Effect of elastic properties of material composition on the fracture response of transversely graded ceramic/metal material

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A B S T R A C T

The effect of material composition and their elastic properties on the fracture behavior of a transversely graded material is investigated. A single edge notched specimen machined from Ti/TiB graded material with a crack perpendicular to the gradient direction is subjected to three-point bending and the displacement fields on both faces of the sample, Ti and TiB-rich, are obtained using full-field 3D digital image correlation. These displacement fields, along with the asymptotic displacement equation, are used to extract fracture parameters using an overdeterministic least square approach. The displacement fields from a 3D finite element model are also used to calculate the stress intensity factor at each layer throughout the thickness. The variation of stress intensity factor in the gradient (thickness) direction is presented as a function of the elastic modulus of the material. A simple semi-empirical model is also proposed to calculate the stress intensity factor as a function of the material elastic properties at any section along the thickness direction, and to predict the effective fracture toughness of the material.

1. Introduction

Functionally graded materials (FGMs), owing to their unique properties, have recently been utilized in a wide range of applications including high temperature aerospace and automotive components, wear/oxidation resistant parts, high ballistic efficiency lightweight armor materials, and biomedical applications [1–3]. Among all FGMs, ceramic/metal FGMs have received extensive attention due to their growing potential interest in high temperature applications. In this class of FGMs, the ceramic part offers superior corrosion resistance and high wear resistance in addition to serving as a thermal barrier, while the material is toughened by its metallic components [4]. Generally, due to variation of material properties along the gradient direction in FGMs, the fracture behavior of these materials is rather complicated. Along with the fact that there has always been a great desire to improve the overall toughness of ceramic/metal FGMs, this has initiated research to investigate the fracture behavior of this class of materials, both theoretically and experimentally.

The majority of the previous studies on the fracture response of FGMs have considered a configuration wherein a crack is aligned with the gradation direction [5–9]. Recently, variation of stress intensity factor (SIF) for transversely graded materials (crack is perpendicular to the gradation) was studied in the works conducted by Wadgaonkar and Parameswaran [10] and Kommana and Parameswaran [11]. In these works, variation of material properties along the crack front was observed to cause the stress intensity factor to vary along the gradient direction, following a trend similar to the elastic modulus of the material. It was also shown that the first three terms in the asymptotic expansion of the stress field in a graded material with exponential variation of elastic modulus along the crack tip is identical to that given by Williams’s solution for a homogenous material.

The quasi-static and dynamic fracture behavior of the proposed material (Ti/TiB FGM) under different temperature conditions and crack geometries were investigated by Kidane and Shukla [12]. Three-point bend experiments were performed in their work to evaluate the fracture toughness of the material containing different crack configurations and under various temperature and loading conditions. However, in their work only the critical far-field load was considered and only the apparent fracture toughness of the material was obtained. The effect of elastic material properties on the fracture behavior of the material and across the thickness was not investigated. It is expected though, that the fracture behavior across the thickness would vary due to the variation of mechanical properties along the crack line. To overcome this issue and to provide a better understanding of the fracture mechanics of transversely graded materials, optical measurement techniques with the capability of observing deformation behavior at the specimen surfaces have shown great potential. These approaches, when coupled with analytical and numerical
models, can give a clearer understanding of the fracture behavior of a material [9,13–17].

The present work focuses on studying the fracture behavior of a Ti/TiB graded material system. Emphasis has been applied to investigation of the variation of the stress intensity factor through the thickness and exploration of the effect of elastic properties of each layer on the fracture properties of the material. To our knowledge, this is the first detailed work that presents the variation of the stress intensity factor as a function of material elastic properties across the thickness in ceramic/metal graded materials. A Ti/TiB specimen has been subjected to quasi-static loading with a crack front configuration perpendicular to the gradient direction. Displacement fields have been obtained on both metallic-rich and ceramic-rich sides of the specimen using DIC. An overdeterministic least-square analysis was utilized to determine the stress intensity factor present on both sides of the specimen. A full-scale 3D finite element simulation was also performed to obtain the internal displacement fields in the crack tip vicinity along the gradient direction. Using the displacement fields obtained from DIC and the FE simulation, the stress intensity factors present throughout the thickness direction were calculated and compared to the material’s apparent fracture toughness determined from the far-field load. The fracture surface was also examined using scanning electron microscopy (SEM) to provide a more detailed understanding of the fracture response of the material system examined in this work.

