Research paper

Effects of cell-wall instability and local failure on the response of closed-cell polymeric foams subjected to dynamic loading

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**A R T I C L E   I N F O**

Article history:
Received 31 October 2016
Revised 16 March 2017
Available online 30 March 2017

Keywords:
Polymeric foam
Meso-scale
Brittle fracture
High strain rate
Cell-wall instability
Digital image correlation

**A B S T R A C T**

Dynamic deformation response of closed-cell rigid polymeric form is explored at different length scales. This study is facilitated by ultra-high speed photography in conjunction with digital image correlation (DIC). In-plane strain components developed over cell-walls are measured via DIC and compared with the global deformation response of the material. Local strain rates measured at mesoscopic scale are found to be at least one order of magnitude greater than the global strain rates applied on the specimen. Such substantial variation across different scales is used to explain the contribution of the strength and modulus of parent polymer material in the cellular scale deformation and failure response of the specimen, as well as load bearing and strong strain rate sensitivity of the foam. Experimental results are also used in conjunction with idealized models to identify the dominant cellular scale failure mechanisms in the examined material.

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1. Introduction

Polymeric foams are the material of choice in applications that require high energy absorption and light structural weight (Gibson and Ashby, 1997). Polymeric foams have shown great potential applications in sandwich structures, cushioning, packaging, automotive and aerospace industries due their excellent energy dissipation, as well as acceptable specific strength and modulus (Gardner et al., 2012; Avalle et al., 2001). Usually, the general trend in designing of components and structures from polymeric foams is based on their energy absorption performance. Due to the fact that the application of polymeric foams in structures is usually in cases where dynamic loading conditions are dominant, characterizing and understanding the response of these materials at high strain rate loading conditions have always been of great interest.

Characterizing the macroscale deformation response of polymeric foams at various time scales (quasi-static, medium and high strain rate conditions) has attracted great attention for decades. It is well documented that, at macroscale, polymeric foams are strongly sensitive to strain rate. In particular, exponential relationships between the crush (failure) stress and strain rate have been documented for a variety of polymer foams (Oulet et al., 2006). It has also been well-documented that a transition in the strain rate sensitivity of these materials occurs usually at strain rates beyond $10^2$ s$^{-1}$, above which both failure stress and modulus of elasticity of the material increase rapidly with strain rate (Zhao, 1997; Nagy et al., 1984; Song et al., 2005; Yi et al., 2006; Mulliken and Boyce, 2006; Koohbor et al., 2016a).

The strong strain rate sensitivity in polymer foams basically has two sources: (a) release of the gas entrapped inside the closed-cell structure of the foam, and (b) strain rate dependent mechanical response of the solid parent material (Sun and Li, 2015). The former mainly contributes to the significant strain rate sensitivity and strain hardening response in closed-cell foams and has been investigated in some previous research works (Bouix et al., 2009; Mondal et al., 2009; Di Landro et al., 2002). For instance, Bouix et al. (2009) experimentally assessed the contribution of gas release to the strain rate sensitivity and hardening response of closed-cell EPP foams by conducting quasi-static and dynamic experiments on submerged specimens. The analysis conducted in that work was based on the formation of gas bubbles on the surface of the specimen as a result of gas release following the deformation and cracking of closed cells inside the material. More recently, the effects of entrapped gas on dynamic compressive response of cellular structures were studied by Sun and Li (2015, 2016) through 2D and 3D finite element analyses. Numerical observations indicated that the gas release effect is significant at the densification stage. In addition, the stress enhancement due to increased strain rate was found to arise from not only the gas release, but also the cell deformation and gas-solid interactions.

The contribution of the mechanical behavior of parent polymer material on the macroscale deformation response of foams has also been investigated, mostly by studying the deformation behavior...
at cellular scales. However, majority of the research conducted at such small length scales is either based on micromechanics simulations with no experimental evidence (Pal et al., 2010; Alkholder and Vural, 2008; Brydon et al., 2005; Mills et al., 2009; Li et al., 2003), or using ex-situ experimental analyses (Chan et al., 1997; Vural and Ravichandran, 2003; Gupta and Kishore, 2002; Godara and Raabe, 2007). In addition, most of meso-scale experimental investigations are focused on the quasi-static deformation and failure mechanisms in cellular materials, the reason for which might be due to a number of challenges faced in dynamic experimentation.

