A DIC-based study of in-plane mechanical response and fracture of orthotropic carbon fiber reinforced composite

Behrad Koohbor, Silas Mallon, Addis Kidane*, Michael A. Sutton

Department of Mechanical Engineering, University of South Carolina, 300 Main Street, Columbia, SC 29208, United States

A R T I C L E I N F O

Article history:
Received 21 February 2014
Received in revised form 9 May 2014
Accepted 16 May 2014
Available online 5 June 2014

Keywords:
A. Carbon fiber
B. Fracture
B. Mechanical properties
D. Mechanical testing
Digital image correlation (DIC)

A B S T R A C T

The in-plane elastic properties and quasi-static fracture response of an orthotropically woven carbon fiber reinforced composite are investigated using 3D digital image correlation. The elastic properties are determined as a function of fiber orientation, and the effect of fiber angles on the crack extension direction is investigated. The modified maximum hoop stress criterion is used to predict the crack extension angle of the examined material. The predicted angles are in very good agreement with those found experimentally. Optical observations and local strain data from the quasi-static fracture studies reveal that crack initiation occurs well before the maximum far-field load is reached.

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1. Introduction

The in-plane elastic properties and quasi-static fracture response of an orthotropically woven carbon fiber reinforced composite are investigated using 3D digital image correlation. Of particular interest in this study are graphite/epoxy composites, which are especially suitable for many applications because of their exceptional thermo-mechanical attributes [1]. While this material may possess many desirable characteristics, a complete set of mechanical properties, including fracture parameters, is requisite for proper engineering use.

Failure and damage as a result of fracture is a major concern with the use of any composite material provoking substantial research in the area [2]. Crack propagation along the laminar interfaces, including the effect of strain rate, has been most thoroughly studied [3–6]. In-plane fracture in composite has been also documented in the literature [7–11]. Most early experimental work relied on conventional material testing techniques to obtain mechanical and fracture properties of composites, point measurements, load data and specimen geometry. These experimental techniques become more robust with the advent of the full field measurement technique, digital image correlation (DIC). Most recently Pollock et al. [12] utilized such full-field measurements to extract elastic properties of woven glass/epoxy composites through tensile loading of specimens extracted at different angles relative to the primary fiber direction.

The advent of DIC has also proven beneficial to the study of fracture mechanics, as seen in pioneering work by McNeill et al. [13], in which DIC data is used to extract stress intensity factors. Pataky et al. [14] utilized full-field measurements in conjunction with a least squares algorithm to investigate mixed mode stress intensity factors in an anisotropic, single crystal stainless steel. Such work demonstrates the applicability and accuracy that full-field measurements offer to fracture studies.

However, the examination of criterion that predicts the direction of crack propagation in woven composites was missing from previous work. Early work by Erdogan and Sih [15] that considered crack propagation plane loading of a brittle material was one of the first to describe such a methodology, suggesting that the crack will initiate along a path perpendicular to the maximum tensile direction at the crack tip. Kidane et al. [16,17] extended the method by employing the minimum energy density criteria to study crack propagation in functionally graded materials subjected to thermo-mechanical loading. Saouma et al. [18] extended the maximum circumferential tensile stress theory presented by Erdogan and Sih [15] to predict the direction of crack propagation in a homogeneous, anisotropic specimen subjected to mixed mode loading, documenting the substantial influence that material anisotropy has on the direction of crack propagation. Most recently, Cahill et al. [9] presented the use of the extended finite element method (XFEM) in conjunction with a modified maximum hoop stress criterion to describe the direction of crack propagation in a unidirectional fiber reinforced composite.
composite that has a high degree of anisotropy. Although this model appears applicable in unidirectional composites that exhibit extreme degrees of anisotropy, the use of this criterion has not been documented in woven composites, which can often exhibit nearly orthotropic properties.

The present study focuses on the development of a general and robust methodology for extraction of mechanical properties and fracture parameters for orthogonally woven composites using full-field measurements. To our knowledge, this is the first detailed study of woven composites where the elastic properties, stress intensity factor and crack direction are investigated as a function of fiber orientation using full field measurement technique. The elastic properties of the examined composite are first determined experimentally with the help of 3D DIC, based on which the elastic stress fields are extracted. Stress fields extracted from fracture experiments, in conjunction with the maximum modified hoop stress criterion, are then utilized to predict the angle of crack propagation as a function of fiber orientation angle.