2. Material and methods

2.1. Material preparation and experimental setup

A Ti/TiB graded material system fabricated by BAE Systems Inc. was examined in this work. The as-fabricated material was a 3.175 mm thick plate composed of seven layers of different material compositions with no distinct interface between layers. The layers ranged from pure Ti on one side to 85% TiB (15% Ti) on the other side as given in Table 1. Further details on the fabrication process and of the material can be found elsewhere [18,19]. Three-point bend test specimens were extracted from the plate to the dimensions shown in Fig. 1. An initial notch was made on the mid-section of the specimen using an Electrical Discharge Machining (EDM) process with a wire of 0.05 mm diameter. Note that the crack front is located along specimen thickness, i.e., the gradation direction.

2.2. Experimental procedure

A quasi-static three-point bend test setup made of hardened steel (manufactured by Wyoming Test Fixtures Inc.) was used to study the fracture response of the material. The hardened steel loading tip eliminates the possibility of elastic deformation of the bar during the loading process. Testing was performed at room temperature using displacement control at a rate of 0.05 mm min⁻¹. Full-field displacement measurements on each side of the specimen were obtained simultaneously using digital image correlation. To facilitate the DIC, the specimen surfaces were first painted with a thin layer of white paint using an air brush, and a black speckle pattern was then applied on top of the white surface. A typical speckle pattern applied to the sample is shown in Fig. 2a. Two sets of 5 Megapixel stereo cameras were used to capture the deformation of the material at each side of the specimen (see Fig. 2b). All four cameras were equipped with 60 mm lenses, and were synchronized during the experiment to simultaneously capture the deformation present on both sides of the specimen at any given time. To obtain the displacement components on each side of the specimen, each pair of cameras were calibrated prior to the test. The calibration parameters obtained for each side are detailed in Table 2. The load cell was also synchronized with both cameras so that the magnitude of load corresponding to each image would be known afterwards. During loading, 2448 × 2048 pixel images were taken at a rate of 1 frame per second. Images were then exported to Vic-3D® (digital image correlation software by Correlated Solutions Inc.) to quantify and analyze the data. In the analysis, a 35 × 35 pixel correlation window with a 3-pixel step size was considered and the full field lateral and axial displacement fields are extracted. These displacement fields were later used to extract fracture parameters. It should be mentioned that the whole experimental procedure was repeated twice in order to assure the repeatability of the experimental results.

Finally, the characteristics of the fracture and failure surface were investigated using field emission scanning electron microscopy (FESEM) and optical microscopy.

2.3. Calculation of stress intensity factor (SIF)

Stress intensity factor, in the case of a three-point bending experiment, can be obtained from the critical load at fracture along with the specimen geometry, using linear elastic fracture mechanics formula given by the following equation:

$$K_i = \frac{P_0}{B \sqrt{W}} f(a/W)$$

(1)

where, $P_0$ is the critical load at fracture, $B$ and $W$ are specimen thickness and width, respectively, $a$ denotes the initial crack length and $f(a/W)$ is the geometric factor defined as

$$f(a/W) = \frac{3S/W}{2(1+2(a/W))(1-(a/W)^3)}$$

$$[1.99 - (a/W)(1-(a/W))(2.15 - 3.93(a/W) + 2.7(a/W)^2)]$$

(2)

Table 1. Composition and material properties of the Ti/TiB material system [12].

