in this stage. These parameters are listed in Table 3.

Fig. 5 illustrates the degree of correlation as a function of subset and step dimensions. Vic-3D restricts the use of step sizes larger than 30 pixels; thus, the results shown here only include cases with $h_{\text{step}} < 30$ pixels. It is observed that the highest degrees of correlation are achieved at larger subset and smaller step sizes. Generally, it is stated that $h_{\text{sub}}$-to-$h_{\text{step}}$ ratios between 3 and 4 result in an optimal correlation [23,33]. The observations made in the present research contradict this general rule of thumb. The reason behind this inconsistency might be due to the extremely nonhomogeneous full-field deformation patterns present at mesoscopic scales in woven composites, the conditions which are completely different from the general macroscale DIC measurements [21]. A combination of the smallest subset and largest step dimensions for which $> 95\%$ correlation is achieved is considered for the full-field strain measurements in this work. Accordingly, $h_{\text{sub}}=95$ pixels and $h_{\text{step}}=9$ pixels are selected, and the values of the desired deformation variable (displacement and/or strain) at the uncorrelated points are interpolated using the neighboring correlated data points and a cubic spline scheme. Note that the physical size of the subset size used in this work is $685 \times 685 \mu m^2$, the value which is slightly smaller than the dimensions of a single unit cell. It must be emphasized that there are several other parameters in a DIC approach that impact the accuracy of full-field measurements, e.g. the order of the shape function. These parameters are particularly important in cases where strain gradients are substantial [28]. Our main objective in the present work is to utilize DIC-based full-field strain measurements to identify a key material characteristic parameter, i.e. RVE size.

<table>
<thead>
<tr>
<th>$\beta$ (°)</th>
<th>$E$ (GPa)</th>
<th>$\nu$</th>
<th>$\sigma_Y$ (MPa)</th>
<th>$\epsilon_f$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>68.1</td>
<td>0.076</td>
<td>-</td>
<td>0.014</td>
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<tr>
<td>45</td>
<td>11.9</td>
<td>0.779</td>
<td>75</td>
<td>0.15</td>
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</table>

* The on-axis specimen shows no plastic deformation.

Fig. 2. (a) Schematic configuration of the on-axis and off-axis tensile specimen with respect to the principal fiber directions. An actual tensile specimen is shown in (b).

Table 1

Global material properties determined for the on- and off-axis specimens.

Fig. 3. (a) Schematic of the experimental setup used in this work. High intensity white LED lights is used as the illumination source and the stereo camera system is calibrated using the calibration plate shown in (b).

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Therefore, an in-depth study of the influence of all DIC parameters will divert us from our main objective; however the study of other image correlation parameters in full-field DIC measurements has been extensively studied in the literature [23,28,29,34].

3.3. Strain measurements performance at meso-scale

Displacement fields obtained from an image correlation process are numerically differentiated to obtain full-field strain distributions. The resultant strain fields usually contain significant levels of noise. To reduce this noise, strain fields are usually filtered by a Gaussian-weighted filtering scheme over an N×N array of data points [33]. Note that each two data points are separated by the value of step size; hence, the gage length over which the strain filtering is applied will be N×h_step, with N denoting the filter size (in terms of data points) and h_step being the step size. The parameter N×h_step is a measure for a virtual strain gage (VSG), the length of which indicates the spatial resolution of strain measurements. Selection of a smaller VSG enables the calcula-

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Camera 1</th>
<th>Camera 2</th>
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<tr>
<td>Center - x (pixels)</td>
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<td>682</td>
</tr>
<tr>
<td>Center - y (pixels)</td>
<td>1299</td>
<td>597</td>
</tr>
<tr>
<td>Focal length - x (pixels)</td>
<td>52005</td>
<td>55023</td>
</tr>
<tr>
<td>Focal length - y (pixels)</td>
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<td>55043</td>
</tr>
<tr>
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</tr>
<tr>
<td>Alpha (°)</td>
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<tr>
<td>Beta (°)</td>
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<td></td>
</tr>
<tr>
<td>Gamma (°)</td>
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</tr>
<tr>
<td>Tx (mm)</td>
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<td></td>
</tr>
<tr>
<td>Ty (mm)</td>
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<tr>
<td>Tz (mm)</td>
<td>38.75</td>
<td></td>
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Table 2
System parameters obtained from calibration of the stereo camera arrangement.