2. Experimental procedure

2.1. Material and specimen geometry

The material examined in the present work is a three-layer orthogonally woven carbon fiber-reinforced epoxy resin matrix composite with a nominal density of 1384 kg/m$^3$. The plain weave structure of the laminate consisted of two mutually orthogonal directions (warp and weft) with an approximate volume fraction of 70% for the reinforcing carbon fibers. The weave structure can be seen in a magnified image of the material surface in Fig. 1. Rectangular 175 × 25 (mm$^2$) coupon specimens were cut from sheets of 0.8 mm thick material using a CNC water jet. Specimens were extracted from the plate at fiber orientation angles ranging from 0° to 90° in 15° increments. The schematic orientation of such specimens is given in Fig. 2.

2.2. Mechanical testing

To obtain mechanical properties, tensile tests were performed on rectangular coupon specimens. Loading was applied monotonically until failure using displacement control at a rate of 1.5 mm/min. Loading rate was chosen such that sufficient number of data points and images could be collected, at a sampling rate of 2 Hz, to characterize each specimen, while the rate is sufficiently low to be considered quasi-static. Tests were conducted on an MTS 810 tensile load frame fitted with hydraulic grips. Specimens were fastened into the machine with approximately 30 mm in length held by each of the hydraulic grips, with 6 mm thick aluminum tabs placed on either side of the specimen thickness within the grips to protect against damage from the serrated, hardened steel grips. Load and displacement data were acquired from the load cell and displacement sensors, while stereovision was simultaneously used to acquire pairs of images during the loading process from one side of the specimen that had been painted to obtain a speckle pattern with random contrast variations (see Fig. 3c).

2.3. Quasi-static fracture testing

Quasi-static experiments were performed on initially flawed specimens using an MTS 810 tensile frame. The geometry of the fracture specimens is shown in Fig. 3. The 160 × 50 (mm$^2$) single-edge notched specimens were extracted from 0.8 mm thick sheets using a procedure similar to the aforementioned procedure. Again, fiber orientation angles ($\theta$) were varied from 0° to 90° in 15° increments. An initial 25 mm long notch was cut into each specimen using the CNC water jet to produce an $a/W$ ratio of 0.5. Prior to testing, the notch on each specimen was struck with a razor blade to effectively produce a sharp crack; approximate crack length was 26 mm in all experiments. To perform each fracture experiment, specimens were fastened into the hydraulic grips utilizing aluminum tabs to protect the roughly 30 mm length portion of the specimen held in each grip. Loading was applied monotonically at a loading rate of 0.72 mm/min until failure. It is noted that the loading rate for each fracture test is kept lower than the rate used for the unflawed sample test so that enough data and images are acquired at 2 Hz sampling rate, as the test duration for fracture test is shorter than the unflawed tensile test. Synchronized load, axial displacement and stereo images were collected at 2 Hz. Each experiment was tested at least twice to ensure the results obtained during mechanical and fracture tests are repeatable.

2.4. Digital image correlation (DIC) method

During the entirety of the present experimental study, full-field strain and displacement fields were obtained with the use of stereovision in conjunction with 3D DIC. To facilitate use of the DIC method, the surface of each specimen was first coated with a thin layer of white paint. A fine, high contrast speckle pattern of black paint was then sprayed on the white surface. A typical speckle pattern applied on the specimen’s surface, along with its gray scale histogram, is shown in Fig. 3. Specimen response during tensile and fracture experiments was observed using a pair of Point Grey Grasshopper 5 megapixel cameras fitted with 35 mm lenses, operating at a resolution of 2448 × 2048 pixels. External lights sources were used to promote uniform illumination of the specimen in the region of interest.
System parameters obtained from calibration for the stereo camera arrangements used in both the mechanical testing and the fracture testing are shown in Table 1. The camera 1 and camera 2 coordinate systems, as well as the camera orientation angles $\alpha$, $\beta$, and $\gamma$ are also depicted, with the Z-axis nominally along the optical axis of each camera. From the calibration data, stereo angles ($\beta_s$) for the mechanical and fracture testing were 21.8° and 36.5°, respectively. Images collected were analyzed using commercial software Vic-3D (VIC-3D website: www.correlatedsolutions.com) with subset and step sizes of 25 and 3 pixels, respectively.