<table>
<thead>
<tr>
<th>Layer no.</th>
<th>Ti (vol%)</th>
<th>TiB (vol%)</th>
<th>Layer thickness (mm)</th>
<th>E (GPa)</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>0</td>
<td>0.2032</td>
<td>106</td>
<td>0.340</td>
</tr>
<tr>
<td>2</td>
<td>85</td>
<td>15</td>
<td>0.3810</td>
<td>170</td>
<td>0.278</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>30</td>
<td>0.3810</td>
<td>227</td>
<td>0.238</td>
</tr>
<tr>
<td>4</td>
<td>55</td>
<td>45</td>
<td>0.3810</td>
<td>262</td>
<td>0.205a</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>60</td>
<td>0.3810</td>
<td>289</td>
<td>0.177</td>
</tr>
<tr>
<td>6</td>
<td>25</td>
<td>75</td>
<td>0.3810</td>
<td>303</td>
<td>0.152</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>85</td>
<td>1.0668</td>
<td>316</td>
<td>0.140</td>
</tr>
</tbody>
</table>

* Interpolated using chemical composition and the rule of mixture.
where, \( S \) is the span between the centers of the rigid supports. Using the critical load, \( P_{Q} \), the apparent fracture toughness, i.e., \( K_{IC} \), of the examined material can be determined. However, in this approach, the fracture parameters on each side of the sample and through the thickness cannot be evaluated. In the case of graded materials, however, due to the variation of material properties along the graded direction, knowing the fracture response on each side of the specimen is essential.

One approach suitable for the material system examined in this work is the use of full-field displacement measurements in the vicinity of the crack tip to estimate the stress intensity factor present on each side of the sample. This can be accomplished by using the asymptotic displacement field equations along with measured displacement components in a least-square scheme. In this case, the horizontal \((u)\) and vertical \((v)\) displacement components in a mixed mode condition are expressed as the following infinite series [20]:

\[
\text{Mode I:} \quad \begin{align*}
u_{II} &= \sum_{n=1}^{\infty} \frac{P_{Q}n^{2/2}}{2\mu}b_{n}\left[\frac{n+\frac{n}{2}}{2}(-1)^{n}\cos n\theta\frac{n}{2} - \cos \frac{(n-4)\theta}{2}\right] \\
u_{II} &= \sum_{n=1}^{\infty} \frac{P_{Q}n^{2/2}}{2\mu}b_{n}\left[\frac{n-\frac{n}{2}}{2}(-1)^{n}\sin n\theta\frac{n}{2} - \sin \frac{(n-4)\theta}{2}\right]
\end{align*}
\]

\[
\text{Mode II:} \quad \begin{align*}
u_{II} &= -\sum_{n=1}^{\infty} \frac{P_{Q}n^{2/2}}{2\mu}b_{n}\left[\frac{n+\frac{n}{2}}{2}(-1)^{n}\sin n\theta\frac{n}{2} - \sin \frac{(n-4)\theta}{2}\right] \\
u_{II} &= \sum_{n=1}^{\infty} \frac{P_{Q}n^{2/2}}{2\mu}b_{n}\left[\frac{n-\frac{n}{2}}{2}(-1)^{n}\cos n\theta\frac{n}{2} - \cos \frac{(n-4)\theta}{2}\right]
\end{align*}
\]
where the subscripts (I) and (II) refer to the Mode-I and Mode-II conditions, respectively. μ is the shear modulus of the material, and ϵ is a function of the material’s Poisson’s ratio, ν, written as

\[ \epsilon = \frac{(3 - ν)/(1 + ν)}{3 - 4ν} \text{ Plane – stress} \]

Eq. (5)

Eqs. 3 and 4 can be written in the matrix form as [20]

\[
\begin{bmatrix}
\mathbf{u}_1 \\
\vdots \\
\mathbf{u}_m \\
\mathbf{v}_1 \\
\vdots \\
\mathbf{v}_m
\end{bmatrix} =
\begin{bmatrix}
\mathbf{f}_1 \\
\vdots \\
\mathbf{f}_m \\
\mathbf{g}_1 \\
\vdots \\
\mathbf{g}_m
\end{bmatrix} \begin{bmatrix}
\mathbf{g}_1 \\
\vdots \\
\mathbf{g}_m
\end{bmatrix} + 
\begin{bmatrix}
\mathbf{a}_n \\
\vdots \\
\mathbf{b}_n
\end{bmatrix} \quad (6)
\]

where the subscripts (I) and (II) refer to the Mode-I and Mode-II conditions, respectively. μ is the shear modulus of the material, and ϵ is a function of the material’s Poisson’s ratio, ψ, written as

\[ \epsilon = \frac{(3 - ν)/(1 + ν)}{3 - 4ν} \text{ Plane – strain} \]