Table 3
Image correlation software (Vic-3D) parameters.

<table>
<thead>
<tr>
<th>Interpolation type</th>
<th>Optimized 8-tap</th>
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</thead>
<tbody>
<tr>
<td>Matching criterion</td>
<td>Normalized squared differences</td>
</tr>
<tr>
<td>Shape function</td>
<td>First order (affine)</td>
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<tr>
<td>Strain tensor type</td>
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<td>Maximum matchability threshold margin (pixel)</td>
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</tr>
<tr>
<td>Maximum consistency threshold margin (pixel)</td>
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</tr>
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Fig. 4. (a) Speckled tensile specimen with a high magnification of the speckled area illustrated in (b). The gray scale histogram of the speckled area is plotted in (c).
tion of highly localized strain magnitudes, but a noisier strain distribution. On the other hand, a large VSG tends to smooth the strain distribution at the cost of smearing out local strain information. It is expected that for applications in which highly localized strains and/or high deformation gradients exist, smaller VSG’s might be more useful in understanding of the underlying material response. Such conditions are present in the mesoscopic deformation study of woven composites, where high degrees of strain localizations might occur due to the complex load bearing mechanisms and/or fiber-matrix interactions [5,21].

Meso-scale strain measurement performance is conducted in this work by first determining the strain noise and bias levels. For this purpose, a set of 5 still images are recorded before the onset of the tensile experiments. Keeping the subset and step dimensions constant, the full-field strain distribution over the area of interest (see Fig. 4) is determined using different strain filter sizes \(N\). It must be emphasized here that the strain measurement error is indeed a function of subset and step size, as well. However, to maintain the simplicity of the analyses in this work, only the effect of strain filter size is discussed here. Subset and step sizes used hereafter have dimensions of 95×95 pixel\(^2\) and 9 pixel, respectively, based on the discussions provided earlier.

Ideally, it is expected that the strain field obtained from the stress-free stationary images show zero values over the entire zone. However, in practice, the strain fields indicate non-zero parasitic patterns. These non-zero values are associated with the levels of noise (standard deviation) and bias (mean) in the DIC strain measurement. Fig. 6 shows the variation of standard deviation and mean as functions of the strain filter size. The standard deviation of the entire population of the strain values in the stress-free specimen is a measure of the level of noise in the calculated strain [35]. It is clearly shown in Fig. 6 that the noise (standard deviation) is reduced as the strain filter size is increased. This is due to the fact that by increasing the strain filter dimension, the size of the VSG is increased; therefore, strain averaging is performed over a larger number of data points, smearing out the spatial noise in the measurement points. On the other hand, the mean values indicate no significant sensitivity to the size of the strain filter. This indicates that there must be a systematic bias in the strain measurements in this work. The source of this strain bias is not known at this point, however, the bias value (~2.83 m\(\varepsilon\)) can be added to all the measured strain values in order to eliminate the effect of bias on the strain measurements.