### Table 1

<table>
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<tr>
<th>Parameter</th>
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<th>Fracture testing</th>
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3. Data analysis

3.1. Elastic properties

Elastic stress-strain relation of an orthotropic lamina under plane-stress conditions can be written in matrix form as [12]:

$$
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{E_1} & -\frac{v_{12}}{E_1} & 0 \\
-\frac{v_{12}}{E_1} & \frac{1}{E_2} & 0 \\
0 & 0 & \frac{1}{G_{12}}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{xy}
\end{bmatrix}
$$

(1)

where $\varepsilon_{ij}$ and $\sigma_{ij}$ are the components of the strain and stress tensors, respectively. In order to determine the in-plane mechanical properties of the woven composite as a function of fiber orientation, off-axis tensile specimens were tested. The tensile load, however, was applied along the specimen axis, making an angle $\theta$ with the principal horizontal fiber orientation (see Fig. 2).

The stress components along the material axes 1 and 2 can be written in terms of the axial stress, $\sigma_{ax}$ and the fiber orientation angle, $\theta$, as:

$$
\sigma_{11} = \sigma_{ax} \cos^2 \theta \\
\sigma_{22} = \sigma_{ax} \sin^2 \theta \\
\sigma_{12} = -\sigma_{ax} \sin \theta \cos \theta
$$

(2)

Using the quadratic elasticity potential function and the stress components shown in Eq. (2), the stored energy, $w$, can be written as a function of axial stress, in-plane elastic constants and the fiber orientation angle as [6, 12]:

$$
w = \frac{1}{2} \sigma_{ax}^2 \left( \frac{1}{E_1} \cos^2 \theta + \frac{1}{G_{12}} \sin^2 \theta \cos^2 \theta + \frac{1}{E_2} \sin^4 \theta \right)
$$

(3)

where subscripts 1 and 2 in the present work denote the material property at $\theta = 0^\circ$ and $\theta = 90^\circ$, respectively. Using the definition of axial strain for a linear elastic material, $\varepsilon_{ax}$ can be written as:
\[ e_0 = \frac{\partial w}{\partial \sigma_0} \frac{\sigma_0}{E_1} h^2(\theta) \quad \text{and} \quad E_0 = \frac{E_1}{h^2(\theta)} \]  

\[ h^2(\theta) = \cos^4 \theta + \frac{d_1^2}{d_1^2} \sin^4 \theta + \frac{d_2^2}{d_2^2} \sin^2 \theta \cos^2 \theta \]

\[ d_1 = 1, \quad d_2 = 1 - c_{12} \frac{2 \delta_{12}}{E_1} \]

where

By performing tensile experiments to obtain \(\sigma_0 - \varepsilon_0\) curves in the linear region for \(\theta = 0^\circ\) and \(\theta = 90^\circ\), \(d_1\) and \(d_2\) can be determined, respectively. At this point, the only unknown parameter would be \(d_3\), which can be numerically assessed using a linear least square optimization process which involves the minimization of the following metric \([12]\):

\[ \phi = \sum_{j=1}^{M} \sum_{i=1}^{N} \left( \frac{c_{ij} - c_{ij}^*}{\sigma_0^*} \right)^2 \]

\[ h^2(\theta, d_1, d_2, d_3) = \cos^4 \theta + \frac{d_1^2}{d_1^2} \sin^4 \theta + \frac{d_2^2}{d_2^2} \sin^2 \theta \cos^2 \theta \]

where \(M\) is the number of specimens tested at angle \(\theta\) and \(N\) denotes the number of \(\sigma_0 - \varepsilon_0\) data obtained for each specimen. From values of \(\sigma_0\) and \(\varepsilon_0\) measured at several load magnitudes for specimens with \(\theta = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ\), the value of the parameter \(d_3\) can be obtained from the optimization process. Additionally, the Poisson’s ratio as a function of fiber angles, \(\nu(\theta)\), can be experimentally obtained by finding the ratio of transverse to longitudinal strain components for each specimen. Once \(d_3\) and \(\nu(\theta)\) are obtained, shear moduli at different angles, \(G(\theta)\), of the material can be determined from Eq. (5). Note that the level of accuracy of the results depends on the number of specimens tested at each \(\theta\), as well as the number of data points obtained in the linear region of \(\sigma_0 - \varepsilon_0\) curves. The procedures that are followed to obtain the mechanical properties can be summarized as follows:

- Measure the linear portion of tensile \(\sigma - \varepsilon\) of a specimen loaded along with fiber angle \(\theta = 0^\circ\) and determine \(d_1\) according to Eq. (5).
- Measure the linear portion of tensile \(\sigma - \varepsilon\) of a specimen loaded along with fiber angle \(\theta = 90^\circ\), and determine \(d_2\) according to Eq. (5).
- Determine the Poisson’s ratio from tensile experiments of specimens with fiber angles are oriented at \(\theta = 0^\circ; 15^\circ; 30^\circ; 45^\circ; 60^\circ; 75^\circ; 90^\circ\).
- Measure \(\sigma_0 - \varepsilon_0\) values at several loads within the linear deformation region from tensile experiments for specimens with fiber angles are oriented at \(\theta = 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ\).
- Since \(d_1, d_2\) and \(\sigma_0 - \varepsilon_0\) values at different angles are known use Eq. (6) to determine the unknown \(d_3\) using a linear least square optimization process.
- Once \(d_3\) is obtained, Eq. (5) can again be used to find the values for shear moduli at different angles.

### 3.2 Crack-tip displacement fields

The in-plane Cartesian displacement components, \(u\) and \(v\), in the vicinity of stationary and propagating cracks in orthotropic materials can be written as \([7,19]\):

\[ u = \frac{K_I}{\sqrt{\pi}} A_{c66} \left[ -\frac{1}{2} t_1 s_{11} + p^2 t_1 s_{12} \right] \sqrt{\cos^2 \varphi + \frac{1}{p^2} \sin^2 \varphi + \cos \varphi} \]

\[ + \left( \frac{p l_1 l_2}{2 q q_0} s_{11} + \frac{p q l_1 l_2}{q q_0} s_{12} \right) \sqrt{\cos^2 \varphi + \frac{1}{q^2} \sin^2 \varphi + \cos \varphi} \sqrt{r} \]

\[ v = \frac{K_I}{\sqrt{\pi}} A_{c66} \left[ -p \left( \frac{1}{2} s_{12} + p^2 t_2 s_{22} \right) \right] \sqrt{\cos^2 \varphi + \frac{1}{p^2} \sin^2 \varphi - \cos \varphi} \]

\[ + q \left( \frac{p l_1 l_2}{2 q q_0} s_{12} + \frac{p q l_1 l_2}{q q_0} s_{22} \right) \sqrt{\cos^2 \varphi + \frac{1}{q^2} \sin^2 \varphi - \cos \varphi} \sqrt{r} \]

where \(K_I\) is the stress intensity factor for Mode-I fracture, and \(s_{ij}\) denotes the components of the material’s compliance matrix as presented in Eq. (1). The results obtained from the tensile experiments were used to determine the material’s compliance matrix. Quantities \(r\) and \(\varphi\) in Eq. (7) are the radial and angular coordinates of an arbitrary point. Expressions for \(p\) and \(q\) are defined as:

\[ p = \sqrt{a_1 - a_1^2 - a_2}, \quad q = \sqrt{a_1 + a_1^2 - a_1} \]

Where the quantity \(a_1\) is given as:

\[ 2a_1 = \alpha + \alpha_1 - 4\beta_1 \beta_2, \quad a_2 = \alpha \alpha_1 \]

With

\[ \alpha = \frac{c_{66}}{c_{11}(1 - M_1^2)}, \quad \alpha_1 = \frac{c_{22}}{c_{66}(1 - M_2^2)} - 2\beta_1 \]

\[ = \frac{c_{12} + c_{66}}{c_{11}(1 - M_1^2)} 2\beta_2 = \frac{c_{12} + c_{66}}{c_{66}(1 - M_2^2)} \]

\(c_{ij}\) in Eq. (10) are the elements of the material’s stiffness matrix. In this case also, the \(c_{ij}\) matrix was obtained as the inverse of the material’s compliance matrix \((s_{ij})\), which was determined earlier based on the elastic properties found from the tensile experiments. \(M_1\)

![Fig. 4. Typical tensile stress–strain curves as a function of fiber orientation (a) and a close-up view of the linear elastic region (b). A linear regression method was used to find the elastic moduli of the specimens in this region.](image-url)
and $M_2$ are non-dimensional parameters, defined as the ratio of the crack tip velocity to the longitudinal wave velocity, $c_l$, and shear wave velocity, $c_s$, of the materials as:

$$M_1 = \frac{c}{c_l} \quad M_2 = \frac{c}{c_s} \quad c_l = \sqrt{\frac{E_{11}}{\rho}} \quad c_s = \sqrt{\frac{E_{66}}{\rho}}$$ (11)