Eq. (5)

Eqs. 3 and 4 can be written in the matrix form as [20]

\[
\begin{bmatrix}
\mathbf{u}_1 \\
\vdots \\
\mathbf{u}_m \\
\mathbf{v}_1 \\
\vdots \\
\mathbf{v}_m
\end{bmatrix} =
\begin{bmatrix}
\mathbf{f}_1 \\
\vdots \\
\mathbf{f}_m \\
\mathbf{g}_1 \\
\vdots \\
\mathbf{g}_m
\end{bmatrix} \begin{bmatrix}
\mathbf{g}_1 \\
\vdots \\
\mathbf{g}_m
\end{bmatrix} + 
\begin{bmatrix}
\mathbf{a}_n \\
\vdots \\
\mathbf{b}_n
\end{bmatrix} \quad (6)
\]

with

\[
f_{m,n} = \frac{n^{n/2}}{2\pi m} \left[ \kappa + \frac{n^2}{2} (n - 1)^2 \right] \cos \frac{n\theta_m}{2} \frac{n}{2} \cos \frac{(n - 4)\theta_m}{2} \]  

\[ (7-a) \]

\[
g_{n,m} = -\frac{n^{n/2}}{2\pi m} \left[ \kappa + \frac{n^2}{2} (n - 1)^2 \right] \sin \frac{n\theta_m}{2} \frac{n}{2} \sin \frac{(n - 4)\theta_m}{2} \]  

\[ (7-b) \]

\[
h_{m,n} = \frac{n^{n/2}}{2\pi m} \left[ \kappa - \frac{n^2}{2} (n - 1)^2 \right] \cos \frac{n\theta_m}{2} \frac{n}{2} \cos \frac{(n - 4)\theta_m}{2} \]  

\[ (7-c) \]

\[
l_{m,n} = \frac{n^{n/2}}{2\pi m} \left[ \kappa - \frac{n^2}{2} (n - 1)^2 \right] \cos \frac{n\theta_m}{2} \frac{n}{2} \cos \frac{(n - 4)\theta_m}{2} \]  

\[ (7-d) \]

where \( f_{m,n} \), \( g_{n,m} \), \( h_{m,n} \) and \( l_{m,n} \), are parameters that represent the material properties at any given location \((r, \theta)\) relative to the crack tip. Since displacement components \(u\) and \(v\) can be found using DIC, the only remaining unknowns in Eq. (6) are the constants \(a_n\) and \(b_n\). These constants can be determined using overdeterministic approach. In this approach, as long as the number of data points \((m)\) exceeds the number of terms \((n)\) in Eq. (6), the system of equations can be solved using a least-square scheme to obtain values for \(a_n\) and \(b_n\). Once the unknowns \(a_n\) and \(b_n\) are obtained, the stress intensity factors can be calculated as

\[ K_I = a_1\sqrt{2\pi} \quad K_{II} = -b_1\sqrt{2\pi} \quad (8) \]

where \(K_I\) and \(K_{II}\) represent the stress intensity factors (SIF) in Mode-I and Mode-II, respectively.

In the case of a Mode-I loading condition, all terms associated with Mode-II can be neglected. In the present work, due to the nature of the loading conditions, only Mode-I fracture is considered. For a higher level of accuracy, the first 5 terms in the series expansion \((n=5)\) and a minimum of 150 \((m \geq 150)\) data points were considered during the least-square analysis. The displacement components within a radius of \(r < 2B\) (with \(B\) being the specimen thickness) were extracted using DIC and used as the input to a MATLAB® code, written in house, to estimate the stress intensity factor present at the instance of fracture as well as its evolution during the loading stage.

2.4. Finite element analysis

In order to extract the fracture parameters across the specimen thickness, a full-scale 3D finite element analysis was performed using ABAQUS. Due to the symmetry of the loading condition and the specimen geometry only one half of the specimen was modeled, applying proper boundary conditions at the symmetry plane. The model was sectioned into seven layers, corresponding to the actual seven layers of the FGM sample, and material properties were assigned to each layer.