The current study is motivated by a need to understand the role of solid polymer material instability, as well as cellular-scale deformation and failure on the macroscale response of closed-cell polymeric foams. Due to the importance of experimental validations and the fact that studies dealing with in-situ cellular-scale deformation measurements in polymer foams are very scarce in the available literature, our main objective in this work is to devise a novel approach that facilitates full-field experimental measurements at cellular scales. It should be noted that in-situ full-field measurements at mesoscopic scales have been successfully implemented in the past to characterize the deformation behavior of various types of materials, e.g. metals and fiber composites deformed in quasi-static conditions (Ravindran et al., 2017; Koohbor et al., 2015; Elstathieu et al., 2010). On the other hand, there are only a few studies that take advantage of in-situ full-field measurements at submillimeter scales and in dynamic loading conditions (Bodelot et al., 2015; Ravindran et al., 2016a,b). In fact, to the best of our knowledge, no experimental full-field deformation characterization at meso-scales has ever been conducted on dynamically deformed foams and other cellular materials. Therefore, the fundamental idea in the present study is to take advantage of meso-scale full-field measurements to reveal the underlying deformation and failure mechanisms in a closed-cell polymeric rigid foam at cellular scales. To this purpose, we propose an experimental approach that utilizes ultra-high speed photography in conjunction with digital image correlation (DIC) to capture the deformation response of cell-walls in a PU-based closed-cell foam. Experimental results are then analyzed in conjunction with well-established simplified failure models to provide a deeper quantitative insight on the meso-scale instability mechanisms in the material.

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2.1. Rigid material

A rigid closed-cell PU-based polymeric foam is examined in this work. The foam used in this work is supplied by Sandia National Laboratories under the commercial name TufFoam35 (Lu, 2014). Macro-scale constitutive response of the examined material at various strain rates, from quasi-static to 5000 /s, has been obtained and studied in detail in our previous studies (Koohbor et al., 2016a, b, c). Apparent bulk density and yield stress of the foam were measured as 560 kg/m$^3$ and 20 MPa, respectively. Fig. 1 illustrates cellular structure of the examined foam. Fig. 1 exemplifies a closed-cell structure with average cell size and wall thickness of 200 μm and 150 μm, respectively. The nominal density of the parent polymeric material is 1329.5 kg/m$^3$. Yield stress and elastic modulus of the solid polymer were also obtained at quasi-static conditions (0.01 /s) as 75.84 MPa and 2413.2 MPa, respectively. Cubic specimens of 14 × 14 × 14 mm$^3$ are extracted from a single foam billet supplied by Sandia National Laboratories and subjected to dynamic loading, as described in the forthcoming section.

Digital image correlation is used to enable full-field measurements at cellular scales. To facilitate DIC, a suitable speckle pattern must be applied on the surface over which full-field measurements are to be conducted. Therefore, in order to perform DIC measurements at cellular scales, a fine high contrast speckle pattern must be applied on the cell walls. The rule of thumb adopted in the DIC literature states that a minimum number of 3 speckle particles are required inside a subset to enable proper strain mapping (Rajan et al., 2012; Sutton et al., 2008). Therefore, based on the average thickness of the cell-walls in the foam specimen, a speckle size range of 24–40 μm was used in this work. To apply a pattern containing speckles particles of this size range, a water-based black paint was applied on the well-polished surface of the specimen using a high-pressure airbrush. The speckle pattern produced in this way is illustrated in Fig. 2.

2.2. Dynamic loading and imaging

Dynamic loading of the sample is performed using a conventional split Hopkinson pressure bar (SHPB). A standard SHPB apparatus typically consists of a striker, incident and transmitter bars, whereas the specimen is sandwiched between the two bars. In this study, aluminum bars are used to obtain sufficient transmitted signals and to attain the stress equilibrium conditions during loading. Both the incident and the transmitter bars are made of 1830 mm long and 25.4 mm diameter 2024 aluminum alloy. No pulse shaping technique is applied. To reduce the friction between the specimen and the bars, a thin layer of molybdenum disulfide is applied on the contacting surfaces. The waves in the incident and transmitter bars are measured with the help of strain gages located at the mid-length of the incident and transmitter bars. The average strain rate (δ$^i$), strain (ε$^i$) and stress (σ$^i$) on the samples are obtained using one-wave equations (Chen and Song, 2011). Fig. 3a illustrates typical signals measured from a SHPB experiment conducted on the foam specimen in this work. Force balance has been checked and validated by measuring the forces on both specimen ends, as shown in Fig. 3b. It is clearly seen that the forces measured on incident and transmitter bars show substantial differences during the first 30 μs. Such difference is due to the presence of inertia loads developed during early stages of deformation as discussed in details in our previous works (Koohbor et al., 2016a, b, c).