In the case of stationary and/or quasi-static crack growth, the crack tip velocity, $c$, is assumed to be zero. The parameters $l_i(i = 1 \ldots 6)$ and $AC_{66}$ in Eq. (7) are also written as:
l_1 = \frac{2\beta_1 p^2}{(\alpha - p^2)(1 - M_1^2)} + 2\beta - \alpha \quad l_2 = \frac{2\beta_1 q^2}{(\alpha - q^2)(1 - M_1^2)} + 2\beta_1 - \alpha
\begin{align*}
l_3 &= 1 - M_2^2 - \frac{2\beta_1}{\alpha - p^2} \\
l_4 &= 1 - M_2^2 - \frac{2\beta_1}{\alpha - q^2} \\
l_5 &= -l_3 - M_2^2 \\
l_6 &= -l_4 - M_2^2
\end{align*}
(12)

The displacement components at arbitrary radial and angular positions in the crack tip vicinity were used to find the stress intensity factor, $K_I$, based on an over-deterministic least square algorithm [20–22]. To avoid any possible error introduced by the crack tip nonlinearity effects, data points were selected to be sufficiently far from the location of the crack tip. Regions with $0.5 < (\epsilon) < 2$, where $B$ is the specimen thickness, were considered as this criterion [17]. In addition, to increase the accuracy, a large number of data points (minimum 100) were used in the calculation of $K_I$ for each specimen. The numerical analysis was performed using an in-house computer code written on the MATLAB™ platform.

3.3. Criterion for crack propagation direction

The modified maximum hoop stress criterion is used to predict the direction of crack growth. The maximum hoop stress criterion states that the crack will propagate perpendicular to the direction

Fig. 9. Crack bridging as a result of fiber pull-out and fiber cross-over at early stages of crack growth for specimens of (a) $\theta = 45^\circ$ and (b) $\theta = 90^\circ$.

Fig. 10. (a) Crack tip location at different load magnitudes for the specimen with $\theta = 45^\circ$. The crack tip location is marked on each image. (b) Crack tip location vs. far-field load magnitude for the same specimen. Here, $P_{init}$ refers to the magnitude of the load at which matrix cracking initiates.
and cos \( r \) sin \( r \) are functions of angular position, and denotes the fiber orientation angle with respect to denote the mate-

in which the hoop/circumferential stress is maximum [23]. The modified criterion, as proposed by Sauma et al. [18] takes into account the angular variation of the fracture toughness of the material, making it effective for orthotropic materials considered in this work.

The modified hoop stress for orthotropic material can be expressed as [18]:

\[
\sigma_{\text{hoop}}^\text{mod} = \frac{\sigma_y \sin^2 \varphi + \sigma_x \cos^2 \varphi - \sigma_{xy} \sin 2\varphi}{E_x \cos^2(\varphi - \theta_{\text{material}}) + \sin^2(\varphi - \theta_{\text{material}})}
\]  

(13)

where \( \theta_{\text{material}} \) denotes the fiber orientation angle with respect to perpendicular to the loading direction. \( E_x \) and \( E_y \) denote the material's elastic moduli along its principal fiber directions, obtained from the tensile experiments in the present work. Note that, the stress components \( \sigma_y, \sigma_x \) and \( \sigma_{xy} \) are functions of angular position, \( \varphi \), and the fiber orientation angle, \( \theta_{\text{material}} \). [19].

4. Results and discussion

4.1. Elastic properties

Following the experimental procedure described earlier, the elastic properties of the examined materials were determined as a function of fiber orientation angles. Typical stress–strain curves for the composite as a function of fiber orientations angle are shown in Fig. 4. The elastic portion of the stress–strain relation has been magnified and shown in Fig. 4. As clearly seen in Fig. 4, the stress–strain relations are consistent with previous studies [12] and show different trends for different fiber orientations. For the case of 0° and 90° fiber orientations, the failure was abrupt with small strain and high failure stress; no progressive failure was observed. On the other hand, failure in the case of off-axis fiber orientation is progressive, leading to a relatively large strain before complete failure. This is expected since the fibers are in the principal loading direction and are the primary load carrying members. Since the matrix has little or no contribution to the tensile strength for 0° and 90° specimens, the stress strain results exhibit a monotonic linear response. In the case of off-axis specimens, the matrix takes the load initially, as none of the fibers are aligned with the loading direction. As the loading process progresses, the fibers begin sharing the load and a non-linear stress-strain response with progressive damage occurs. A similar response was observed by Pollock et al. [12] for an orthotropic woven glass–epoxy composite.