To be consistent with the experimental procedure, as shown in Fig. 3, a constant vertical displacement \((\delta)\) was applied on the top edge of the finite element model. The value chosen for \(\delta\) in this case was identical to that found during the actual experiments. A static finite element analysis was conducted in the present work and the material properties were assumed to be rate-independent, assuring that the loading rate in the finite element simulation would not affect the results. The ABAQUS default meshing tool was used to discretize the model, with a total number of 146,945 elements of the type C3D8. In addition, each layer was discretized...
3. Results and discussion

Typical far-field load as a function of loading tip displacement is shown in Fig. 4. The load curve is characterized by two distinct stages. The magnitude of the load constantly increases with the loading tip displacement during the first stage, reaching a local maximum at which the ceramic-rich side fractures, followed by a step-wise load drop. The step-wise load drop observed in the load displacement diagram in this stage could be related to the fracture process occurring at different layers near the ceramic-rich side of the specimen. The magnitude of the load then increases once again, until it reaches another local maximum followed by a sudden drop. The load curve observed in Fig. 4 indicates that, for transversely graded materials with cracks perpendicular to the gradient direction, each layer could have fractured at a different time, resulting in a non-uniform crack propagation through the thickness. It is also noticed that a large portion of the external work is applied between the two local maxima in the load curve. After the onset of crack propagation on the ceramic-rich side, the crack front is no-longer a straight line; instead an inclined crack front with a leading edge towards the ceramic-rich side. The crack path can be clearly seen on the image of the fracture surface shown in Fig. 5a. The crack front locations on the ceramic-rich side and metal-rich side of the specimen, well after the crack initiation on the ceramic-rich side, are also displayed in Fig. 5b and c. The crack is shown to have propagated completely through the specimen ligament on the ceramic-rich side, while the crack propagation has ceased at some distance shorter on the metallic side.

The scanning electron microscopy (SEM) images of the fractured surface shown in Fig. 6 give a clear insights regarding the fracture behavior of the material through the thickness direction. The inclined crack front can be observed in the low magnification SEM image on the left. The quasi-cleavage fracture surface evident on the metal side indicates that some extent of plastic deformation has occurred on this side [21]. The fracture surface changes remarkably from apparent cleavage to a surface indicating brittle fracture in the regions towards the ceramic-rich side. The occurrence of plastic deformation within the metallic side of the specimen can be also observed from the load curve shown in Fig. 4. The extended displacement between the first and second local maxima shown in the load–displacement curve could be...
attributed to the aforementioned local plastic deformation in the metallic layers of the specimen. The formation of the inclined crack front, corresponding to the first peak load makes the linear elastic fracture analysis invalid once the crack has initiated on the ceramic-rich side. Thus, all the mathematical analyses as well as the fracture toughness calculations are performed up to the first peak load, i.e., the crack initiation on the ceramic-rich side.

3.1. Apparent fracture toughness

Using the peak load, 204 N, at the onset of fracture on the ceramic-rich side, the apparent fracture toughness of the material was calculated according to Eqs. (1) and (2), and a value of 5.2 MPa√m was found accordingly. The apparent fracture toughness of the material calculated in the present work agrees well with the value reported earlier by Kidane and Shukla [12].

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Fig. 5. (a) A magnified view of the fracture surface of the material, showing inclined crack propagation. (b) The crack path on the TiB-rich side and (c) the crack path on the Ti side. The final crack tip location is marked on (b) and (c).