In-situ full-field deformation measurement is facilitated in this work by the use of a single ultra-high speed imaging system in conjunction with DIC. The camera system used in the present work is a HPV-X2 camera (Hadland Imaging Inc.) recording images at a framing rate of 1 million fps and at a full resolution of 400 × 250 pixel$^2$. To conduct high magnification imaging, an extension tube (by Navitar) is used as the objective lens. The greatest challenges in obtaining images at high magnification and high rates are mainly
associated with low depth of field in the optical system and the need for high intensity illumination. The magnification of the optical system for the meso scale experiments in this work is selected by considering the resolution and depth of field required for the experiment. An optical resolution of 8 μm/pixel is achieved with sufficient depth of field for the experiment. For illumination, a high intensity flash light is used along with auxiliary illumination. The flash lamp takes 200 μs to reach its highest intensity, after which its intensity remains constant for 1 ms; therefore, the triggering of the camera and the flash unit are performed carefully to utilize the maximum illumination without any significant variation in lighting. Fig. 4 shows the experimental setup used in this work.

Processing of the images acquired during loading stage is performed in VIC-2D (Correlated Solutions, Inc.). In this software, displacement distribution is first determined using subset size of 9 × 9 pixels (72 × 72 μm²) and step size of 3 pixel. Strain distribution is then derived from the full-field displacement with the use of a Gaussian filtering and a filter size of 9.

3. Possible failure modes at cellular scales

In order to give an insight to the dominant failure modes at cellular scales, mathematical models proposed for cellular structures by Gibson and Ashby (1997) are presented here. Note that the relative density of the material used in this work is 0.42, and it slightly exceeds the value commonly accepted for the definition of a cellular solid (Gibson, 2005). The relative density measured for the examined material basically falls within the range for porous materials. In this case, the load transfer mechanisms may not be represented by the idealized equation proposed by Gibson and Ashby (1997). Therefore, the instability equations described here are merely presented to facilitate a more in-depth understanding of the possible cellular scale failure mechanisms in a polymeric rigid foam specimen subjected to dynamic loading conditions. Three principal failure mechanisms are regarded: (1) elastic buckling (2) plastic collapse and (3) brittle failure. Our focus, hereafter, will be on investigating the possibility of the occurrence of each failure mechanism at a function of relative density in the material. To avoid the repetition of well-established mathematical models already available in the literature, we will directly focus on the final equations proposed by Gibson and Ashby (1997). These equations are provided in Appendix A.

Simple failure mode expressions presented in Appendix A can be evaluated using actual material properties measured for the parent polymer. Accordingly, Fig. 5 illustrates the variation of critical stress magnitudes for the three most probable cellular-scale failure modes with respect to different t/l ratios. Curves shown in Fig. 5 indicate that in conditions where there is no significant strain rate effect in the strength and elastic modulus of the parent polymer, there will be a competition between elastic buckling and failure brittle within thinner cell-walls. In this regard, elastic buckling is shown to be a more probable failure mode over the areas with very thin cell-walls. On the other hand, brittle failure mode is evidently the predominant failure mode in thicker cell-walls. For
the material examined in this work, due to its larger t/l ratio, brittle failure is more likely to be the dominant failure mode when the material is subjected to slow rates of deformation. The predominance of brittle failure in the examined material is confirmed by applying quasi-static compression on foam specimens and observing the cellular structure after unloading. Fig. 6 depicts the cell structure of a specimen after 10% and 28% quasi-static compression applied at a constant global strain rate of 0.01/s. As shown in Fig. 5, at lower global strain (10%), buckling is a dominant failure mode; while upon exertion of larger global strains (28%) brittle failure is observed to become dominant, as indicated by circles in Fig. 6b.

4. Identification of cellular-scale failure in dynamic loading conditions

Similar to the approach presented for the case of quasi-static loading, a study is conducted attempting to identify the failure and cellular-scale instability modes in dynamic loading conditions, as well. To this purpose, the same set of equations described for the case of quasi-static loading (Eqs. A1 to A7) are regarded. However, in order to incorporate the influence of strain rate in the analysis, elastic modulus and yield stress of the parent polymer material are taken to be strain rate dependent in this case. This is essentially a valid assumption, since the parent polymer is a viscoelastic material, therefore shows significant strain rate dependence in its elastic modulus (Mulliken and Boyce, 2006). Moreover, the rate-dependence of polyurethane has been well established in the literature, indicating strong rate sensitivity of the yield stress of the material (Yi et al., 2006; Sarva et al., 2007).