From the full-field displacement data obtained using 3D DIC, longitudinal and transverse strain components were also determined for each tensile specimen. Fig. 5 depicts the longitudinal and transverse strain components for specimens of \( \theta = 0^\circ \) and \( \theta = 45^\circ \). The ratio of transverse to longitudinal strains increases significantly as fiber orientation angle increases from 0° to 45°, indicating that the apparent Poisson’s ratio will vary substantially as a function of fiber orientation relative to loading axis. The experimentally determined values of the apparent Poisson’s ratio exhibited by the material are plotted with respect to the fiber orientation angle in Fig. 6. The values obtained here are in a good agreement with literature data [24].

Once the elastic moduli and the apparent Poisson’s ratio of the composite specimens at different fiber orientation angles are obtained, the method described earlier was used to extract the
material’s shear modulus. Using a least-square approach and Eq. (6), the values for $d_i$’s in Eq. (5) are calculated as:

\[
d_1 = 0.02167 \text{ GPa}^{-1} \quad d_2 = 0.01733 \text{ GPa}^{-1} \quad d_3 = 0.31432 \text{ GPa}^{-1}
\]

Then using $d_3$ along with the other elastic constants found earlier, the material’s apparent in-plane shear modulus $G(\theta)$ as a function of fiber orientation angle $\theta$ are obtained, as shown in Fig. 7a.

Having all the necessary parameters, the theoretical variation of $E(\theta)$ was calculated from Eq. (4) and the results were compared to the experimental findings. Fig. 7b demonstrates the comparison between the theoretical and experimental results. A numerical comparison of the experimental and analytical results shows a relative small difference, less than 3%, in all the cases, confirming that the theoretical formulation presented above has a high level of accuracy which was achieved in all of the results found in this work. Another point worth mentioning is that the material

Fig. 13. Stress contours for specimens of (a) $\theta = 45^\circ$ and (b) $\theta = 90^\circ$. The original crack tip is located at (0, 0) and the stress values are in MPa. Dimensions are also in mm.
properties of the specimens at $0°$ and $90°$ fiber orientation angles are not the same, and as shown in Fig. 7 the elastic moduli are not distributed symmetrically with the fiber orientation. A similar behavior was reported in the work performed by Pollock et al. [12], with this phenomenon explained as being the result of a different number of reinforcement fibers used along the $0°$ and $90°$ directions.

4.2. Stress intensity factor and crack propagation direction

Several typical far-field quasi-static loading histories for single edge cracked specimens with different fiber orientations with respect to the initial crack direction are shown in Fig. 8. A constant increase in the load magnitude prior to fracture, followed by a sudden drop, was observed for specimens with $θ = 0°$ and $θ = 90°$, typical of a brittle fracture response. By increasing the specimen fiber orientation angle from $0°$ to $45°$, both the peak value and the loading slope were remarkably reduced, reaching minima for the specimen with $θ = 45°$. The same trend was found when varying $θ$ from $90°$ to $45°$ as well. The gradual crack growth observed for specimens with $θ$ approaching $45°$ is consistent with the failure mode observed in unflawed samples in Section 4.1, indicating a change in failure mechanism for specimens with fiber orientation angle deviating from the loading axis. From a magnified image of a typical failure surface, shown in Fig. 9, it was observed that for specimens with fibers aligned with the loading axis, the failure is predominantly by fiber fracture. In the case of off-axis specimens, failure is associated with matrix cracking and fiber pullout. Fig. 9 shows a typical failure mode during early stages of crack propagation for two different cases with fiber angles $θ = 45°$ and $θ = 90°$.