Fig. 6. SEM observation of the fracture surface of the transversely graded specimen, showing inclined crack front and the quasi-cleavage fracture surface on the Ti side.
3.2. Stress intensity factor on each side of the specimen

In order to experimentally determine the SIF values on both sides of the specimen, the numerical approach explained in Section 2.3 was utilized. Typical displacement components on the two sides of the specimen are shown in Fig. 7. The evolution of SIF obtained from the displacements fields using Eqs. (6)–(8), on the metallic and ceramic sides of the specimen, are displayed in Fig. 8. The stress intensity factors on the ceramic and metallic sides of the specimen at the first peak load, i.e., the moment at which fracture initiates on the ceramic side, were 6.2 MPa√m and 2.12 MPa√m, respectively. It is important to note that the SIF value found on the metallic side, i.e., 2.12 MPa√m, does not represent the fracture toughness of the metallic component (Ti) of the examined FGM, rather it is the SIF value on the metallic side at the instant of crack initiation on the ceramic-rich face. On the other hand, comparing various studies in the literature reveals that the SIF value on the ceramic side found in this work is actually very close to the value of the fracture toughness of the monolithic ceramic component of the examined material, i.e., 85% TiB–15% Ti [18,22,23]. This, as well as the onset of crack initiation, which initially takes place from the ceramic side, indicates that the parameter controlling the fracture toughness of the material in this work has indeed been its ceramic component. In the case of a transversely graded material the far-field load will not be uniformly distributed along the layers, and a larger portion of the load will be supported by the stiffer section compared with the more compliant section. The effective fracture toughness of this material, i.e., 5.2 MPa√m, which was found to be relatively closer to the SIF value obtained for the ceramic side, also confirms the above statement.

3.3. Stress intensity factor across the thickness

Next, the SIF variation along the specimen thickness was calculated following the same procedure explained earlier, but instead using the displacement fields from the FE simulation. To validate the numerical results obtained from the finite element model, the opening displacement component history obtained from DIC and FE model are shown and compared in Fig. 9. As clearly shown in this figure, numerically calculated displacement values are in good agreement with those measured experimentally. Also, typical variation of displacement components within a 3 mm area around the crack tip, obtained from the finite element model, on both ceramic and metal sides are shown in Fig. 10. It is observed that prior to the fracture initiation, the variation of u and v displacement components on both metallic and ceramic sides are quite similar throughout the loading process. This is consistent with the results found in the literature for the transversely graded materials [11]. Using the experimentally measured SIF values at the two end faces as a benchmark, the SIF values were calculated at different sections along the thickness direction of the specimen utilizing the displacement fields obtained from 3D FE simulation. The SIF values calculated using the FE displacement fields are shown in Fig. 11, as a function of the normalized thickness.
3.4. Semi-empirical model to estimate the variation of stress intensify factor across the thickness

The variation of the SIF along the gradient direction is shown to follow the trend of elastic modulus, as also previously reported in literature [10,11]. This indicates that the variation of SIF through the gradient direction in transversely graded structures depends highly on the layers’ elastic constants. In addition, as shown earlier in Fig. 10 and previously documented by Kommana and Parameswaran [11], the displacement distribution at the crack tip neighborhood does not undergo any remarkable variation throughout the gradient direction. Considering these two facts as the guideline, a simple approach is proposed to determine the SIF value at any position along the gradient direction, having the material’s elastic constants at any given position, as well as the SIF value on either side of the specimen, e.g., ceramic side. In this approach, using the first term in the series expansion representation of the opening displacement component in Eq. (3), the ratio of $K_I(x)$, i.e., the stress intensity factor at position $x$ along the thickness, to the $K_I$ value found on the ceramic side, can be expressed as

$$\frac{K_I(x)}{K_{I}^{\text{ceramic}}} \approx \frac{E(x)}{E_{\text{ceramic}}} \times \frac{K_{I}^{\text{ceramic}}(1 + \nu_{\text{ceramic}})}{K(x)(1 + \nu(x))} \tag{9}$$

where the sub/superscript ‘ceramic’ refers to the material properties on the ceramic side, and $x$ denotes the position along the gradient direction. The normalized $K_I(x)$ ratios at each point along the thickness of the specimen obtained from the FE simulation and Eq. (9) are compared in Fig. 12. A good agreement between the two sets of data is observed, indicating that for the examined graded material, the SIF values along the graded direction can be readily calculated with a reasonable accuracy using the magnitude of SIF at one side along with the material’s elastic constants through the gradient direction. This can be considered a computationally efficient method to estimate the variation of SIF through the thickness when the crack line is located along the graded thickness.