To incorporate the strain rate effect into the equations, yield stress and elastic modulus of the polymer are assumed to follow a linear relationship with the logarithm of the applied strain rate, \( \dot{\varepsilon} \), as (Song et al., 2005):

\[
E = E_0 + A \log \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) 
\]

\[
\sigma_{ys} = \sigma_{ys0} + B \log \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right) 
\]

where the subscript “0” denotes the value of the parameter in quasi-static loading; \( A \) and \( B \) are material constants that can be obtained from experiments performed on the polymer. In the present work, material constants \( E_0 \) and \( \sigma_{ys0} \) were obtained from independent experiments conducted on the parent material at quasi-static conditions (\( \dot{\varepsilon} = 2 \times 10^{-3} \)). In addition, constants \( A \) and \( B \) were
mechanisms in dynamic loading conditions:

\[ E(MPa) = 2413.2 + 68.5 \log \left( \frac{\dot{\varepsilon}}{2 \times 10^{-3}} \right) \]  

\[ \sigma_{yy}(MPa) = \begin{cases} 
75.8 + 4.7 \log \left( \frac{\dot{\varepsilon}}{2 \times 10^{-3}} \right) & \dot{\varepsilon} \leq 2 \times 10^{-3} \\
75.8 + 15.8 \log \left( \frac{\dot{\varepsilon}}{2 \times 10^{-3}} \right) & \dot{\varepsilon} > 2 \times 10^{-3}
\end{cases} \]  

Using the modified strain rate sensitive equations for modulus and yield stress of the parent polyurethane material, variation of critical stress for different failure modes with respect to \( t/l \) ratios are plotted for different strain rate conditions in Fig. 7.

It is clearly seen that the trend previously observed for quasi-static loading conditions still exists for the dynamic loading conditions, meaning that the elastic buckling still will be the dominant failure mode for thinner cell-walls with its critical stress being significantly small. The difference between the critical stresses associated with elastic buckling and the other two modes is considerably high at larger \( t/l \) ratios and at quasi-static loading conditions. This means that in real cases with \( t/l \) ratios larger than 0.15, which is the case in this work, elastic buckling may not be regarded as the dominant failure mode in quasi-static loading conditions.

On the other hand, as the applied strain rate escalates, the difference between the critical stresses predicted for all three failure modes becomes smaller. This means that although brittle failure mode is anticipated to be the principal failure mode in all strain rate conditions, the chance of failure by elastic buckling and/or plastic collapse also becomes higher at higher strain rates. The convergence of all three modes at higher strain rate conditions is due to the more significant increase of yield (failure) stress of the parent material with strain rate. Note that the elastic modulus of the solid material does increase with strain rate, however not at the same pace as its yield stress does. Due to the fact that the elastic buckling is governed by the elastic response of the cell-walls, its increase is less significant with strain rate; therefore, the three curves tend to converge at higher strain rates. It is also important to note that, although the critical stresses in the dynamic case are shown to be higher than the quasi-static condition, the specimen under dynamic condition could fail at a lower load if there are initial flaws or cracks in the material (Kidane 2013). This is well in line with the argument stated earlier, at higher strain rate the apparent material become brittle.

It is important to note that the curves shown in Fig. 7 are plotted assuming perfect hexagonal cell configuration, and that the strain rate is distributed uniformly over all cell-walls. This is far from reality. In a realistic case: (1) the cell-wall thickness \( t/l \) ratio is distributed randomly within the material, (2) the orientation of cell-walls with respect to the loading direction is arbitrary, and (3) the global strain and strain rates applied on the specimen are significantly different from those applied locally on the cell-walls. The first two issues are very difficult, if not impossible, to control and study in an accurate way. However, the latter challenge can be a subject of study using in-situ full-field measurements at cellular scales. Accordingly, in-situ meso-scale digital image correlations are conducted in this work to provide insight on the local deformation response and meso-scale instability of the material at cellular scales, as discussed in the following section.

5. In-situ full-field measurements at meso-scales

Fig. 8 illustrates images acquired during the dynamic loading stage, showing the cellular structure of the specimen subjected to various global stress magnitudes. The images shown in
Fig. 8. (a) Gray level in-situ images acquired during dynamic loading at different stress levels shown in (b). Early failure locations are marked with arrows.