4.2.1. Stress intensity factor

To quantify the stress intensity factor, both the load and displacement/strain fields from DIC measurements are considered. However, the comparison made between the load data and the images taken during the loading process revealed that crack initiation takes place well before the load magnitude reaches its peak value. A magnified view of the crack tip area at different load levels for a specimen having $θ = 45°$ is shown in Fig. 10a. The crack extension vs. far-field load was also plotted for this specimen in Fig. 10b. It is obvious from the images that the crack has already initiated some time before the maximum load is reached. To find the correct load magnitude associated with the material’s crack initiation, the history of the opening strain component ($σ_{yy}$) was examined for a point located very near to, and at an angle $φ = 0°$ relative to, the original notch tip. This strain component experiences a distinct jump in its value at the actual moment of crack initiation, as depicted in Fig. 11. This type of strain response can be used to find the exact load magnitude at which the fracture initiates. After measuring the corrected crack initiation load in this way for each specimen, the critical stress intensity factor as a function of fiber orientation angle was calculated using load data and specimen geometry. The stress intensity factor present in each of the specimens was also calculated using the displacement fields obtained from DIC and the asymptotic displacement fields with the over-deterministic approach explained earlier. The calculated $K_I$ values obtained using the over-deterministic approach are compared to those found using specimen load and geometry in Fig. 12a. As shown in Fig. 12a, the $K_I$ values determined using both methods exhibit similar trends and are in relatively good agreement. Noting that the $K_I$ value calculated from the corrected load is in better agreement with the value obtained from strain fields suggests the necessity for optical observation to determine the instant of fracture. By including such observations, the critical stress intensity factor of such materials can be obtained. In cases where only far-field load is considered, the stress intensity factor of the material may be substantially over-estimated (see uncorrected load data for instant of crack propagation in Fig. 12a). Variation of the peak load as obtained from the load cell (uncorrected fracture load) and the corrected fracture load values at the instant of fracture are presented in Fig. 12b for specimens with different fiber orientation angles. A minimum of 16% difference was obtained between the corrected and uncorrected fracture load magnitudes.

4.2.2. Crack propagation direction

In order to predict the angle at which crack propagation takes place, the modified hoop stress criterion described in Section 3.3 is used. Using the full-field strain components obtained from DIC along with the elastic constants determined for the material, the full-field stress components are calculated. These stress components are directly determined from Eq. (1) using known $ε_i$ and $σ_i$ values within the area of interest. Note that the elastic constants $σ_i$ that are used in Eq. (1) are those obtained from mechanical testing, explained earlier in Section 3.1 and the strains $ε_i$ are obtained from DIC measurements. The distribution of the corresponding stress components $σ_{xx}$, $σ_{yy}$ and $σ_{xy}$ for two different specimens obtained through this process are depicted in Fig. 13. Using these stress components, the modified hoop stress ($σ_{hoop}^{mod}$) field was determined within the same region of interest on which the full field stress components were calculated. The distribution of modified hoop stress in the crack tip neighborhood is shown in

![Fig. 14. Contours of the modified hoop stress for the specimens with (a) $θ = 60°$ and (b) $θ = 90°$. Experimental results confirm that the maximum modified hoop stress lies along the angle of crack propagation.](image)
Fig. 14. The experimentally observed results showing the crack initiation angle for all the specimens have been depicted in Fig. 15. Inspection of Figs. 14 and 15 indicates that the crack path follows the direction in which the modified hoop stress is maximized. To give more detailed insight into prediction of the crack path, the material’s elastic constants, along with the stress formulae proposed for orthotropic materials [10,11,18], was used to analytically determine the distribution of the modified hoop stress in the vicinity of the crack for specimens with different fiber orientation. In this case, the stress components, $\sigma_x$, $\sigma_y$ and $\sigma_{xy}$ in the neighborhood of the crack for Mode-I fracture are calculated as:

$$\sigma_x = \frac{K_1}{\sqrt{2\pi r}} \Re \left[ \frac{\mu_1 \mu_2}{\mu_1 - \mu_2} \left( \frac{\mu_2}{(\cos \varphi + \mu_2 \sin \varphi)^{1/2}} - \frac{\mu_1}{(\cos \varphi + \mu_1 \sin \varphi)^{1/2}} \right) \right]$$ \hspace{1cm} (14-a)

$$\sigma_y = \frac{K_1}{\sqrt{2\pi r}} \Re \left[ \frac{1}{\mu_1 - \mu_2} \left( \frac{\mu_1}{(\cos \varphi + \mu_1 \sin \varphi)^{1/2}} - \frac{\mu_2}{(\cos \varphi + \mu_2 \sin \varphi)^{1/2}} \right) \right]$$ \hspace{1cm} (14-b)

$$\sigma_{xy} = \frac{K_1}{\sqrt{2\pi r}} \Re \left[ \frac{1}{\mu_1 - \mu_2} \left( \frac{\mu_1}{(\cos \varphi + \mu_1 \sin \varphi)^{1/2}} - \frac{\mu_2}{(\cos \varphi + \mu_2 \sin \varphi)^{1/2}} \right) \right]$$ \hspace{1cm} (14-c)

where $\mu_1$ and $\mu_2$ are the complex roots of the general characteristic equation:

$$s_{11} \mu_1^4 - 2s_{16} \mu_1^3 + (2s_{12} + s_{66}) \mu_1^2 - 2s_{66} \mu + s_{22} = 0 \hspace{1cm} (15)$$