It is of the utmost importance in the experimental study of meso-scale deformation to determine an optimal value for the strain filter size. As the subset and step sizes are maintained constant in this work, the size of the VSG directly depends on the strain filter dimension. In the study of meso-scale strain field in woven composites, the designated VSG must be small enough to allow for capturing of strain localization, which is essentially a natural characteristic of the material. On the other hand, the VSG has to be sufficiently large such that the measurement noise level (see Fig. 6) remains small compared with the local strain magnitudes in the material. To make it more clear, one should note that the meso-scale strain field data obtained from DIC contains both strain measurement noise and the actual strain information. Owing to the appreciable level of strain heterogeneity and strong spatial gradients in composites, distinguishing between the noise and the real strain value is quite challenging. This challenge intensifies when the measurement noise and the strain magnitudes are of the same order of magnitude. To determine the optimal strain filter size in the present work, the full-field meso-scale strain distributions are

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**Fig. 5.** Variation of the degree of correlation as a function of subset and step dimensions.

**Fig. 6.** Variation of strain measurement noise (standard deviation) and bias (mean) with respect to the strain filter size. The results are obtained at constant subset and step sizes of 95 pixel and 9 pixel, respectively.
obtained at different applied global strains, using various strain filter sizes. Fig. 7 depicts typical strain maps indicating how increasing the strain filter size can alter the strain resolution. Accordingly, significant smoothing is observed when filter size is increased from \( N = 5 \) to \( N = 49 \). In fact, this spatial strain smoothing tends to noticeably reduce the strain gradients measured over the area of interest.

The decrease in strain variability can be quantified by calculating the standard deviation of all the strain data (\( \sigma_{SD} \)) inside the area of interest. Note that the strain variability inside the area of interest exhibits a Gaussian distribution, the range of which depends on the strain filter size. Fig. 8 illustrates typical curves for strain distribution within the area of interest, plotted for two different global strain values. Gaussian distribution of the local strain data is evident in this figure. The standard deviation values calculated using the full-field strain data contain both measurement noise and spatial variability of strain due to the material response. Fig. 9 shows the variation of the standard deviation with global strain at different strain filter sizes and for both on-axis and off-axis specimens. Note that the standard deviation data presented in Fig. 9 are plotted as a function of global strain values ranging from 0 to failure strain (\( \varepsilon_f \)) for each specimen. The standard deviation is shown to decrease with increasing strain filter size, as discussed earlier in this section. Standard deviation in both on-axis and off-axis specimens is increased at larger global strains. Such behavior indicates the progressive increase of the strain heterogeneity within the area of interest at larger applied strains. This type of deformation response is explained in detail in Section 4, where meso-scale strain maps are presented and discussed. One final remark is that although the magnitudes of the standard deviation in \( \beta = 45^\circ \) specimen are roughly 5 times larger than those of \( \beta = 0^\circ \), the on-axis specimen shows a higher level of strain uncertainty at meso-scales. This is not understood intuitively, but can be confirmed by plotting the normalized standard deviation with respect to the normalized global strain. For this purpose, the entire population of strain data points inside the area of interest is first averaged. These average values are shown in Fig. 10 for both on-axis and off-axis specimens. Spatially averaged curves plotted in Fig. 10 indicate no significant sensitivity to the filter size. Next, the standard deviations shown earlier in Fig. 9 are normalized with the \( \varepsilon_f \) values to be later used as a measure of strain uncertainty. Curves showing the evolution of \( (\sigma_{SD}/\varepsilon_f) \) are illustrated in Fig. 11. It is clearly shown that in general, meso-scale strain uncertainty levels for \( \beta = 0^\circ \) are higher than those of \( \beta = 45^\circ \), in spite of the overall smaller \( \sigma_{SD} \) values obtained for \( \beta = 0^\circ \) (see Fig. 9).

In order for the quantitative analyses to be consistent for on-axis and off-axis specimens, it is essential to have the same level of strain uncertainty for both cases. In this regard, the first step is to determine the optimal strain filter size for the \( \beta = 45^\circ \) specimen. The strain uncertainty level corresponding to this optimal filter size is calculated, and the filter size which results in the same degree of uncertainty for \( \beta = 0^\circ \) is then obtained and used for the quantitative analyses.