3.5. A model to predict the material’s fracture toughness

The average stress intensify factor across the thickness, $K_{I}^{\text{average}}$, can be calculated using the mean value theorem as

$$K_{I}^{\text{average}} = \frac{1}{B} \int_{0}^{B} K_I(x) dx \tag{10}$$

where $B$ is the thickness of the FGM plate. At this point, having already known that the fracture response of the ceramic-rich side controls the fracture response of the entire material, Eq. (10) can
be rewritten as

\[ K_{\text{IC}}^{\text{average}} = \frac{K_{\text{IC}}^{\text{ceramic}} (3 - \nu^{\text{ceramic}})}{B E^{\text{ceramic}}} \int_0^b \frac{E(x)}{3 - \nu(x)} dx \]  

(11)

where \( K_{\text{IC}}^{\text{ceramic}} \) denotes the fracture toughness of monolithic ceramic-rich component of the material, i.e., 85% TiB–15% Ti, in the present work. Since the thickness of each layer is small, the plane-stress condition is considered and the corresponding relation for \( K \) give by Eq. (5) has been considered. Using this mathematical expression, the average stress intensity factor \( K_{\text{IC}}^{\text{average}} \) of the FGM material in this work was found to be 5.11 MPa\( \sqrt{m} \), which is very close to the effective fracture toughness of the material determined from the far-field load in Section 3.1, exhibiting less than 2\% difference. This indicates that the effective fracture toughness of such FGM structure in a transversely graded configuration can be predicted using the fracture toughness of its stiffest component, along with the variation of the material properties through the thickness (gradient) direction as

\[ K_{\text{IC}}^{\text{FGM}} = \frac{K_{\text{IC}}^{\text{ceramic}} (3 - \nu^{\text{ceramic}})}{B E^{\text{ceramic}}} \int_0^b \frac{E(x)}{3 - \nu(x)} dx \]  

(12)

where \( K_{\text{IC}}^{\text{FGM}} \) denotes the effective fracture toughness of the FGM. Considering the results obtained here, the following final remarks should be noted:

- The parameter controlling the fracture toughness of transversely graded structures subjected to quasi-static loading conditions, such as the one considered in this work, is their most brittle component. Therefore, any enhancement in the toughness of these materials in such configuration requires the improvement of the fracture toughness of their ceramic component.

- In contrast to the crack arrester configuration [18,19,22], the metallic component will not have any significant effect on toughening of the metal/ceramic FGMs in transversely graded configurations.

- The actual load carrying section of a transversely graded material, before fracture initiation is its stiffer section, i.e.,
ceramic-rich layers. Upon the crack initiation on the ceramic-rich side, the load bearing section suddenly shrinks to the more compliant metallic layers, leading to a sudden development of high stress intensity on these layers, eventually resulting in complete failure. Thus, no beneficial influence is provided by the metallic components in transversely graded metal/ceramic FGMs, at least prior to crack initiation.

4. Conclusion

Fracture response a graded Ti/TiB material system was studied for a specimen having a crack perpendicular to the graded direction. Surface deformation of the cracked specimen was simultaneously examined on both metal and ceramic-rich faces of the specimen using 3D DIC. The stress intensity factors on either side of the specimen were calculated using the displacement fields obtained from DIC, based on an overdeterministic least-square approach. Variation of the SIF through the thickness (gradient) direction was also determined using the displacement fields obtained from finite element analysis. Variation of the SIF value along the gradient direction was shown to follow a trend similar to the variation of the elastic modulus of the material. Considering this, a simple model was proposed to relate the ratio of SIF at any desired location as a function of SIF at ceramic face, elastic modulus, and Poisson’s ratio. A more detailed investigation of the fracture response of the material reveals that the fracture resistance of the material in its present configuration is highly dependent on the fracture resistance of its ceramic-rich layers.

Acknowledgments

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References


Fig. 12. Normalized stress intensity factor (the ratio of stress intensity factor at position ‘\( x \)’ to the SIF on the ceramic side) as function of normalized thickness. (comparison between FE analysis and DIC.)