With $s_{ij}$ denoting the components of the material’s compliance matrix (see Eq. (1)). For an orthotropic case similar to that considered in this work, Eq. (15) reduces to the following form [9]:

Fig. 15. Experimentally observed direction of crack initiation for different specimens with (a) $\theta = 0^\circ$, (b) $\theta = 15^\circ$, (c) $\theta = 30^\circ$, (d) $\theta = 45^\circ$, (e) $\theta = 60^\circ$, (f) $\theta = 75^\circ$ and (g) $\theta = 90^\circ$. 
reinforced composite are determined using DIC measurements. The stress intensity factor at the onset of crack extension is calculated based on both the far-field load and also the displacement fields obtained from 3D-DIC. A detailed study of the material’s fracture response reveals that the far-field load and specimen geometry are insufficient to extract accurate estimates for fracture parameters such as the stress intensity factor. Rather, the experiments should be accompanied with either (a) optical observations to detect the onset of crack extension and/or (b) a corresponding measurement of local strain/displacement fields at a location very close to the initial crack tip to detect the abrupt changes in these components that occurs when crack propagation initiates.

Finally, the modified maximum hoop stress is utilized to analytically predict the angle of crack initiation. The predicted angle of crack initiation is shown to be in very good agreement with the measured crack initiation angle, indicating that this criterion is expected to be reliable for use in both experimental as well as numerical studies conducted on similar material systems.

Acknowledgement

The financial support of NASA through EPSCOR under Grant No. 21-NE-USC_Kidane-RGP, the College of Engineering and Computing and the Department of Mechanical Engineering at the University of South Carolina is gratefully acknowledged.

5. Conclusions

A detailed and robust digital image correlation-based metrology is used to determine the elastic and fracture properties of woven composites. In-plane mechanical parameters are quantified and the fracture response of an orthotropically woven carbon fiber reinforced composite are determined using DIC measurements. The stress intensity factor at the onset of crack extension is calculated based on both the far-field load and also the displacement fields obtained from 3D-DIC. A detailed study of the material’s fracture response reveals that the far-field load and specimen geometry are insufficient to extract accurate estimates for fracture parameters such as the stress intensity factor. Rather, the experiments should be accompanied with either (a) optical observations to detect the onset of crack extension and/or (b) a corresponding measurement of local strain/displacement fields at a location very close to the initial crack tip to detect the abrupt changes in these components that occurs when crack propagation initiates.

Finally, the modified maximum hoop stress is utilized to analytically predict the angle of crack initiation. The predicted angle of crack initiation is shown to be in very good agreement with the measured crack initiation angle, indicating that this criterion is expected to be reliable for use in both experimental as well as numerical studies conducted on similar material systems.

Fig. 16. Variation of normalized hoop stress, $\sigma_{hoop}^\text{mod}/(K_i/\sqrt{2\pi \theta})$, as a function of angular position from the crack tip for some selected specimens.

$$s_{11} \mu^2 + 2(s_{12} + s_{66})\mu^2 + s_{22} = 0 \quad (16)$$

In which the constants, $s_{ij}$, are experimentally determined. Once all the stress components are analytically determined in the crack tip vicinity, Eq. (13) can be utilized to obtain the distribution of the modified hoop stress within the region of interest. The variation of normalized modified maximum hoop stress as a function of angular position around the crack tip is shown for specimens with different fiber orientation angles in Fig. 16. The crack initiation angle was theoretically found as the angle at which the magnitude of the modified hoop stress reaches a maximum. Comparing the results found in this stage, with those obtained experimentally (Fig. 15), indicates very good agreement between both sets of results. Comparison of predicted and measured crack growth initiation angle for different fiber angles is shown in Fig. 17. A very slight discrepancy is observed between the analytical and experimental results, which might be associated with (a) slight errors in the experimentally measured angles or (b) the original fibers might have not been perfectly straight within the composite sheet.

References