A non-dimensional parameter is defined as the ratio of \( \sigma_{SD} \) normalized with respect to the strain measurement noise (shown in Fig. 6). Fig. 12 shows the variation of \( \sigma_{SD} \) normalized with the strain measurement noise (shown earlier in Fig. 6), as a function of the applied global strain and filter size. Note that it is more favorable to have a large \( (\sigma_{SD}/\sigma_{noise}) \) ratio, since a greater \( (\sigma_{SD}/\sigma_{noise}) \) ratio indicates that the noise level has had a smaller contribution on the actual strain data. Accordingly, the \( (\sigma_{SD}/\sigma_{noise}) \) ratio equal to 10 is considered as the acceptance criterion. It is observed that the \( (\sigma_{SD}/\sigma_{noise}) \) ratio increases with global strain magnitudes. This indicates that the level of uncertainty decreases with the applied global strain for \( \beta = 45^\circ \) specimen, particularly at global strains > 5%. In addition, the acceptance criterion \( (\sigma_{SD}/\sigma_{noise}) > 10 \) is reached at a smaller global strain for \( N = 13 \), compared with any other strain filter size; whereas, the maximum \( (\sigma_{SD}/\sigma_{noise}) \) ratio is also obtained for the same filter size, i.e. \( N = 13 \). This means that a relatively more reliable set of data is obtained over a larger extent of global strains for \( \beta = 45^\circ \), when a strain filter size of \( N = 13 \) has been in use. For the global tensile strains < 5%, the noise level is of the same order of magnitude as the real strain values developed at mesoscopic scales. This makes quantitative analyses more challenging and less reliable at earlier stages of deformation for the off-axis \( \beta = 45^\circ \) specimen. The level of strain uncertainty for \( \beta = 45^\circ \) associated with \( N = 13 \) filter size is used as a guideline to determine the

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**Fig. 7.** Full-field variation of vertical strain component \( \varepsilon_y \) at global strain of 15%, plotted for strain filter sizes of (a) \( N = 5 \) and (b) \( N = 49 \). Far-field load is applied in \( y \)-direction and \( \beta = 45^\circ \).
optimal strain filter for $\beta = 0^\circ$. Accordingly, the strain filter using similar uncertainty in mesoscopic strain is obtained for the on-axis specimen as $N=45$. Fig. 13 compares the $(\sigma_{SD}/\text{noise})$ ratios obtained with $N=45$ and $N=13$ for $\beta = 0^\circ$ and $\beta = 45^\circ$, respectively.

It is worth noting that the whole displacement/strain measurement performance is indeed a function of the speckling method, as well [34]. Speckle size and spacing, as well as the gray scale pattern of the speckles also play major roles in the selection of a proper set of image correlation parameters [34,35]. These have been studied in detail in the available literature on the subject, for e.g. see Ref. [34]. The influences of speckling techniques and pattern quality on the deformation measurement performance at meso-scales are out of the scope of the current work. In addition, it is also well-established that image averaging is another approach by which the measurement errors due to image noise can be extensively reduced. A combination of the approach presented in the current study with the image averaging methods proposed elsewhere [29,34] can further improve the reliability and accuracy of full-field strain measurements at mesoscopic scales.

4. Meso-scale full-field strain maps

Using the obtained proper strain filter sizes for on- and off-axis specimens, full-field strain distributions at mesoscopic scales are evaluated. Typical contour maps depicting the evolution of different strain components, at various global strains are shown in for $\beta = 0^\circ$ and $\beta = 45^\circ$ specimens, are shown in Figs. 14 and 15 respectively. Substantial strain heterogeneity can be observed for all cases in Figs. 14 and 15; whereas, the degree of such strain heterogeneity is significantly higher for $\beta = 45^\circ$. In particular, the shear strain distribution obtained for $\beta = 45^\circ$ indicates a noticeably higher spatial variability compared with that of $\beta = 0^\circ$. Such a behavior was previously studied in detail in [31] and was attributed to the occurrence of fiber trellising phenomenon during in-plane tension of the off-axis $\beta = 45^\circ$ specimen.