Fig. 8 are obtained following the procedure described earlier in Section 2.2 using an un-speckled specimen. It is clearly seen that cellular-scale failure occurs at stress levels as low as 4 MPa, a magnitude significantly smaller than the bulk yield stress of the foam (i.e., 20 MPa). It is also evident that the cell structure of the material undergoes complete failure at stress levels above 20 MPa, after which the densification process out-dominates the cellular scale deformation. Accordingly, our full-field measurements will include the time span over which the global stress levels remain well below 20 MPa. Note that achieving stress equilibrium in dynamic experiments is a challenging task, particularly during the earlier deformation times. The challenge intensifies when materials with low mechanical impedance (e.g., foams) are to be tested (Koohbor et al., 2016b,c). As described earlier, our approach in this work is to make a comparison between deformation kinematic (strain and strain rate) at various scales. Therefore, although force balance has been validated, stress measurement has no particular significance in our discussions.

To provide a more quantitative insight on the deformation response of cell walls, distribution of strain at mesoscopic scales is studied. Fig. 9 shows contour maps indicating different in-plane strain components developed over cell walls at different times after the initial impact. It is interesting to note that in spite of the application of uniaxial (along x-direction) global strain, all in-plane strain components show significant values during dynamic deformation, indicative of a complex deformation and strain state at cellular scales. It is worth noting that similar experiments conducted at various nominal strain rates indicated very similar deformation patterns, confirming complex deformation response at cellular scales. The complex distribution of in-plane local strain components makes it difficult to establish a correlation between global and local strain values. To facilitate a direct comparison between the global and local strain values, effective local von Mises strains are determined using the following equation:

$$e_{eff}(x, y, t) = \left[ \frac{2}{3} \left( e_{xx}^2(x, y, t) + e_{yy}^2(x, y, t) + 2e_{xy}^2(x, y, t) \right) \right]^{1/2}$$

(5)

where $e_{eq}$ denotes effective strain.

Fig. 10a shows the distribution of effective von Mises strain over the cell walls in the foam specimen at various times after the impact. It is clearly seen that effective strains of up to 10% are developed locally while the global strain magnitude is less than 1%. More importantly, a comparison between the local and global strain values (see Fig. 10b) indicates that at any given time during the entire course of deformation, the strain magnitudes at the two scales are vastly different.

Different strain magnitudes measured at macro and meso scales gives rise to the variation of strain rate at these two scales. Since the parent polymer is highly strain rate sensitive, local high strain rate magnitudes can alter the meso-scale deformation response as well as the failure mode. Fig. 11 depicts typical strain rate histories obtained at macro and meso-scales. The curves show substantially different strain rate values over the first 30 μs of the deformation. In fact, there is a one order of magnitude difference between the global and the local strain rate histories. Significantly high strain rates applied locally on the cell walls result in the local hardening of the polymer material. Depending on the orientation and the relative thickness of the cell wall located over regions where such high strain rate magnitudes are locally applied, the failure mechanism can be different. This was shown and discussed earlier in Section 4 where critical stresses associated with various failure modes were plotted for different strain rate values.

Full-field strain rate measurements conducted in this work can be used in conjunction with our failure model results (see Fig. 7) to conclude that the local behavior and instability response of cell walls in a dynamically deformed foam specimen are significantly different from what is expected, if the global deformation response is to be considered. In addition, using the modeling results discussed in Fig. 7, the contribution of the elastic buckling at cellular scales is also shown to be of great importance in dynamic loading conditions, particularly over the thinner cell walls. However, even at very high strain rates, the dominant instability mode for examined material is still brittle failure. Based on the fact that the brittle failure mode itself is controlled by the strength of the parent material, in cases where higher strength and higher energy dissipation characteristics are desired, special attention must be drawn toward the strengthening of the parent polymer material. One specific method practiced by different researchers to improve the mechanical strength of cell walls in polymeric foams is the reinforcement of the polymer with nanoparticles (Antunes and Velasco, 2014; Dolomanova et al., 2011; Mahfuz et al., 2004). This along with the ability to measure the local stress and strains using full-field measurements can be used as a guideline to design foam structures with improved energy absorption and load-bearing capacities.
6. Conclusions

Effects of local deformation, failure and cell-wall instability on the global response of closed-cell polymer foams subjected to high strain rate loading were investigated experimentally. The experimental setup in this work was consisted of an ultra-high speed camera equipped with high magnification lens and high intensity illumination system. Full-field measurements were facilitated through the use of digital image correlation, which enabled measuring of in-plane strain and strain rate components developed locally over the cell-walls in the examined polymer foam material.