An interesting point in the study of meso-scale strain maps is the location of the peak local strain in on-axis and off-axis specimens. As evidenced in Fig. 16, the local maximum vertical strain component is developed over the longitudinal fiber tows in the $\beta = 0^\circ$ specimen. Conversely, for the off-axis $\beta = 45^\circ$ specimen, the largest magnitude of vertical strain is developed within narrow matrix-rich regions located

![Fig. 8. graphs showing the distribution of local strain data at global strain values of (a) 5.1% and (b) 14.3%. Graphs are plotted for $\beta = 45^\circ$ specimen using subset =95 pixel, step =9 pixel, strain filter =13.](image)

![Fig. 9. Variation of standard deviation ($\sigma_{SD}$) with the applied global strain at different strain filter sizes (N), for (a) on-axis $\beta = 0^\circ$, and (b) off-axis $\beta = 45^\circ$ specimens.](image)
mainly at the intersection of longitudinal and transverse fiber tows. This observation can be related to different load bearing mechanisms in on- and off-axis woven composite specimens subjected to axial loading. In-depth discussions on the possible load-bearing mechanisms in fiber composites can be found in [5,21,31].

5. Determination of the RVE length scales and discussion

The length scale of the representative volume element in this work is determined for both on-axis and off-axis specimens, focusing on the possible variation of RVE dimensions with the applied global strain magnitude. A very important point to note here is that the approach presented in this work is based on surface measurements; therefore, the results obtained and presented here are basically related with the characterization of representative surface element (RSE). There is still no evidence on a well-established mathematical correlation between RSE and RVE. However, assuming that the deformation takes place in a similar manner within the inner plies of a woven composite as it does

![Fig. 10. Variation of spatially averaged vertical strain $\bar{\varepsilon}_y$ with the applied global strain at different strain filter sizes ($N$), for (a) on-axis $\beta = 0^\circ$, and (b) off-axis $\beta = 45^\circ$ specimens.](image1)

![Fig. 11. $\sigma_{SD}/\bar{\varepsilon}_y$ vs global strain at different strain filter sizes ($N$), for (a) on-axis $\beta = 0^\circ$, and (b) off-axis $\beta = 45^\circ$ specimens. Global strain, $\varepsilon_{global}$, has been normalized with the failure strain, $\varepsilon_f$, for each specimen.](image2)

![Fig. 12. Variation of $\sigma_{SD}$-to-noise ratio with the strain measurement noise (shown in Fig. 6), with the applied global strain and filter size ($\beta = 45^\circ$).](image3)
The representative volume element size in this work is determined using a statistical approach based on average strain method\[20,36\]. In this approach, an $R \times R \ \mu m^2$ square window is first virtually drawn at the center of the field of view (see Fig. 17). The axial strain values of the entire population of data points located inside this square are averaged, and the resulting average value is denoted as $\varepsilon_{local}$. The error associated with this $\varepsilon_{local}$ value is then calculated as:

$$\text{error} = \left| \frac{\varepsilon_{local} - \varepsilon_{global}}{\varepsilon_{global}} \right| \times 100$$  \hspace{1cm} (1)

where $\varepsilon_{global}$ denotes the global (macro scale) axial strain measured by the extensometer in the tensile machine. The dimensions of the virtual square are progressively enlarged, increasing the number of data points encompassed in it; and the error is estimated for each corresponding square dimension. Plotting the error values with $R$, one can expect that the error values become smaller at larger $R$’s, as the average of local strain data tends to converge to the global axial strain magnitudes. This is shown in Fig. 18, where the evolution of $\varepsilon_{local}$ is plotted against $R$, for the off-axis specimen under a global strain of 0.084. The convergence of $\varepsilon_{local}$ and $\varepsilon_{global}$ are clearly demonstrated in Fig. 18, at window sizes larger than 4000 $\mu$m (~554 pixels).

To calculate the length scale of RVE, similar curves are obtained for both on- and off-axis specimens under different global strain magnitudes. For each set of curves the evolution of error (see Eq. (1)) with respect to $R$ is plotted. The convergence criterion in all cases is set as $|\text{error}| < 2\%$. Fig. 19a shows the variation of error with $R$, at selected global strain values for the on-axis specimen. The error is shown to be significantly high at small window sizes. Note that the smallest window size considered here is 80×80 $\mu$m$^2$ (11×11 pixel$^2$), which is significantly smaller than the dimensions of a single unit cell in the material. The curves in Fig. 19a indicate a progressive decrease in error with increasing $R$, up to $R=3000 \ \mu m$ (416 pixels). From this point forward, all curves corresponding to different global strains indicate oscillatory-type variations with $R$. These oscillations tend to die out at larger $R$ values, confirming the convergence of $\varepsilon_{local}$ towards $\varepsilon_{global}$. The $R$ value above which error remains $<2\%$ for each curve in Fig. 19a is selected as the length scales of RVE for the on-axis specimen. In addition, the number of unit cells, $N$, encompassed within the calculated RVE can be determined as:

$$N = (R/\delta)^2$$  \hspace{1cm} (2)

where $\delta$ denotes the average unit cell dimension of the material, or
Fig. 15. Full-field distribution of different strain components at mesoscopic scale for the specimen with $\beta = 45^\circ$.

Fig. 16. Locations of the peak local strain values in (a) on-axis and (b) off-axis specimens.
simply the width of single fiber tow (=0.8 mm). Variation of the length scale of the RVE, along with its associated $N$, has been plotted as a function of global strain in Fig. 19b for the on-axis specimen. This figure shows a strong sensitivity to the globally applied tensile strain. This can be attributed to the relatively high strain measurement uncertainties present in the on-axis woven composites specimens, as discussed earlier in Section 3. In line with this, the number of unit cells required to be present within the gauge area, such that the strain averaging results can represent the macroscale deformation response, is also found to be a function of global strain. At small global strains, e.g. < 0.1%, the largest number of unit cells required to be statistically representative of the material at continuum scale is calculated as ~100. This means that in cases where strain averaging is to be performed on the surface of an on-axis specimen, the dimensions of the averaging area must be at least 10 times larger than the width of a single fiber tow. This lower bound value is reduced at larger global strain. At small global strains, the strain uncertainty is also decreased (see Fig. 11a).

A similar approach is used to determine the RVE length scale for the off-axis specimen. Fig. 20a depicts the variation of error with $R$ for the off-axis specimen. In this case also, an initial high percentage of error is observed, and is followed by a rapid decrease up to $R = 2250 \mu m$ (312 pixels). After this point, all curves demonstrate small variation about the horizontal axis, indicating that after an $R$ value roughly equal to 1500 $\mu m$, the convergence criterion is met. Similar to the on-axis specimen, strain dependence of the RVE length scale is studied for the $\beta = 45^\circ$ specimen. Fig. 20b shows the variation of the RVE length scale and its corresponding $N$, as a function of global strain for the specimen with $\beta = 45^\circ$. A trend completely different from what was previously observed for the on-axis specimen is seen in Fig. 20b. The minimum RVE size for the off-axis specimen deformed at small global strains is shown to be significantly smaller than that observed for $\beta = 0^\circ$ (see Fig. 18b). This can be attributed to the lower overall strain uncertainty in the case of our off-axis specimen.