Simple mathematical approaches were used to identify the possibility of the occurrence of various failure mechanisms in the examined material. It was shown that depending on (1) the strain rate applied locally on the solid structure of the material, and (2) the thickness of cell-walls, failure modes can be switched between the elastic buckling and brittle failure. Local strain rates were shown to be at least one order of magnitude greater than those applied on the specimen at macro scales. The significantly higher local strain rates can give rise to appreciable strain rate hardening of the solid parent polymer, eventually altering the cellular scale failure mode from elastic buckling to brittle failure.
Acknowledgements

The material is provided by Sandia National Laboratories. Sandia National Laboratories is a multi-program laboratory managed and operated by Sandia Corporation, a wholly owned subsidiary of Lockheed Martin Corporation, for the U.S. Department of Energy’s National Nuclear Security Administration under contract DE-AC04-94AL85000. Todd Rumbaugh at Hadland Imaging LLC is gratefully acknowledged for providing high-speed camera used in this work.

Appendix A

Failure modes at cellular scales

A.1. Elastic buckling (Cell wall instability)

According to Gibson and Ashby (1997), the equation proposed for elastic buckling in perfect 2D honeycomb structures is expressed as:

\[ \frac{\sigma_{cr}^*}{E_s} = \frac{n^2 \pi^2 t^3}{24} \frac{1}{1/n^2 \cos \theta} \]  

(A1)

where \( \sigma_{cr}^* \) denotes the critical collapse stress, and \( E_s \) is the elastic modulus of the parent solid material. For a regular hexagon cellular structure (see Fig. A1a) with \( l/h, \theta = 30^\circ \) and \( n = 0.686 \), the final equation for will be simplified as:

\[ \frac{\sigma_{cr}^*}{E_s} = 0.1935 \left( \frac{t}{l} \right)^3 \]  

(A2)

where \( t \) and \( l \) represent wall thickness and cell side dimensions, respectively.

A.2. Plastic collapse

Similar to elastic buckling, the final equation for the plastic collapse in a 2D hexagonal structure is expressed by Gibson and Ashby (1997):

\[ \frac{\sigma_{pl}^*}{\sigma_{ys}} = \left( \frac{t}{l} \right)^2 \frac{1}{2(h/l + \sin \theta) \sin \theta} \]  

(A3)

whereas \( \sigma_{pl}^* \) denotes critical stress in plastic collapse failure mode, and \( \sigma_{ys} \) is the yield stress of the parent solid material. Similar to Sec. A.1, for a regular hexagon cellular structure with \( l/h \) and \( \theta = 30^\circ \), the final expression is (see Fig. A1b):

\[ \frac{\sigma_{pl}^*}{\sigma_{ys}} = \frac{2}{3} \left( \frac{t}{l} \right)^2 \]  

(A4)

A.3. Brittle failure

Again, similar to cases discussed for elastic buckling and plastic collapse, the final expressions for the brittle failure mode are:

\[ \frac{\sigma_{bf}^*}{\sigma_{fs}} = \frac{1}{3(h/l + \sin \theta) \sin \theta} \left( \frac{t}{l} \right)^2 \]  

(A5a)

\[ \frac{\sigma_{bf}^*}{\sigma_{fs}} = \frac{1}{3 \cos^2 \theta} \left( \frac{t}{l} \right)^2 \]  

(A5b)

where subscripts 1 and 2 denote the direction of the far-field stress, and \( \sigma_{bf}^* \) denotes critical stress for the occurrence of brittle failure. For a regular hexagonal cellular structure with the same conditions as those described earlier, both Eqs. A1a and A1b reduce to:

\[ \frac{\sigma_{bf}^*}{\sigma_{fs}} = 4 \left( \frac{t}{l} \right)^2 \]  

(A6)

The modulus of rupture of the solid parent polymer, \( \sigma_{fs} \), is assumed to be roughly 1.1 times larger than the tensile strength, \( \sigma_{ys} \), of the materials (Gibson and Ashby, 1997), therefore Eq. A6 will be rewritten as:

\[ \frac{\sigma_{bf}^*}{\sigma_{fs}} = 0.489 \left( \frac{t}{l} \right)^2 \]  

(A7)
Fig. A1. Schematic view of a honeycomb at (a) undeformed state. Cellular scale deformation and failure are schematically shown for (b) elastic buckling, (c) plastic collapse and (d) brittle failure modes under far-field uniaxial compression.