By increasing the global strain magnitude to approximately 3%, the minimum size of RVE decreases, reaching a local minimum at $\varepsilon_{global}=0.03$. Such behavior was also previously observed in Fig. 13, indicating the smallest strain uncertainty value to have been achieved at $\varepsilon_{global}=0.03$ for the off-axis specimen. At $\varepsilon_{global}=0.03$ the global strain is sufficiently large so that the strain measurement noise level is well below the level of spatial variability of the strain. On the other hand, $\varepsilon_{global}=0.03$ is still too low to cause the formation of highly localized strain domains over the matrix-rich areas (see Fig. 14). Therefore, the smallest dimension of RVE for the off-axis specimen is determined at $\varepsilon_{global}=0.03$.

By further increasing the global strain, the RVE size again continues to rise monotonically. This can be explained through the formation of high and low strain domains on the surface of the material which promote substantial strain gradients and deformation heterogeneity [21], thus requiring the strain averaging to be conducted over a larger area in the specimen. Finally, considering the number of unit cells contained inside the RVE in an off-axis specimen, it can be concluded that strain averaging over an area having 2x2 unit cells should be theoretically sufficient to represent the macro-scale deformation response in axially loaded woven specimens with $\beta = 45^\circ$.

The RVE sizes determined for the woven composite in this work by using <2% convergence criterion are shown to be both angle-dependent and strain-dependent. This contradicts the fundamental definition of the RVE by Hill [13], which indicates that the size of the representative volume element must be “effectively independent of the surface values of traction and displacement”. Therefore, to be consistent with the fundamental definitions, the RVE size corresponding to the lowest global strain value obtained for the on-axis specimen is regarded here as a value for the whole composite, which is unique regardless of the fiber orientation angle and the magnitude of strain applied globally. This RVE size, according to our experimental observations, must be equivalent to the area of a square whose side dimensions are 10 times as large as the width of a fiber tow.

As a final remark, it should be emphasized that the results obtained in this work can be considered as a useful source for the type of research that involves strain averaging over a well-defined region of interest. Full-field strain measurements, followed by strain averaging have been successfully implemented in composite research community as a substitute for contact extensometry, to measure the elastic modulus and Poisson’s ratio in woven composites [2,3,37,38]. Another application of strain averaging is in the rapidly growing use of virtual fields method (VFM). One particular application of VFM is in the identification of constitutive mechanical parameters for non-homogeneous materials, such as orthotropic composites, whereas spatial averaging of strain components is conducted over a predefined field with the purpose of estimating the internal virtual work [39–41]. Results obtained in this work can be used as a guideline for the determination of lower bounds for length scales of the virtual fields, over which strain averaging is to be conducted.
6. Conclusions

The length scale of the representative volume element (RVE) in a plain woven composite is estimated using an optical-based experimental approach. Meso-scale strain deformation measurements are conducted using 3D DIC at millimeter and submillimeter scales to investigate the local deformation response of the material. To facilitate an accurate quantitative measurement and conclusive study, the main image correlation parameters, i.e. subset, step and strain filter size, are optimized through sensitivity analyses. It is found that to maintain the level of accuracy of strain measurements conducted at meso-scales, different strain filter sizes must be utilized for on- and off-axis specimens. A simple algorithm based on the strain averaging method is also implemented to identify the length scales at which spatially-averaged local strains converge to the globally-applied strain magnitudes. Using this as a guideline, the physical dimensions above which locally-averaged strains are equal to the global strains are identified as the representative volume element size for on- and off-axis composite specimens. Accordingly, the smallest RVE dimensions found experimentally are indicated to be strain- and specimen angle-dependent. The unique, strain and orientation insensitive RVE size for the examined composite obtained using the proposed approach corresponds to the area of a square whose side dimensions are equal to 10 fiber tow widths.

Fig. 19. (a) variation of error with $R$, at selected global strain values for on-axis $\beta = 0^\circ$ specimen. The length scales of RVE as a function of global strain for the on-axis specimen is shown in (b).

Fig. 20. (a) variation of error with $R$, at selected global strain values for on-axis $\beta = 45^\circ$ specimen. The length scales of RVE as a function of global strain for the on-axis specimen is shown in (b).
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References